Séminaire de l'IRT SystemX
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Numerical strategies mixing models and data for the real-time monitoring of complex mechanical systems

Ludovic Chamoin (ENS Paris-Saclay, IUF)

with P-B. Rubio, B. Marchand, F. Louf, C. Rey

institut universitaire de France

école normale supérieure paris—saclay

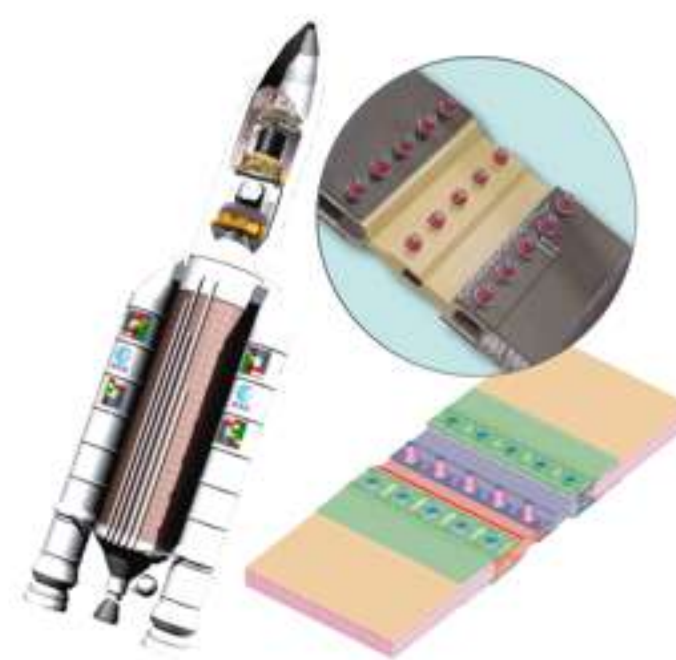
Activities of LMT

- **focused on Structural Mechanics (modeling & simulation)**
- **180 individuals (about 100 PhD students)**
- **interactions with industry (Airbus, EDF, Safran,…, Mitsubishi, Biomodex)**

- **divided into 3 groups : Mechanics and Materials**
  <u>Structures and Systems</u>
  Civil Engineering and Environment

5 research units (Computational Mechanics):

- Model Control, Adaptation & Validation
- Composites, Nano and Microstructures
- Advanced Computation Strategies (PGD, TVRC)
- Coupled problems and Parallelism
- Engineering and Robust Design

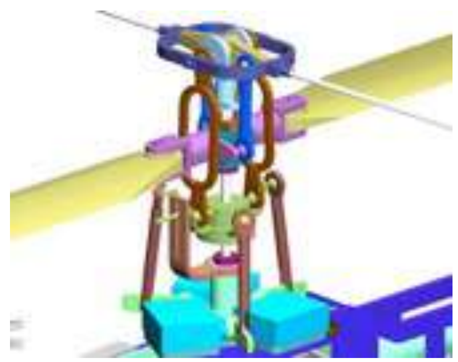


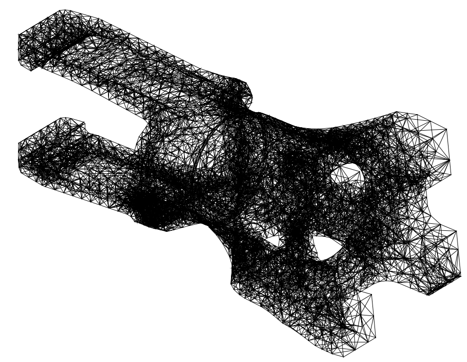
«Essentially, all models are wrong, but some are still useful» **(G.E. Box)**

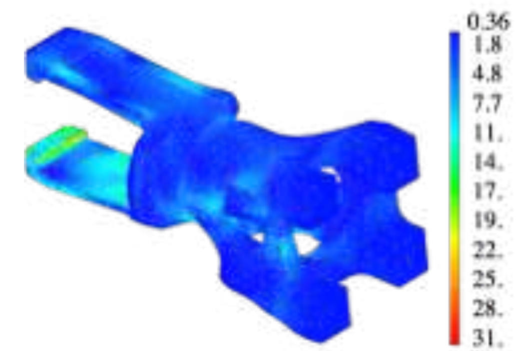
Context

traditional vision of numerical simulation

data → mathematical model → numerical model → *simulation outputs*







- difficult to handle incomplete modeling (variabilities, uncertainties)
- modeling & simulation remains useful for analysis and monitoring of complex systems

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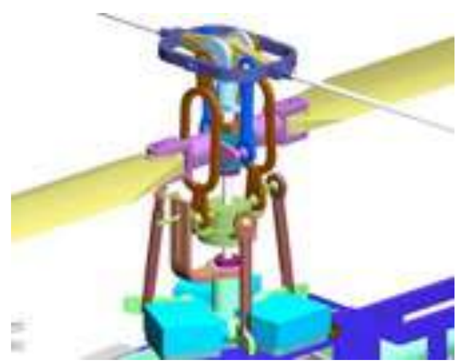
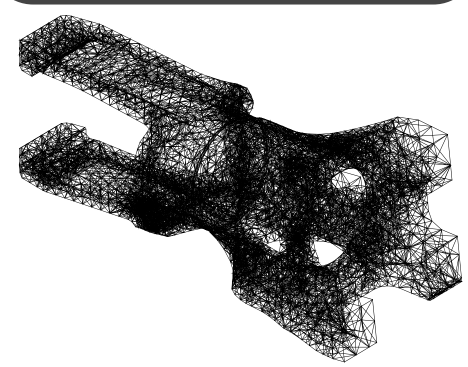
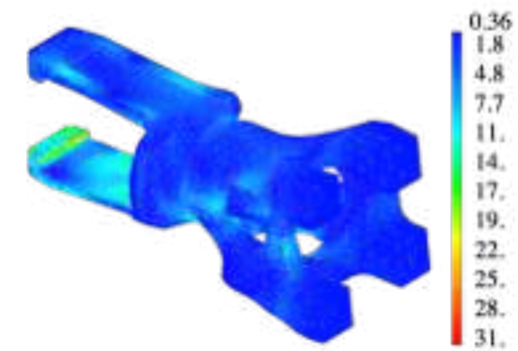
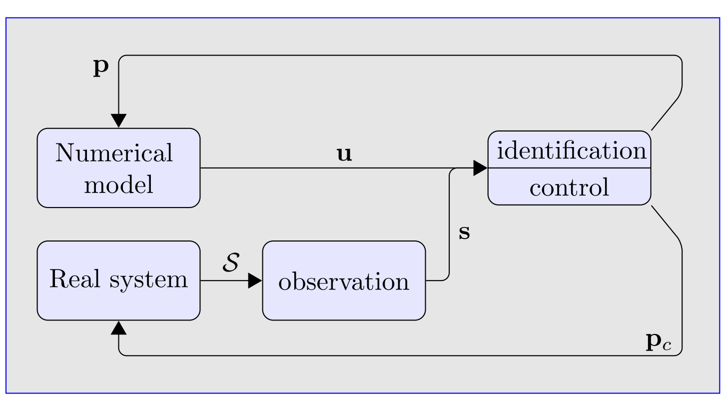






- difficult to handle incomplete modeling (variabilities, uncertainties)
- modeling & simulation remains useful for analysis and monitoring of complex systems

→ *need for stronger interaction between real systems and simulators (numerical twins)*

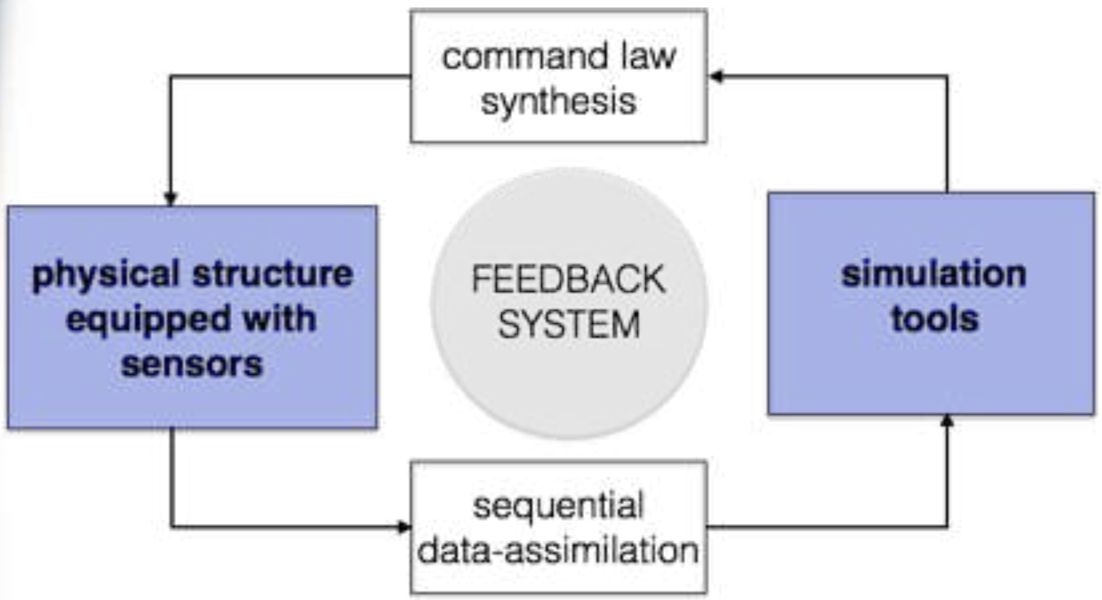


connected systems, cyber-physics,…

→ *multidisciplinary task (applied maths, exp. & comp. mechanics, computer science,...)*

Context

[Darema 04, Darema 15]



- *exploit the best of models & data (knowledge)*
- *increased interpretation*
  prediction
  anticipation
  decision-making

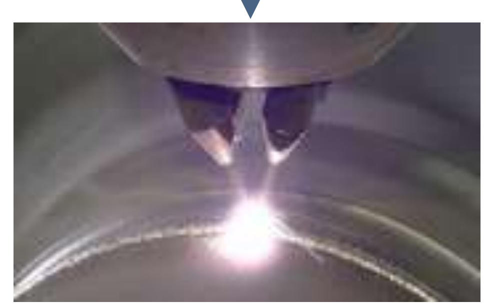
- *economical & societal impact (performance, reliability)*

Dynamic Data-Driven Application Systems (DDDAS)

assisted surgery

renewable energy

building performance



manufacturing

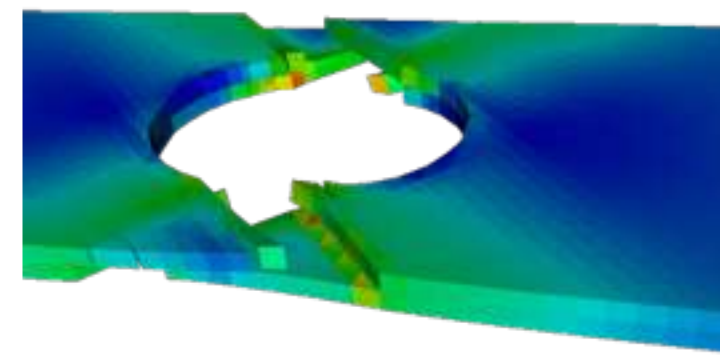
transportation

Context

- Application to **engineering structures** → **optimized maintenance**
  safety (operation in degraded mode)

embedded strain sensors
(defects ∼ 1mm)

complex physics



predictive simulations

→ *next-generation structural health monitoring (SHM)*
[Ding 07, Allaire 12, Prudencio *et al.* 15, Kapteyn *et al.* 20]

CHALLENGES

- **complex nonlinear** large systems with **uncertain environment**
- **real-time** requirements
- numerous, indirect and **noisy data**
- **biased models**

Outline

1. Reduced order modeling
2. Fully stochastic approach for sequential data assimilation
3. Online model bias correction & feedback control
4. Alternative coupled deterministic-stochastic approach
5. Ongoing projects & new challenges

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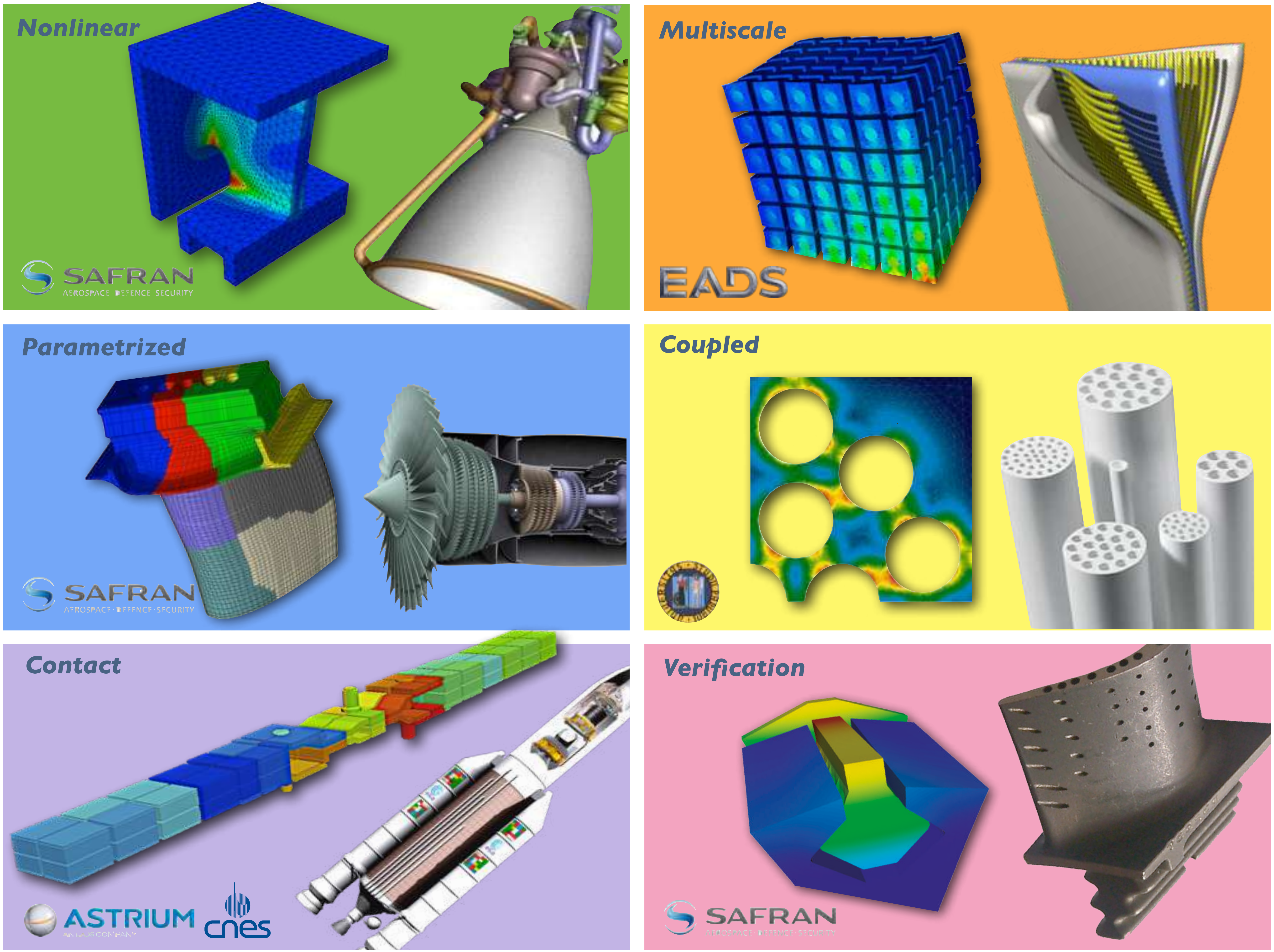
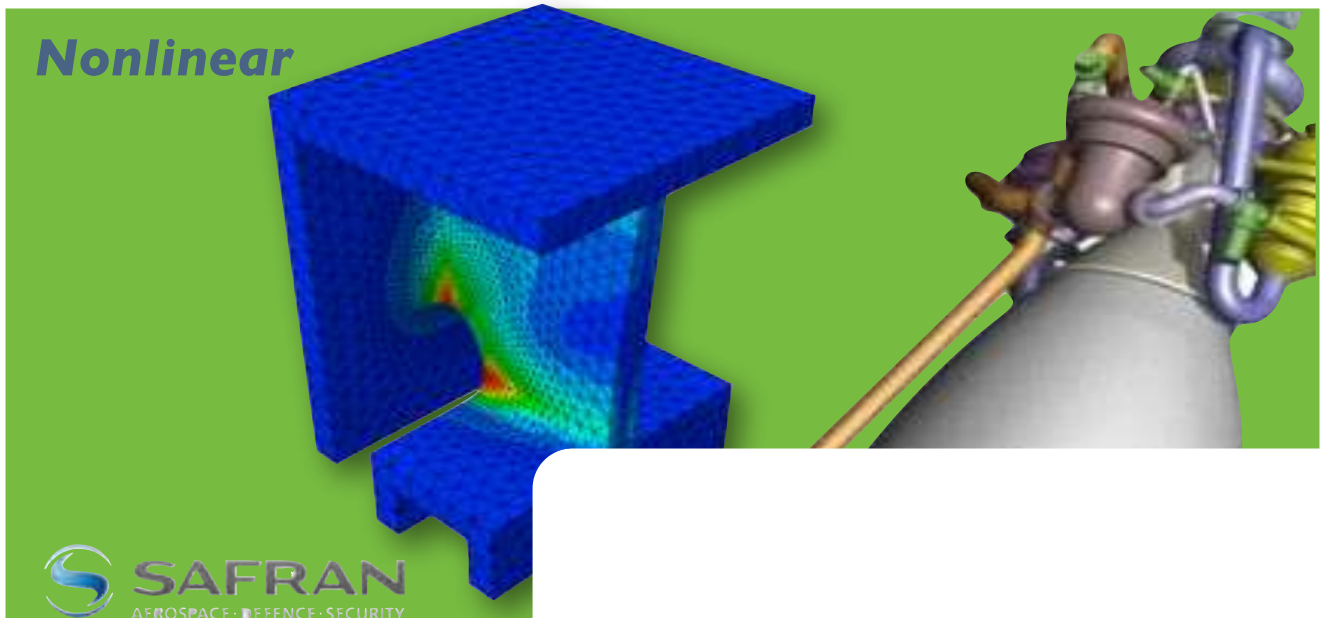
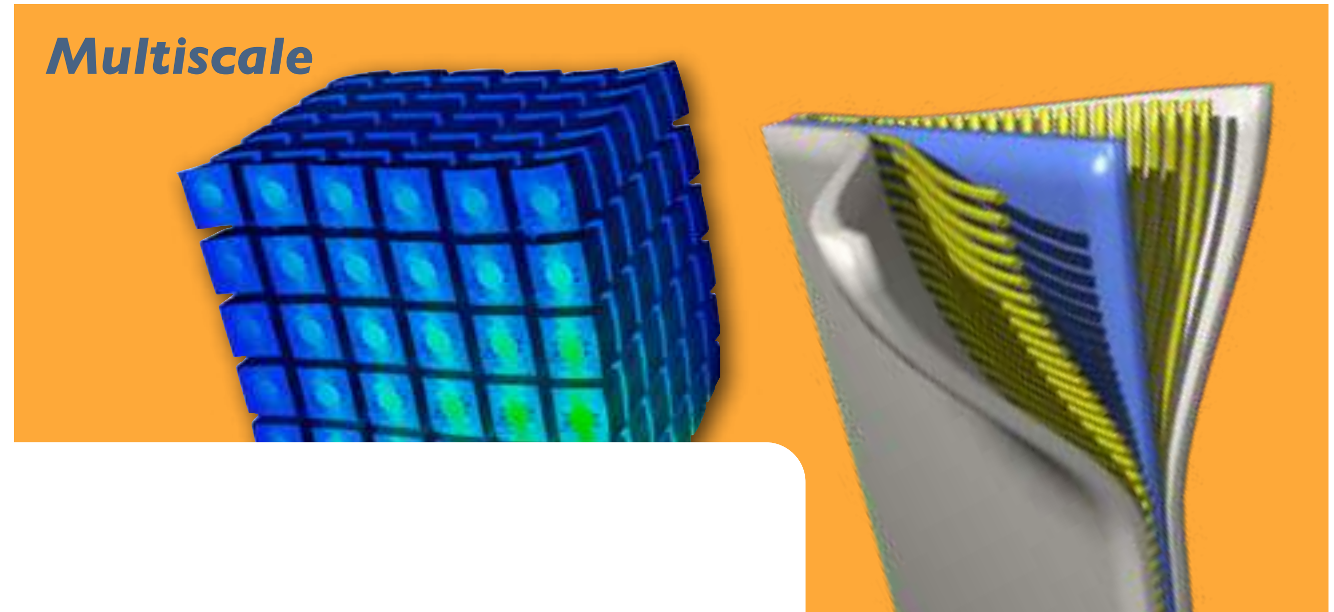
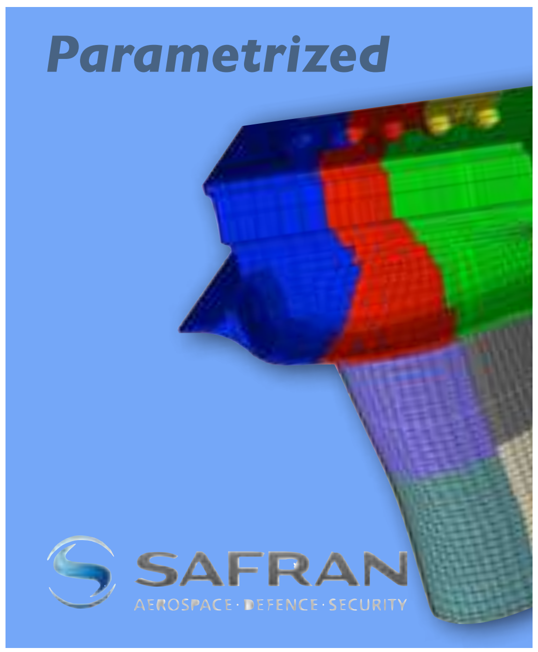

Nonlinear
SAFRAN
AEROSPACE · DEFENCE · SECURITY
Multiscale
EADS
Parametrized
SAFRAN
AEROSPACE · DEFENCE · SECURITY
Coupled
Contact
ASTRIUM
cnes
Verification
SAFRAN





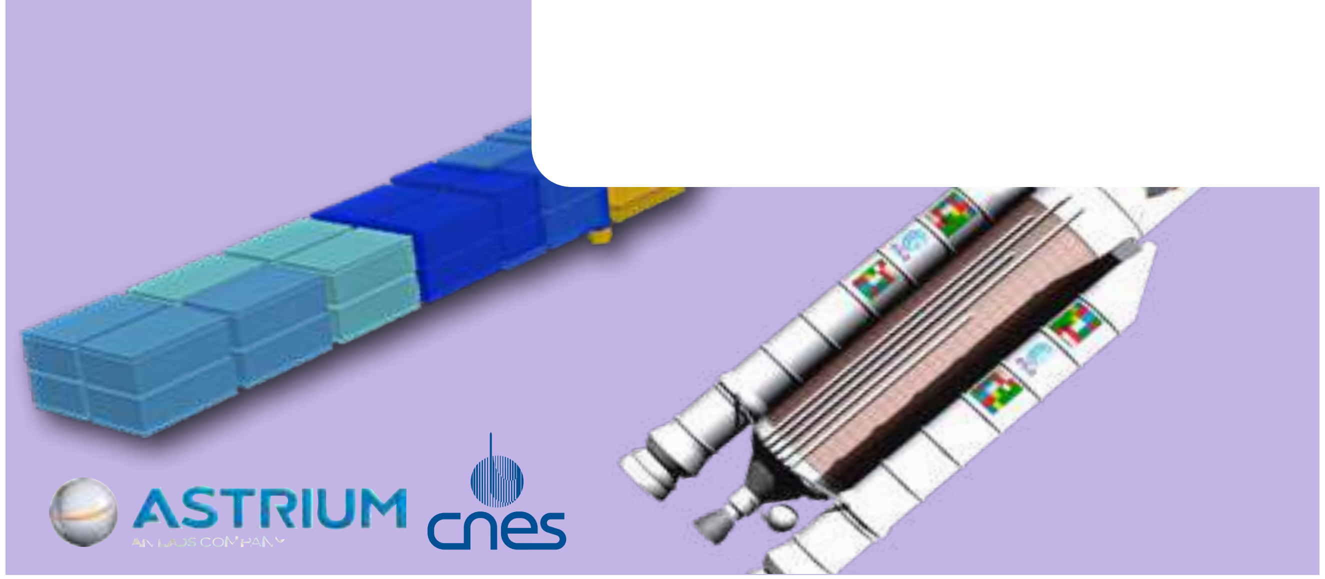


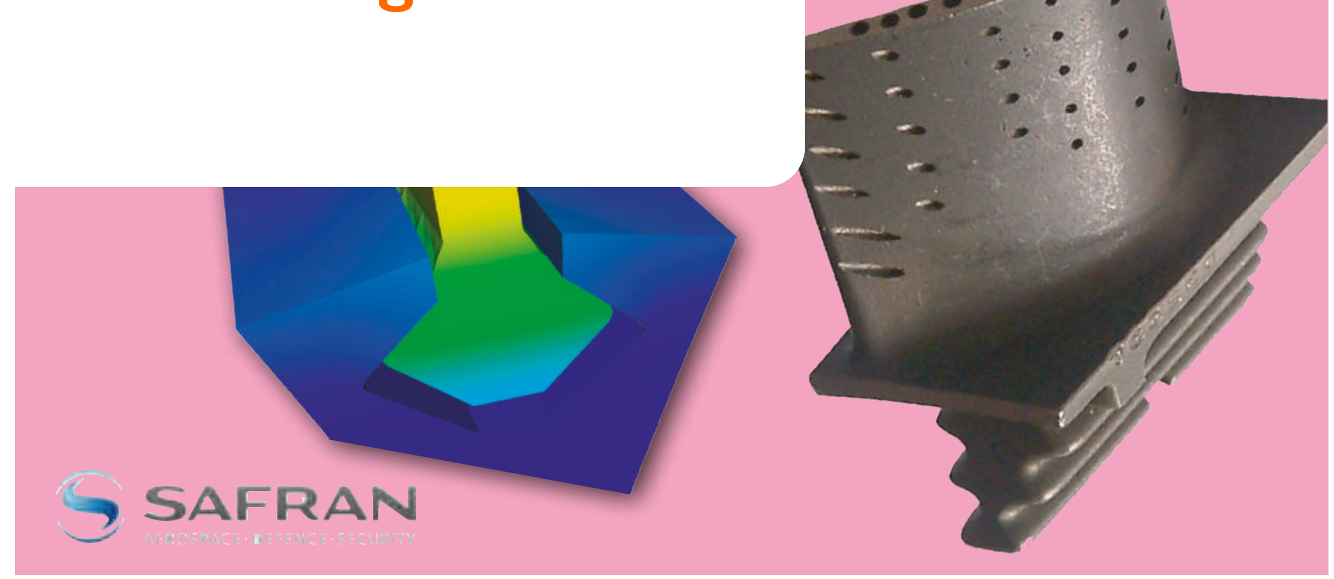
Incredible richness of **physical models**
that engineers want to handle

Due to **CPU time** and **big data** issues
direct techniques are inapplicable

Need for alternative methods dedicated to
high performance computing

Reduced-Order Modeling





Proper Generalized Decomposition

[Ladeveze 99, Nouy 08, Chinesta *et al.* 10, 11, Néron *et al.* 15,...]

Context: multiparameter PDEs: solution $\boxed{u(x,t,p_1,p_2,\dots,p_n)}$

Idea of PGD: *a priori* representation using linear combination of modes with variable separation (tensor product space, low-rank structure, canonical format)

$$u(x,t,p_1,p_2,\dots,p_n) \approx \sum_{i=1}^{m} \psi_i(x)\lambda_i(t)\phi_{1i}(p_1)\phi_{2i}(p_2)\dots\phi_{ni}(p_n)$$

main features of the solution

All model parameters are seen as extra-coordinates [Chinesta *et al.* 2011]

→ decrease of computation/storage costs (linear growth of the number of dof)

→ no need of *a priori* information on the solution (no snapshot)

→ modes are computed offline solving (eigenvalue) problems

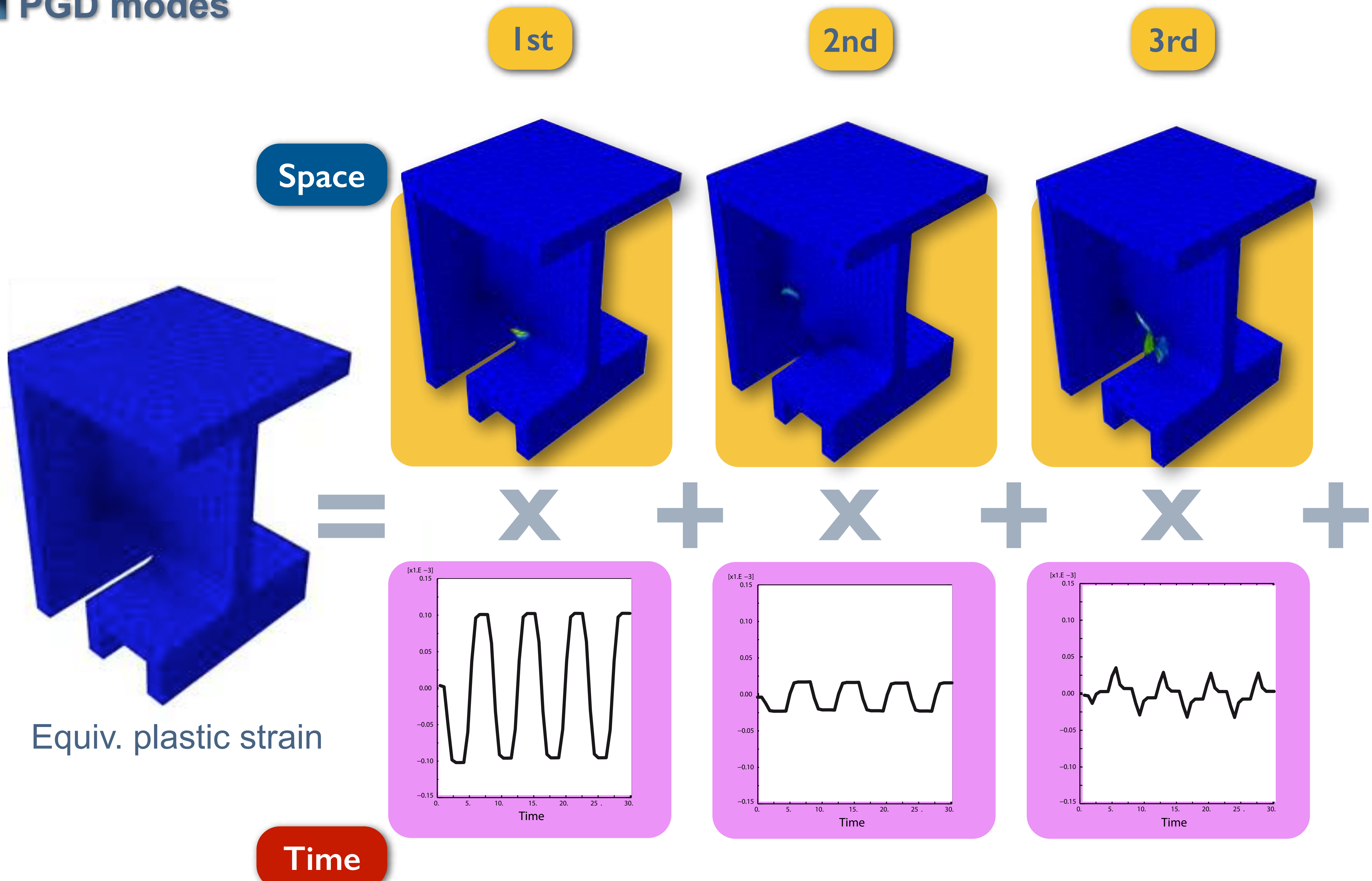
→ various parameters (material, boundary/initial conditions, geometry,...)

→ various applications, in particular for multi-query (inverse pb, UQ,...)

Example 1: Motor Blade

[Relun *et al.* 13]

PGD modes



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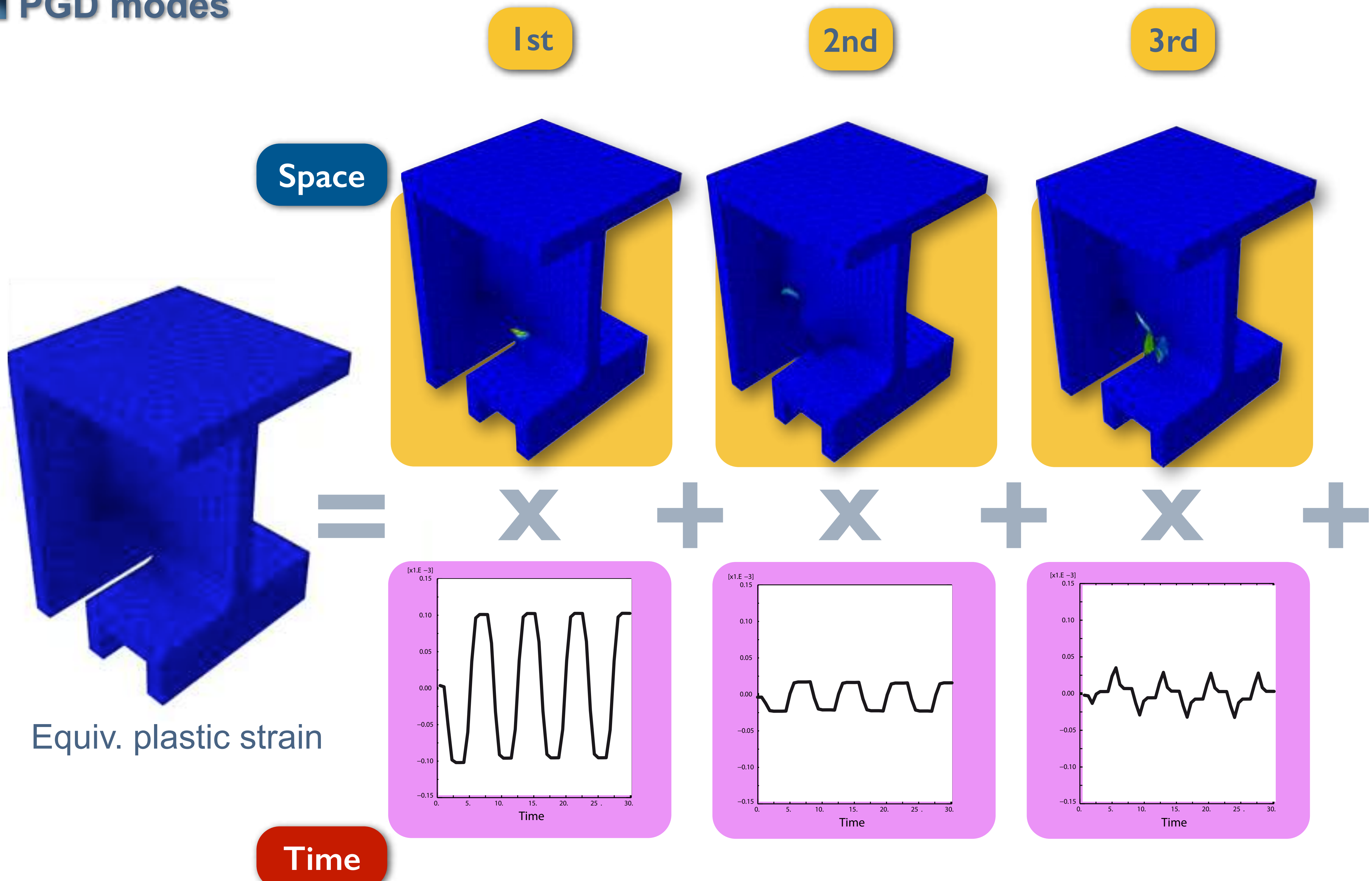
- **PGD modes**



Example 2: 4-points Bending Test

Courtesy Alexandre Michou, LMT

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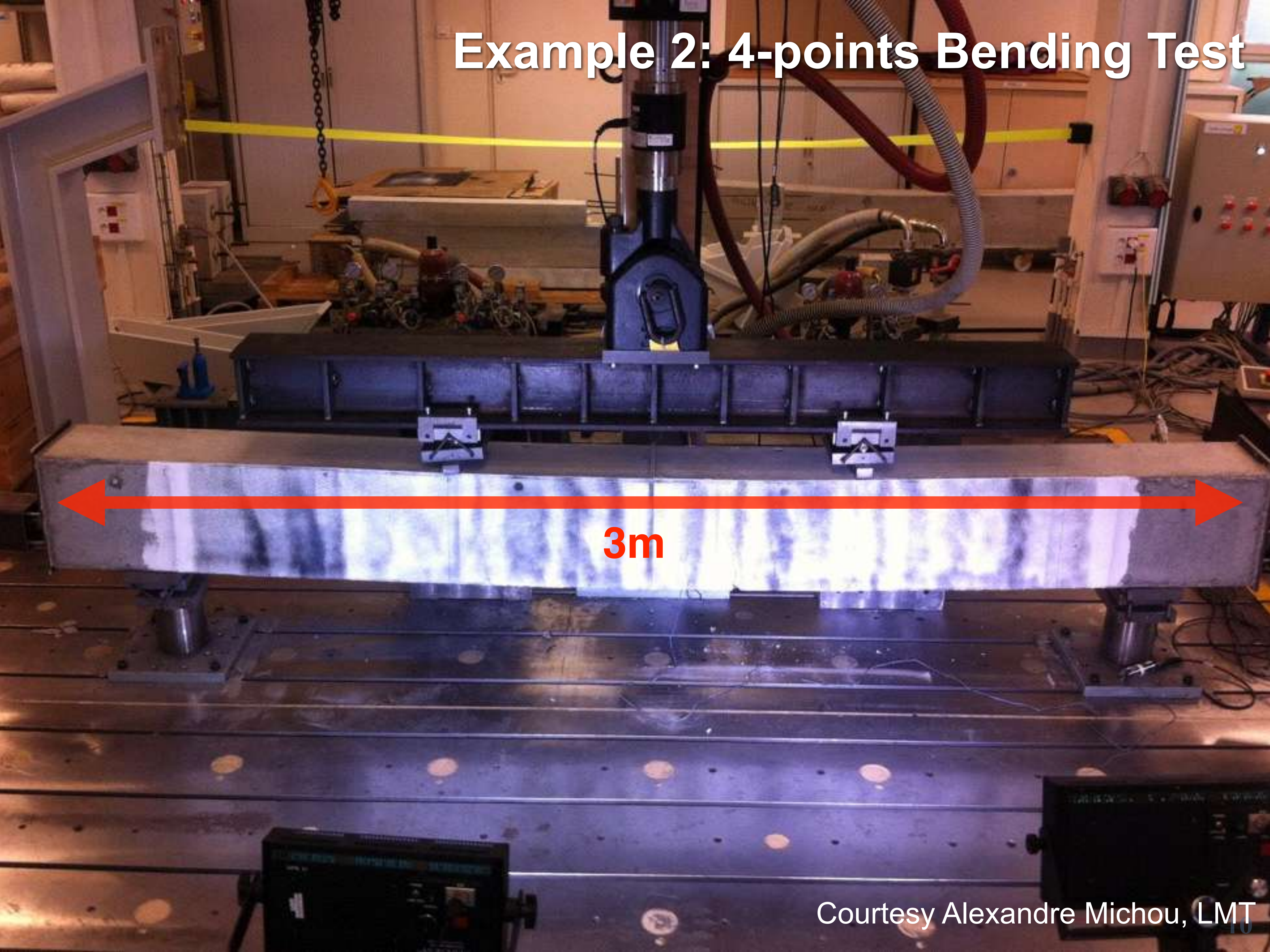

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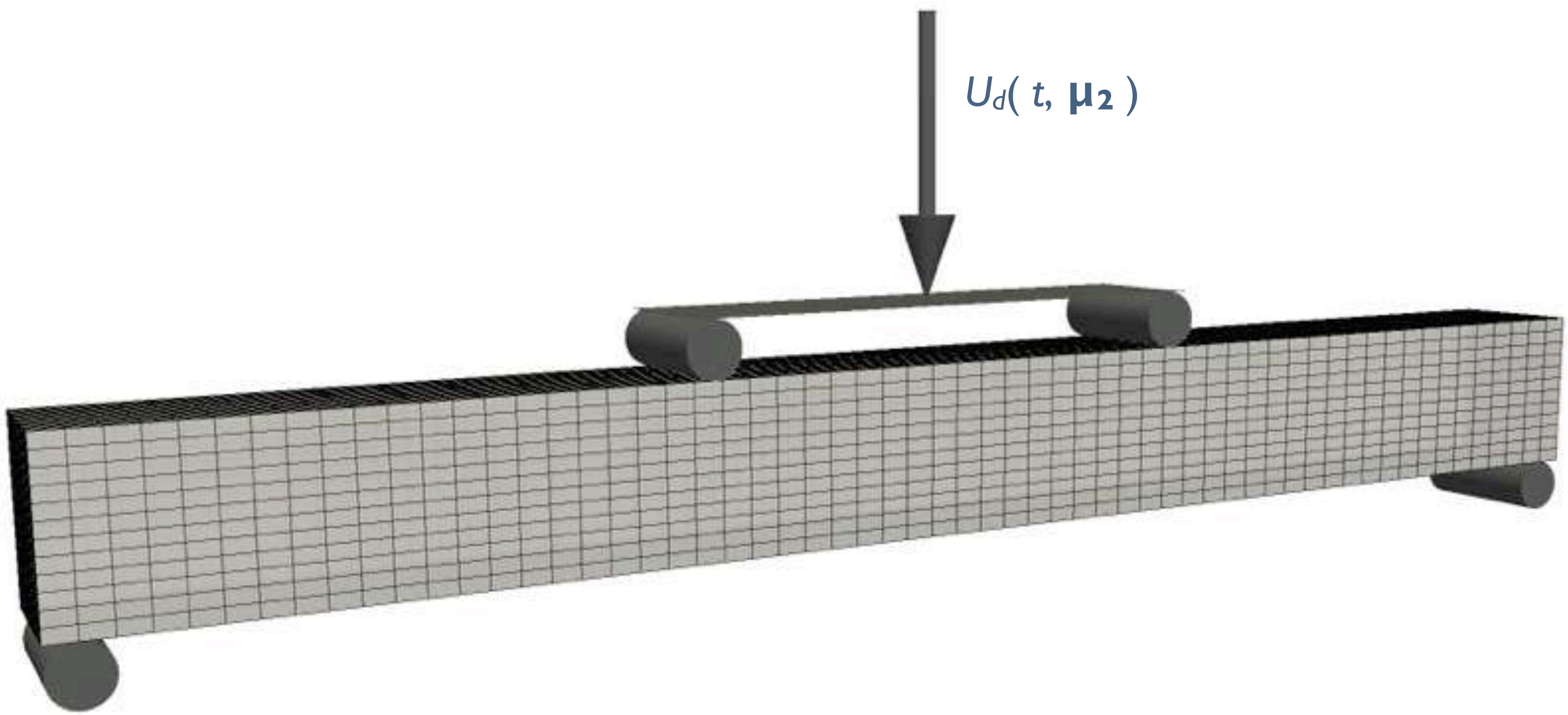
Example 2: 4-points Bending Test

[Vitse *et al.* 19]

- **Reinforced concrete beam**
  - 4-points bending test
  - prediction of damage



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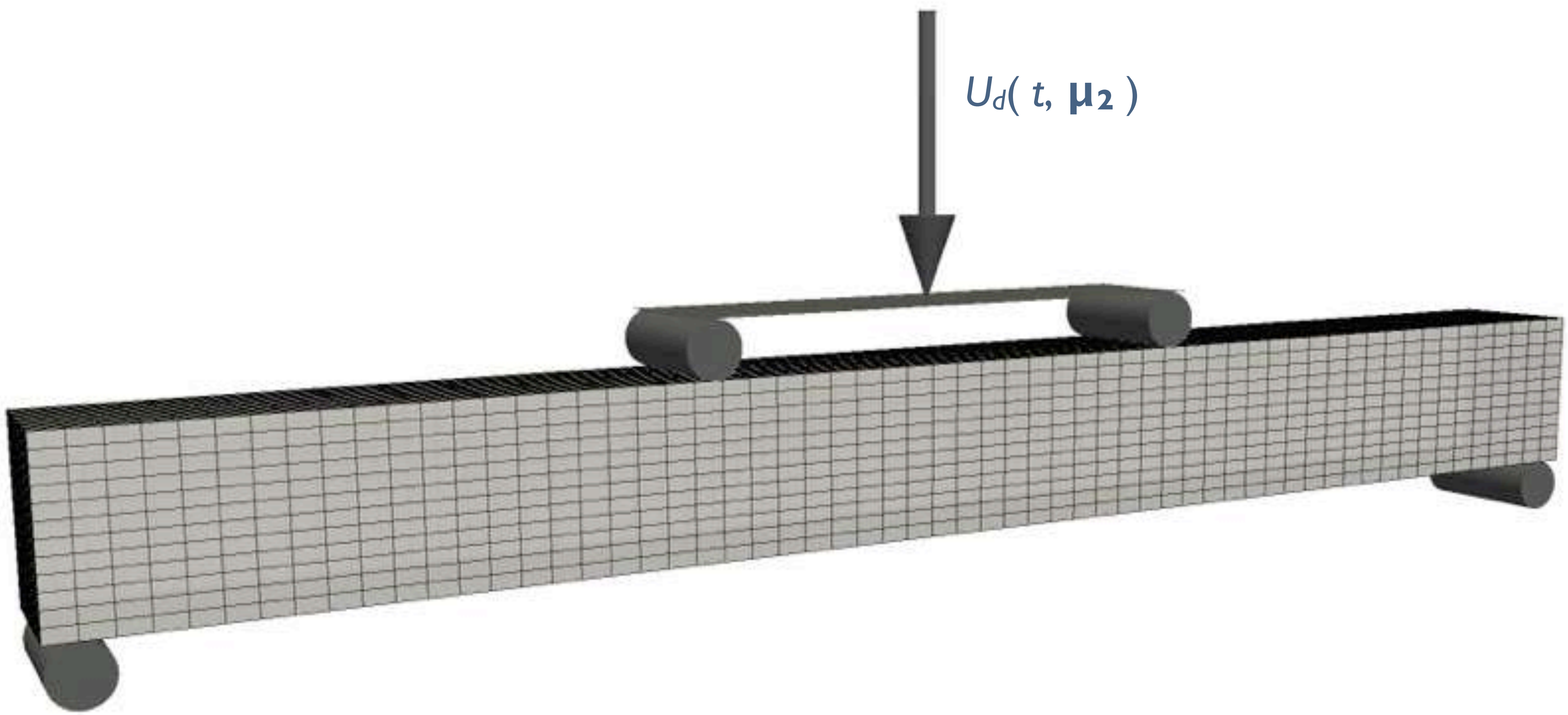

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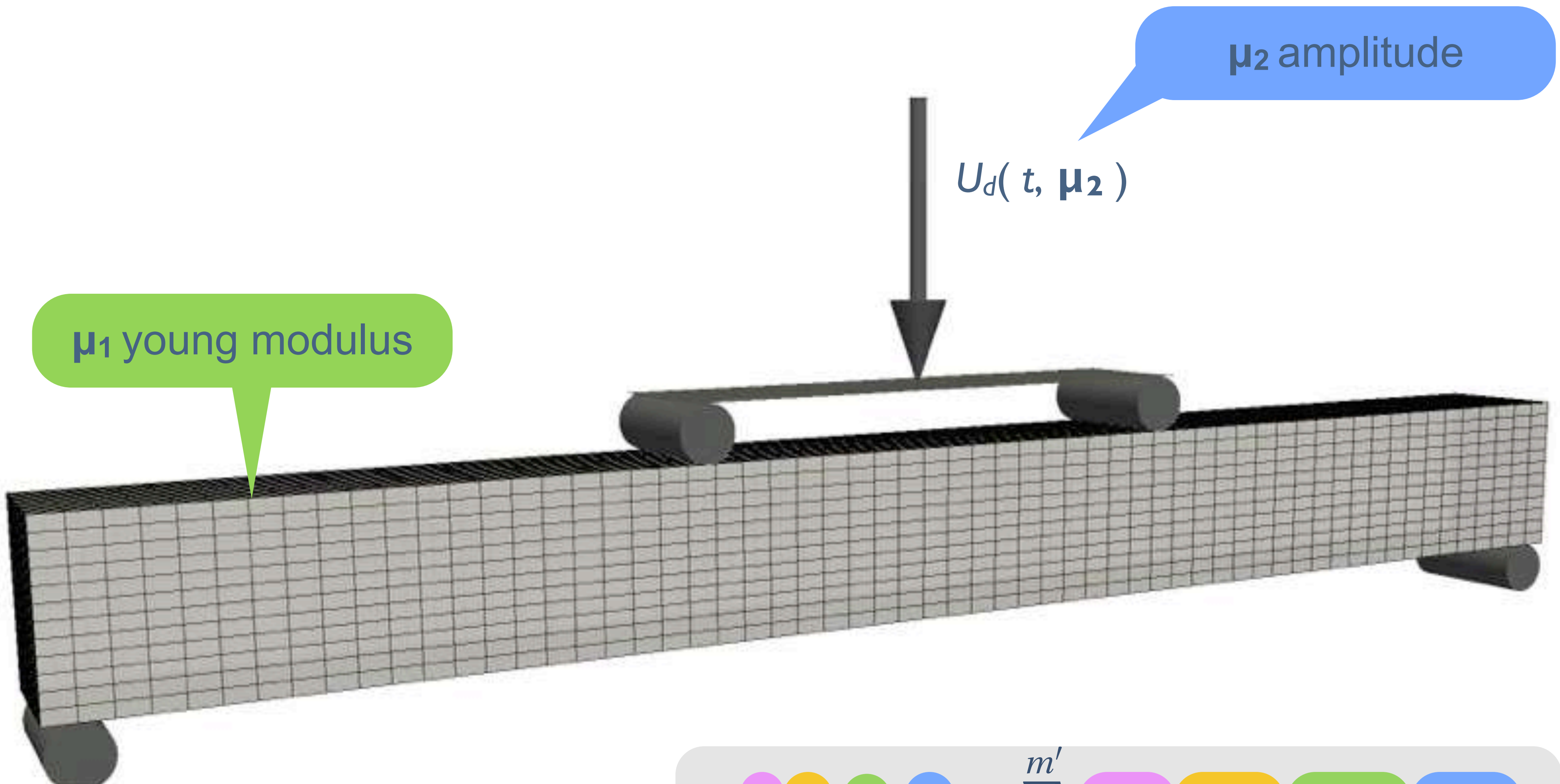

$$u(t, M, \mu_1, \mu_2) \approx \sum_{i=1}^{m'} \lambda_i(t) \Lambda_i(M) \alpha_i(\mu_1) \beta_i(\mu_2)$$

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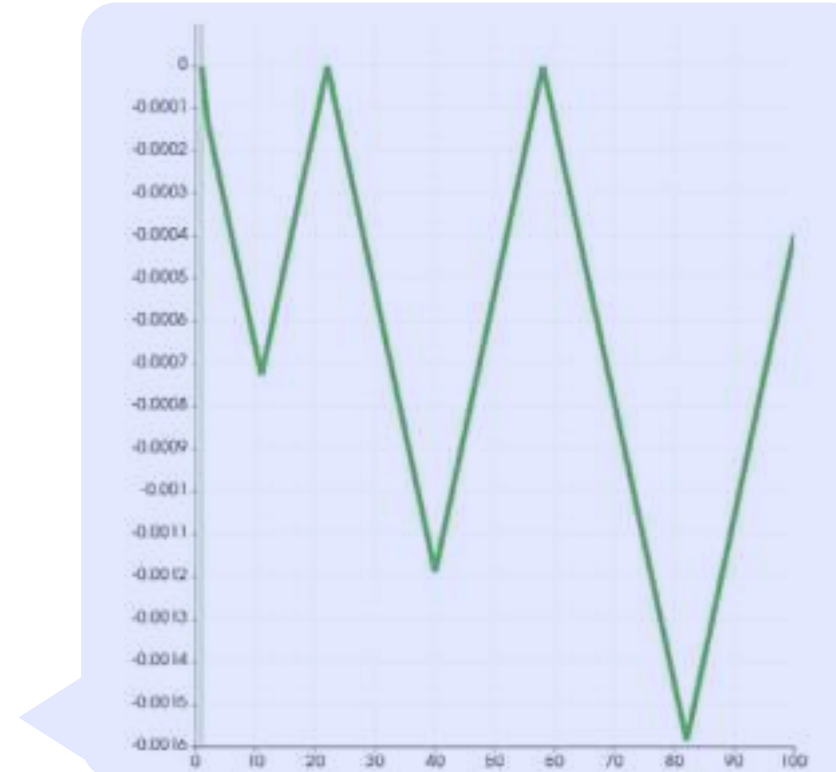

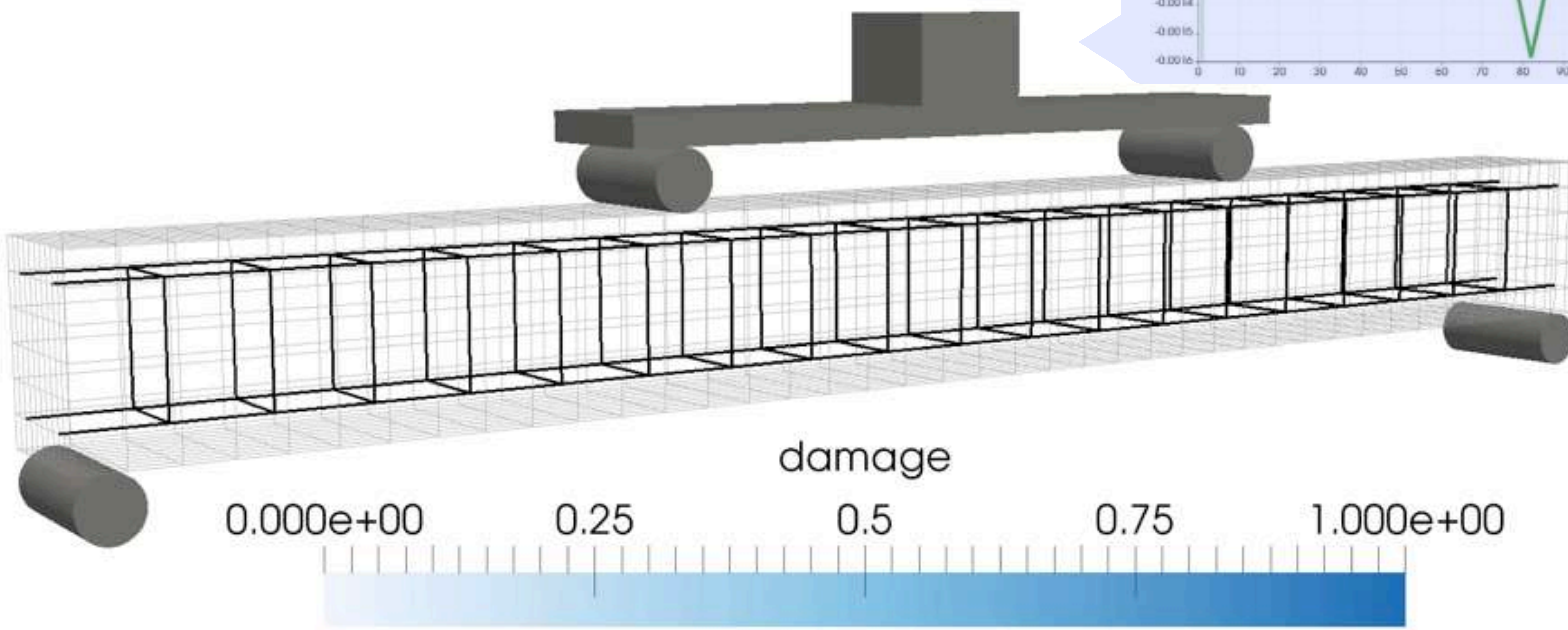

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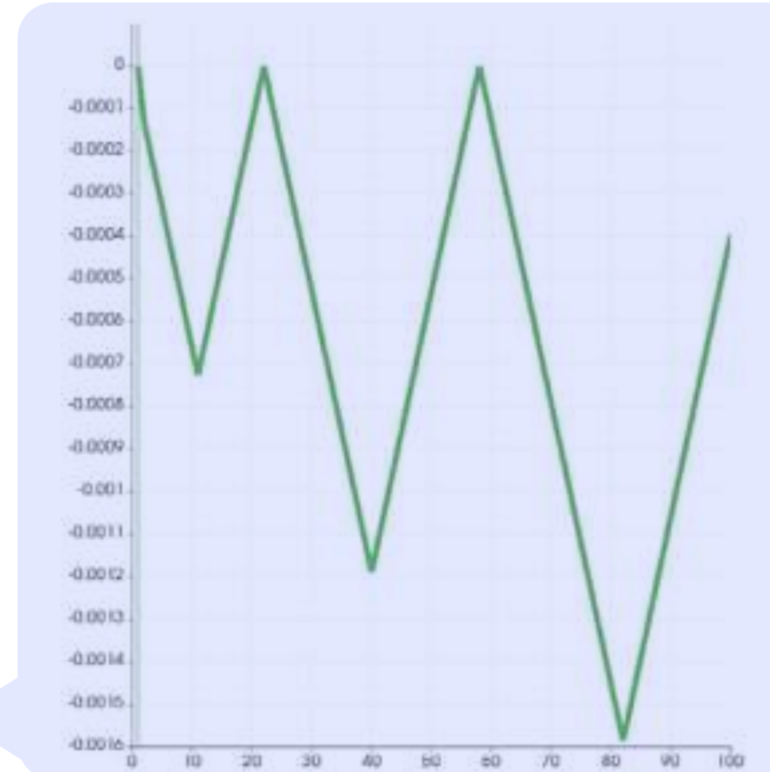
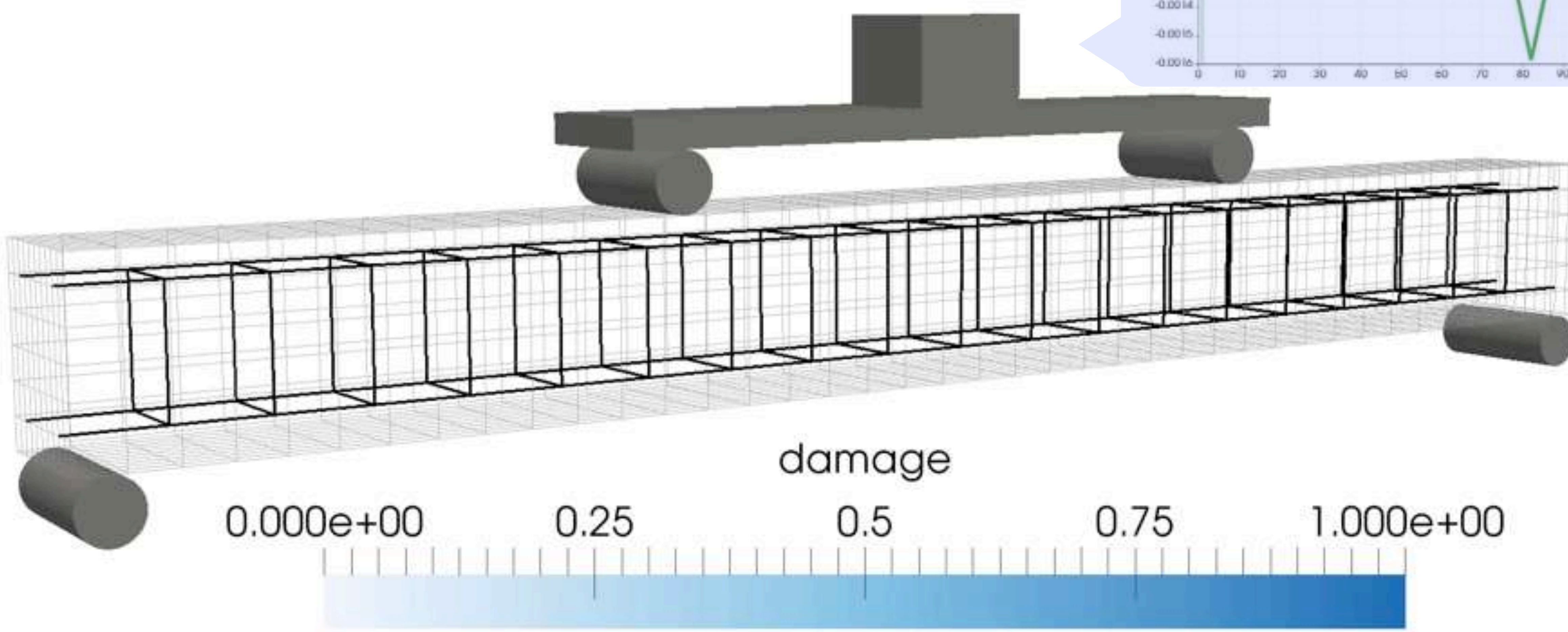
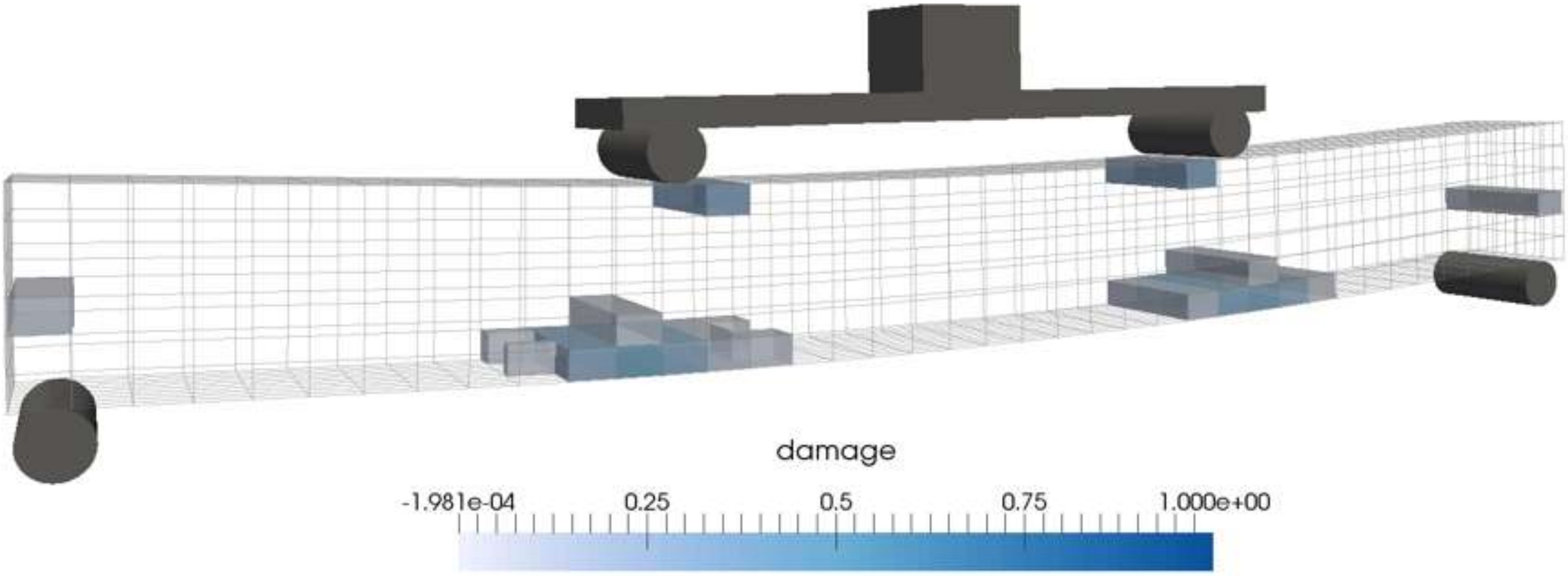
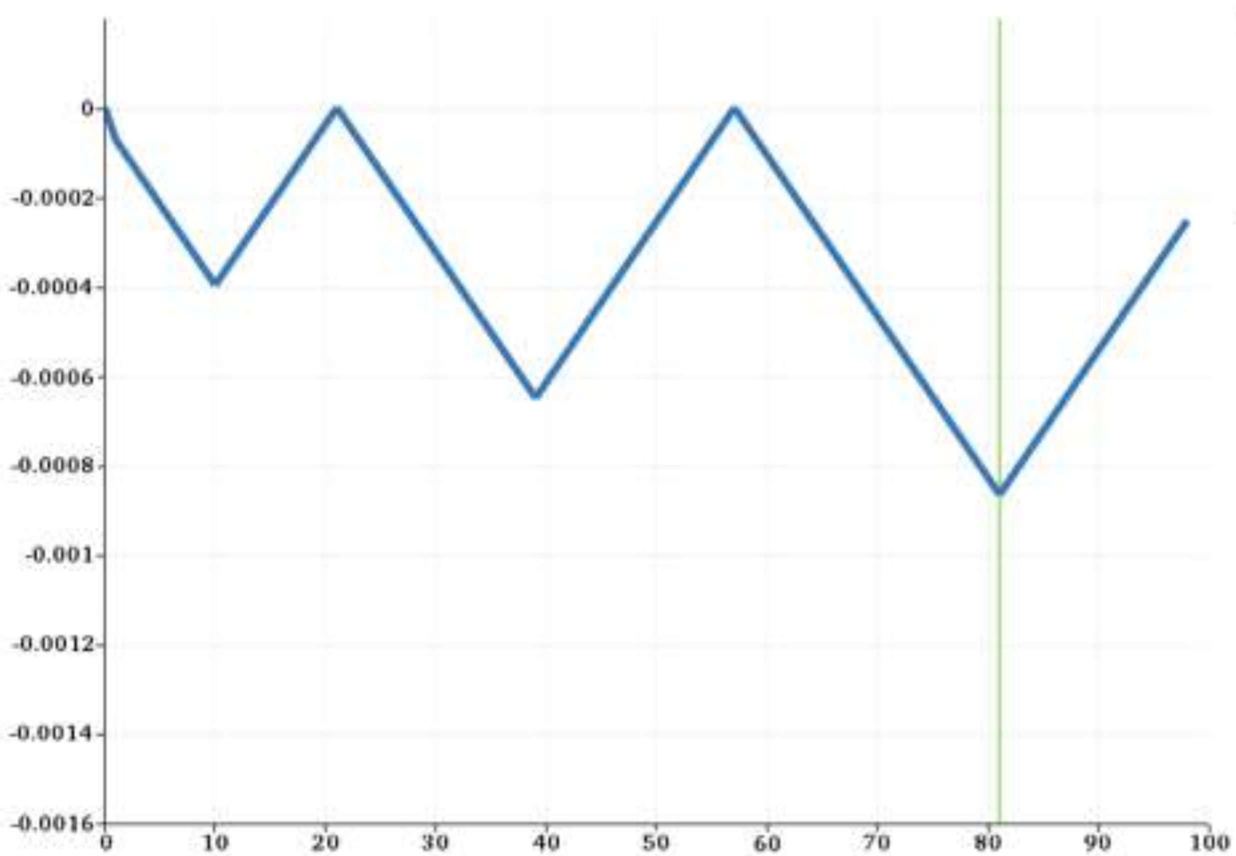
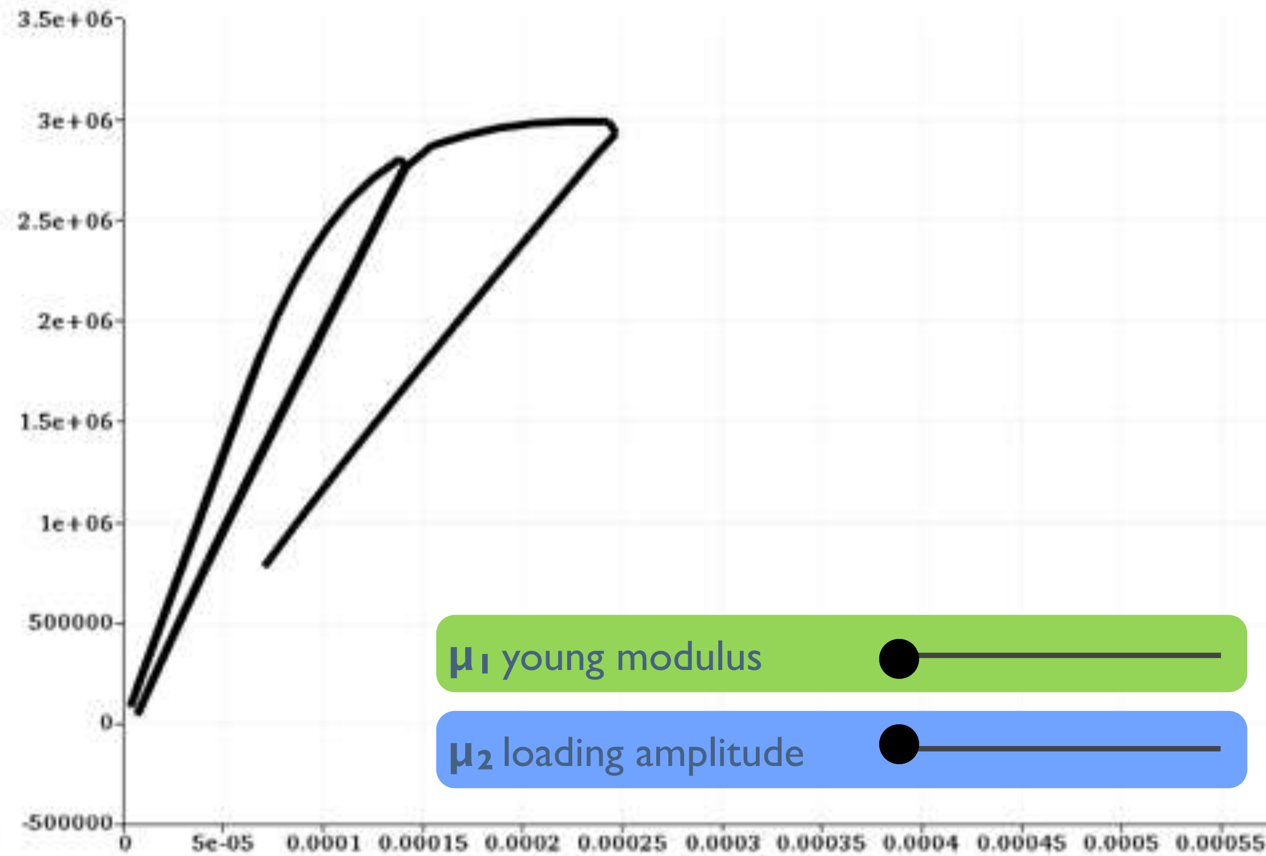




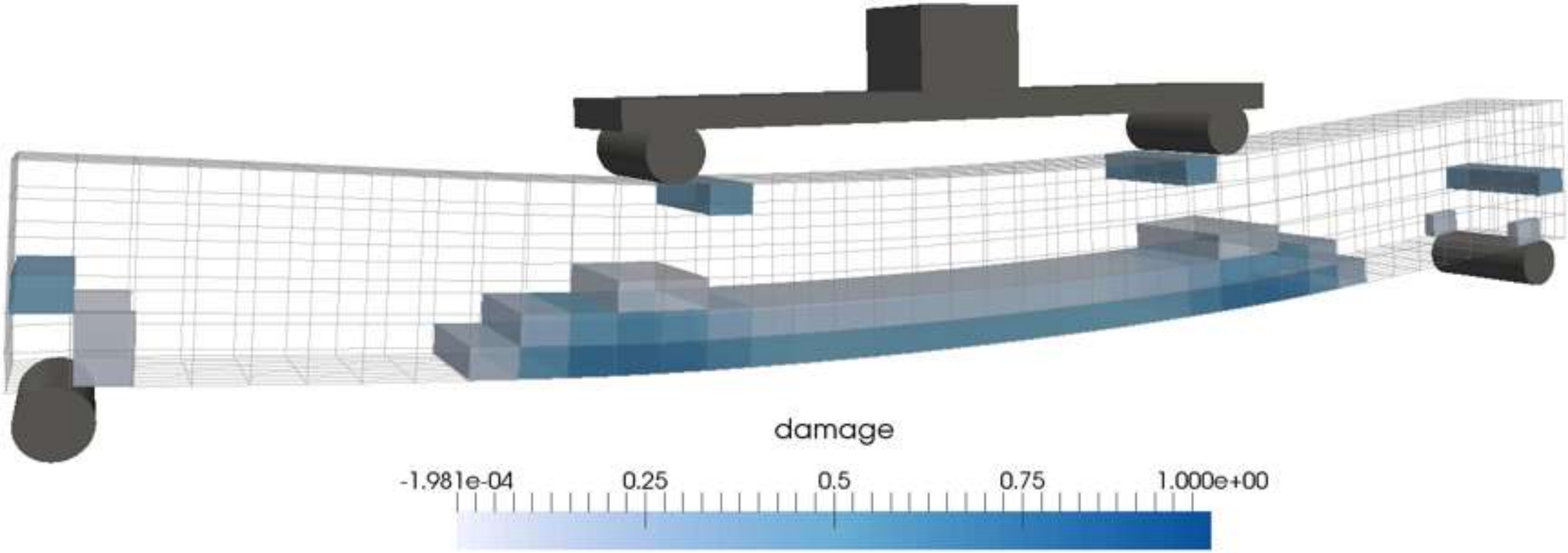


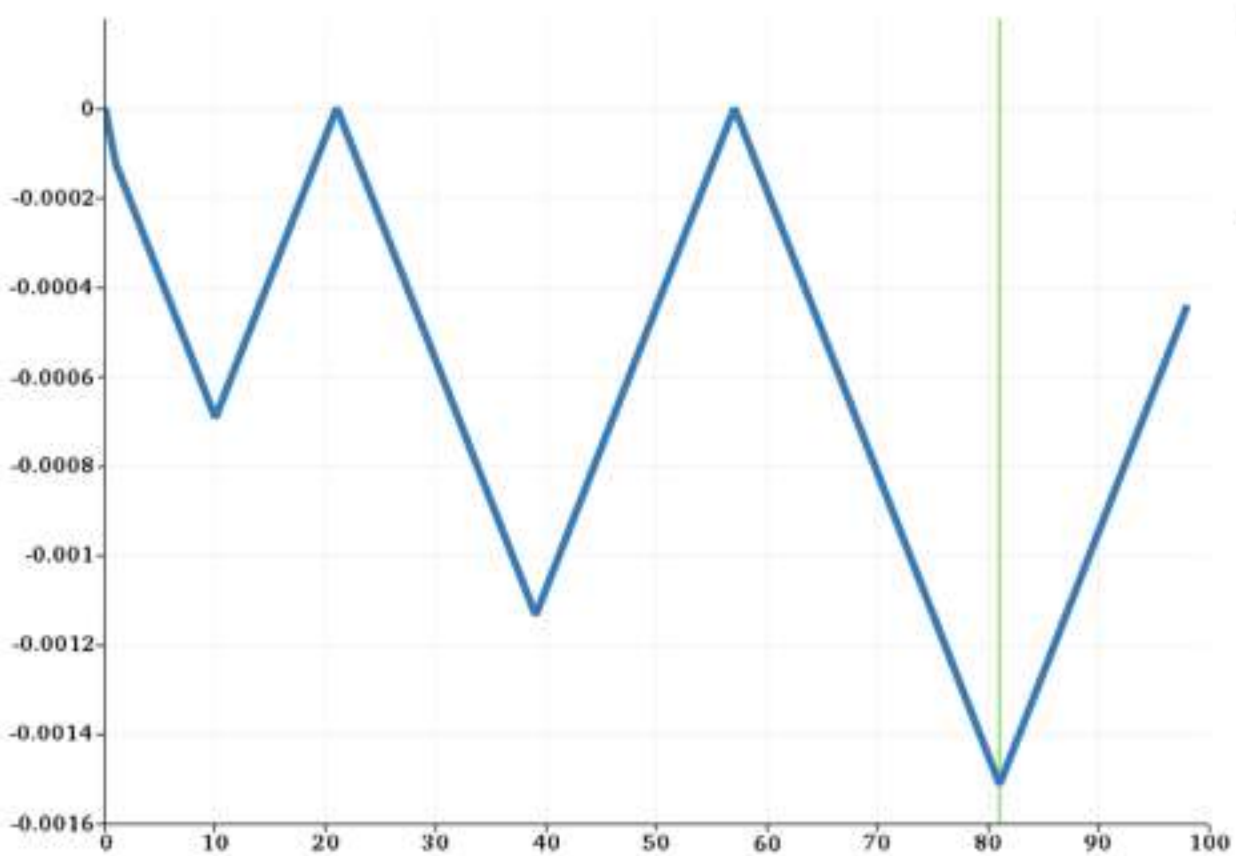


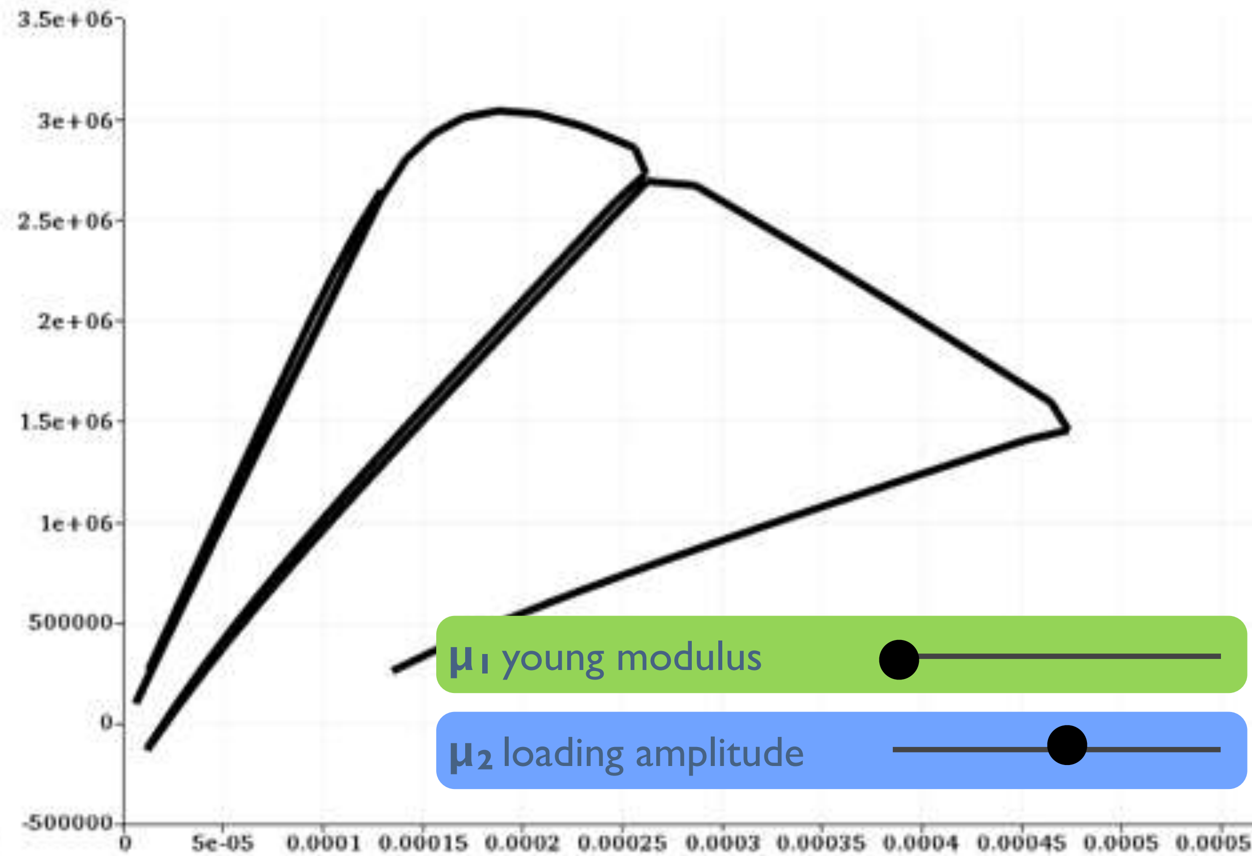


PXDMF Reader

A Separated Variable Reader for ParaView







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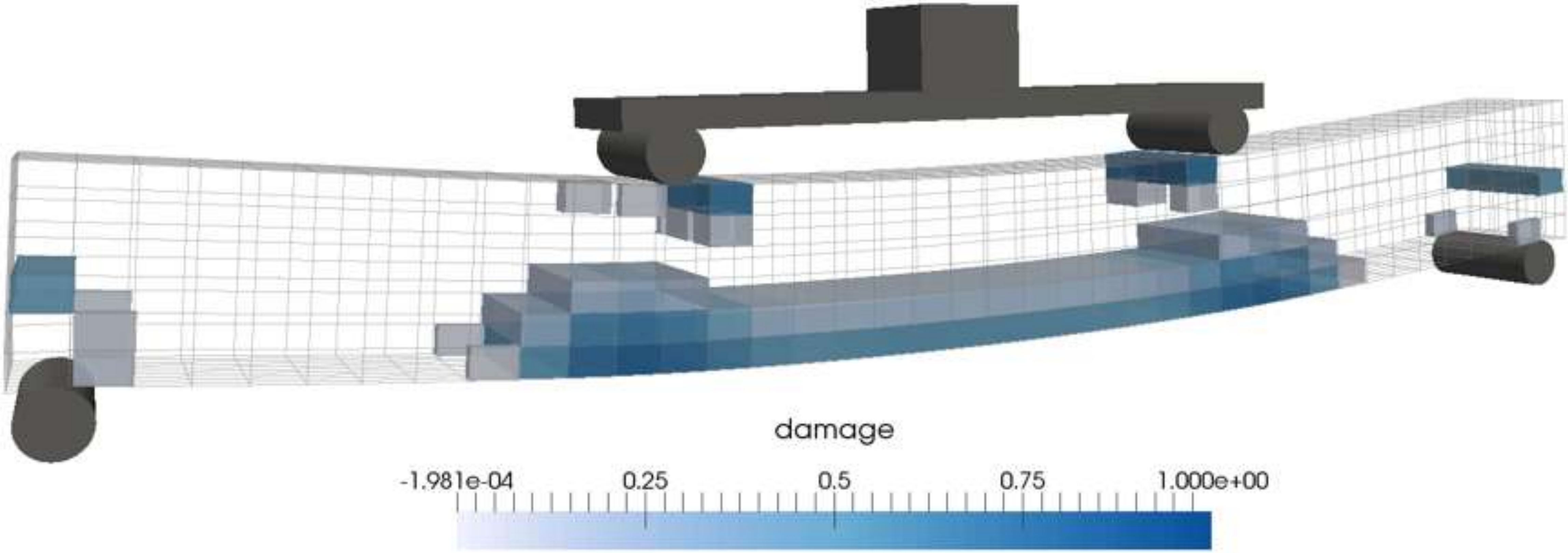

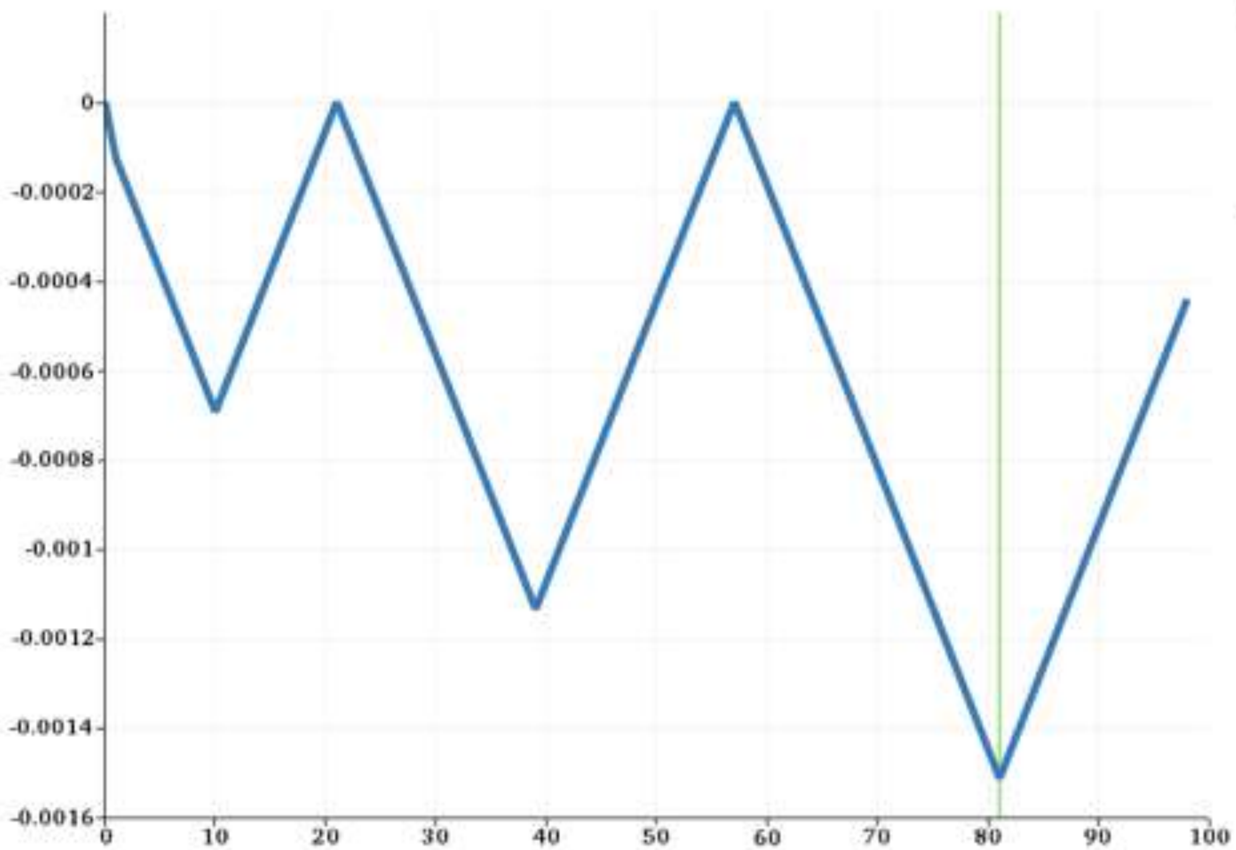

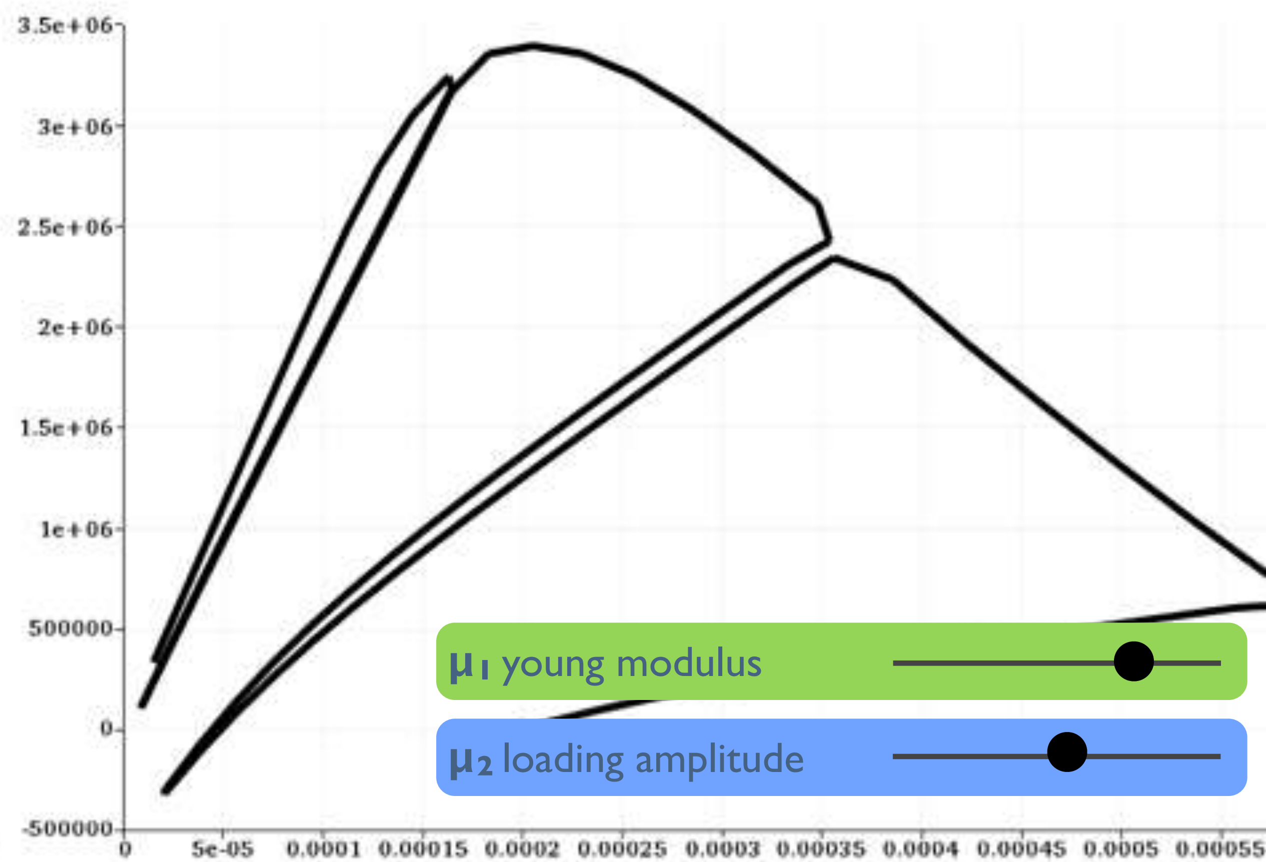

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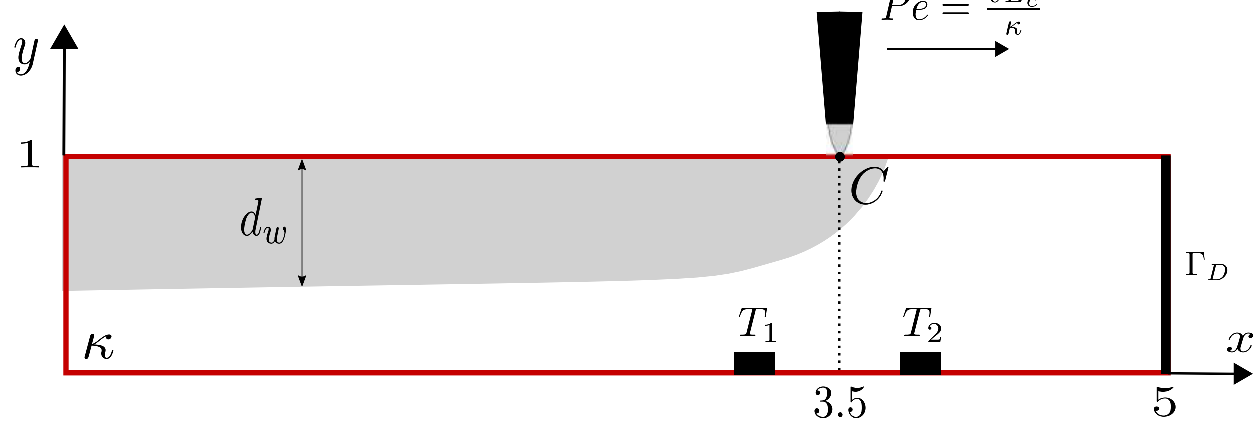
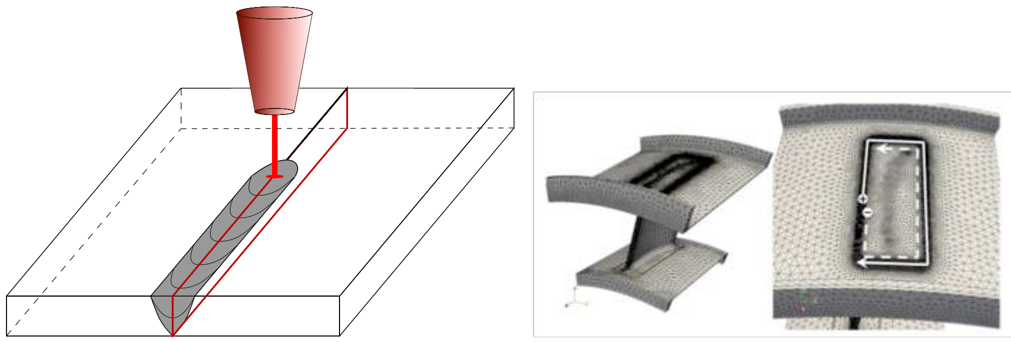
Illustration: Welding Process

[Rubio *et al.* 18]





→ predict & control the welding zone

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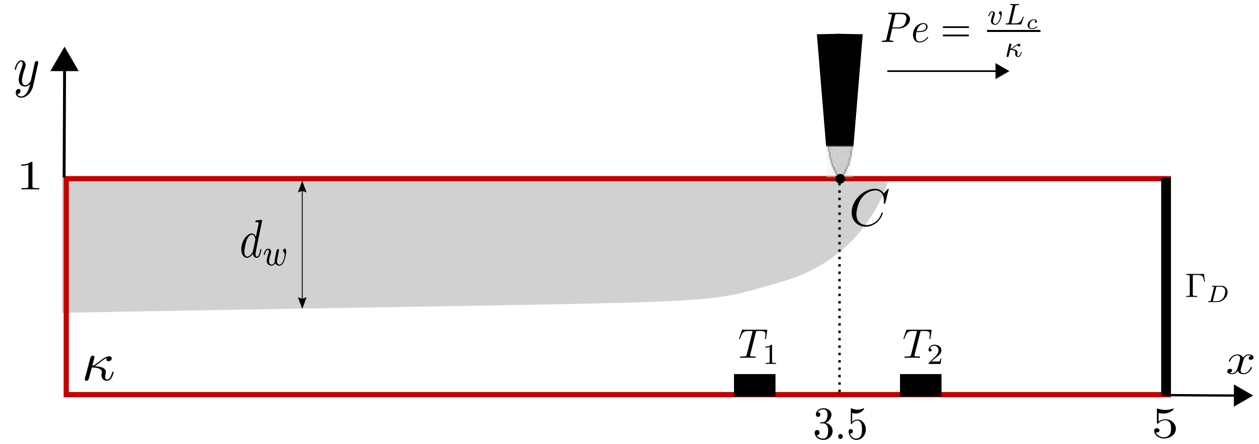
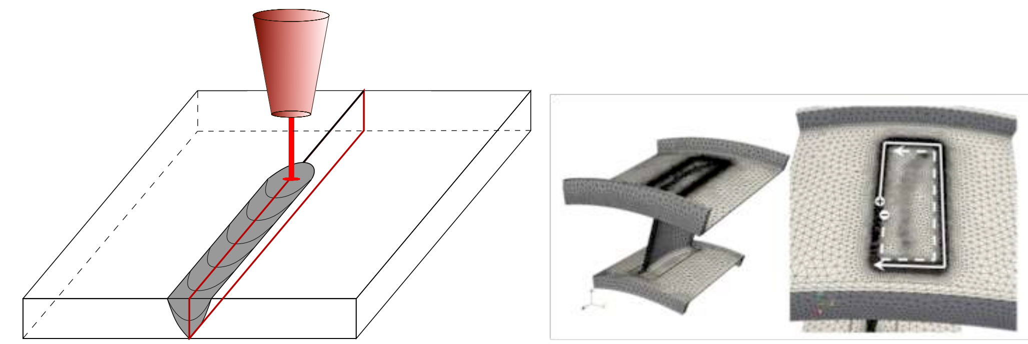
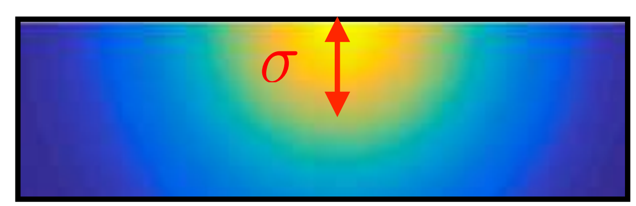
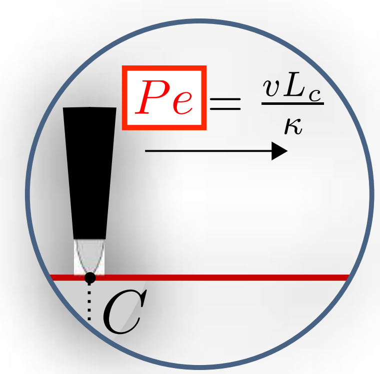




→ predict & control the welding zone

- **Observations:** temperature **T1** and **T2**
- **Model:** $\frac{\partial T}{\partial t} + \underline{v}.\underline{\mathrm{grad}}T - \kappa \Delta T = s(\sigma)$, $\underline{v} = [Pe; 0]$
- **Parameters of interest:** σ, Pe





$$s(x, y; \sigma) = \frac{u}{2\pi\sigma^2} \exp\left(-\frac{(x - x_c)^2 + (y - y_c)^2}{2\sigma^2} \right)$$

→ **dynamical model updating** of (σ, Pe) from data assimilated on-the-fly

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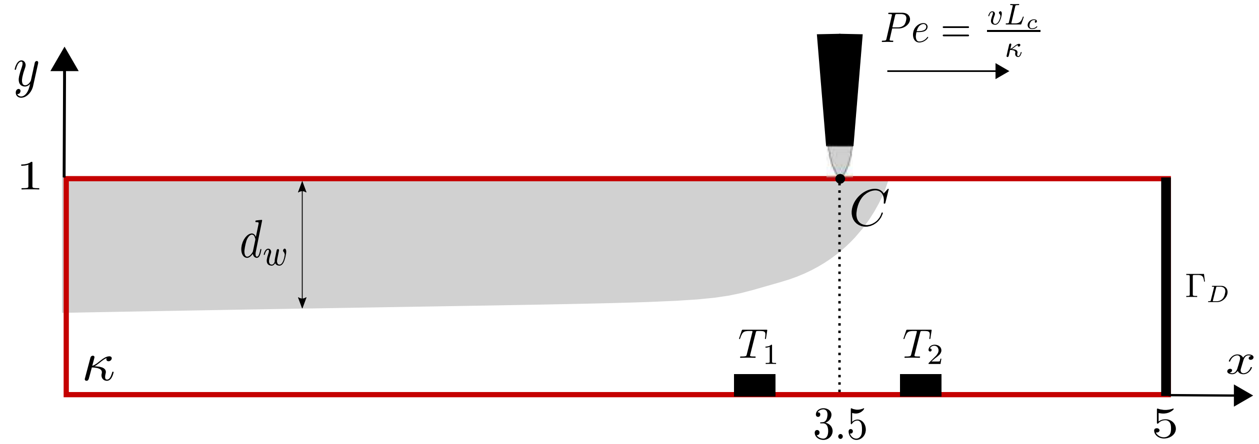

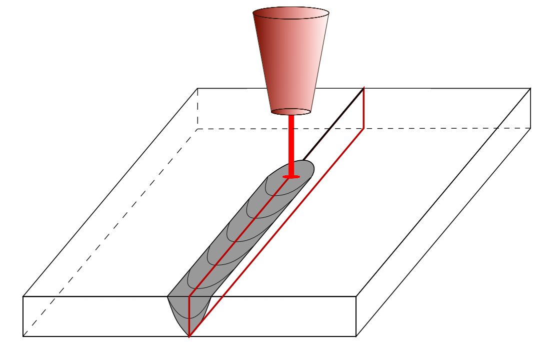

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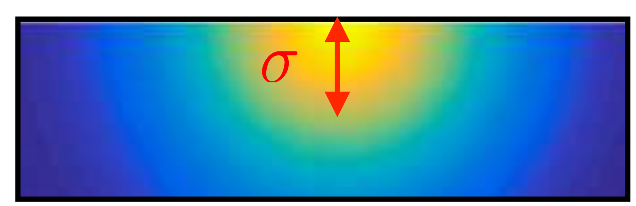

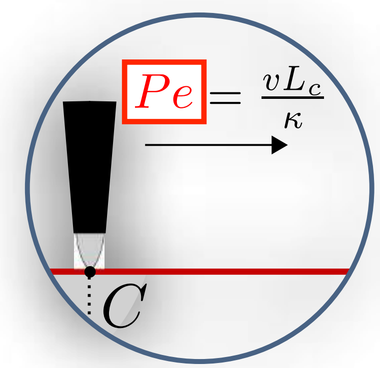

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- robustness with uncertainties (measurement noise,…) → **Bayesian inference**
- real-time constraint → **Reduced Order Modeling (PGD)**

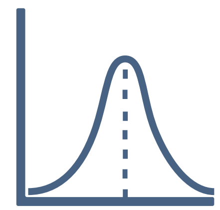
  → **Transport Map sampling**

Bayesian Formulation

[Kaipio & Sommersalo 04, Tarantola 05, Stuart 10]

- **Problem unknown**

$$\boxed{\pi(\mathbf{p}|\mathbf{d}^{\mathrm{obs}})}$$



→ automatic regularization

→ natural framework to consider uncertainties

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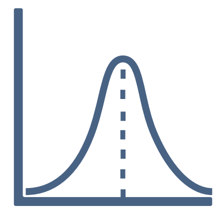

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- **Bayes Theorem:** $$\pi(\mathbf{p}|\mathbf{d}^{\mathrm{obs}}) \propto \pi(\mathbf{d}^{\mathrm{obs}}|\mathbf{p}).\pi(\mathbf{p})$$

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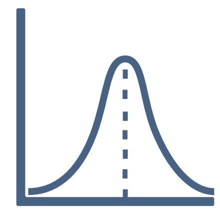

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prior

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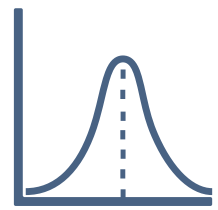

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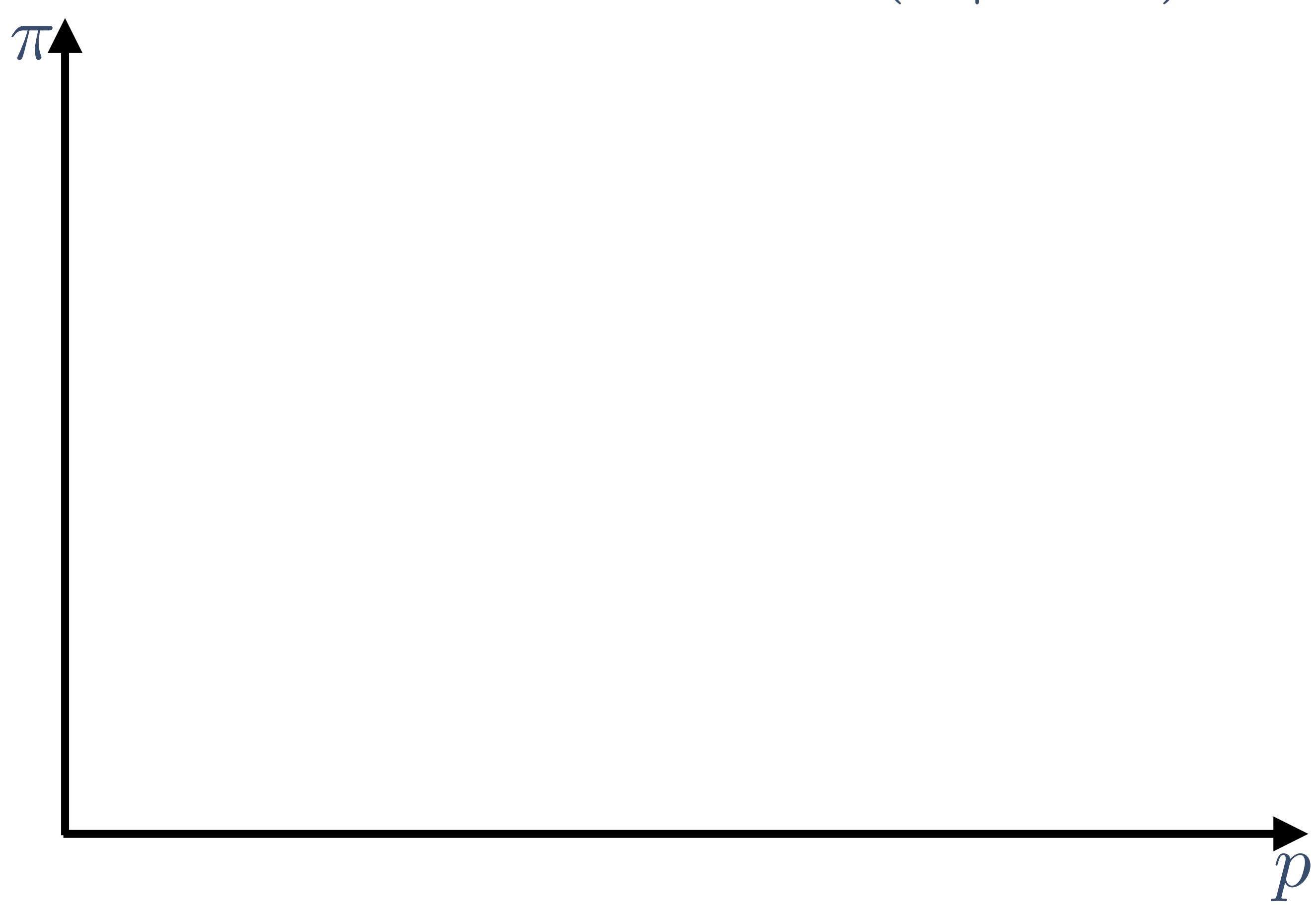

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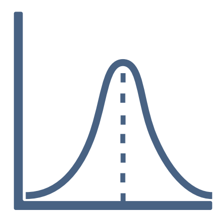

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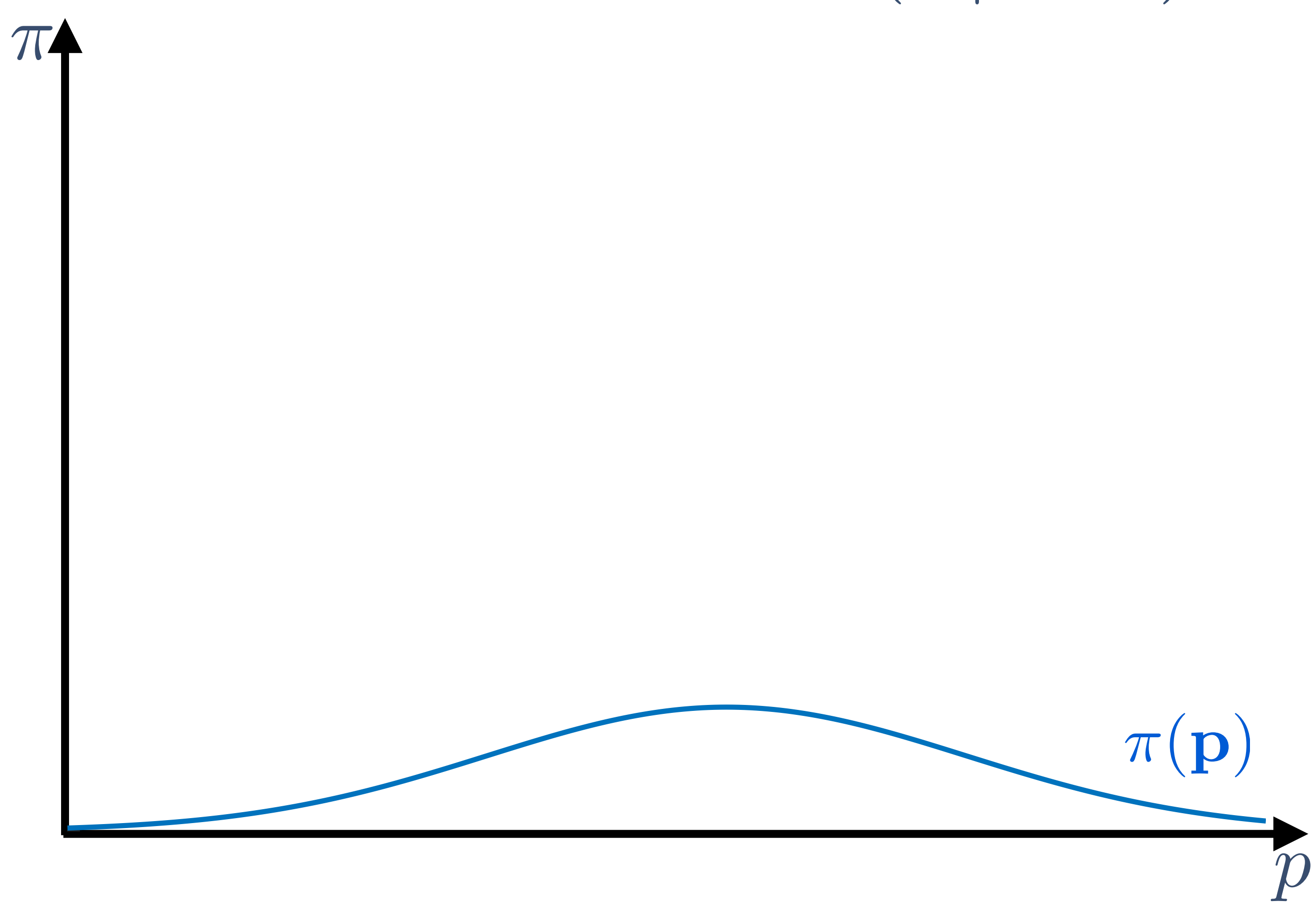
→ natural framework to consider uncertainties

- **Bayes Theorem:**

$$\pi(\mathbf{p}|\mathbf{d}^{\mathrm{obs}}) \propto \pi(\mathbf{d}^{\mathrm{obs}}|\mathbf{p})\,\boxed{\pi(\mathbf{p})}$$

prior

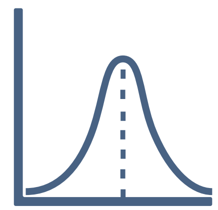


Bayesian Formulation

[Kaipio & Sommersalo 04, Tarantola 05, Stuart 10]

- **Problem unknown**

$$\boxed{\pi(\mathbf{p}|\mathbf{d}^{\text{obs}})}$$



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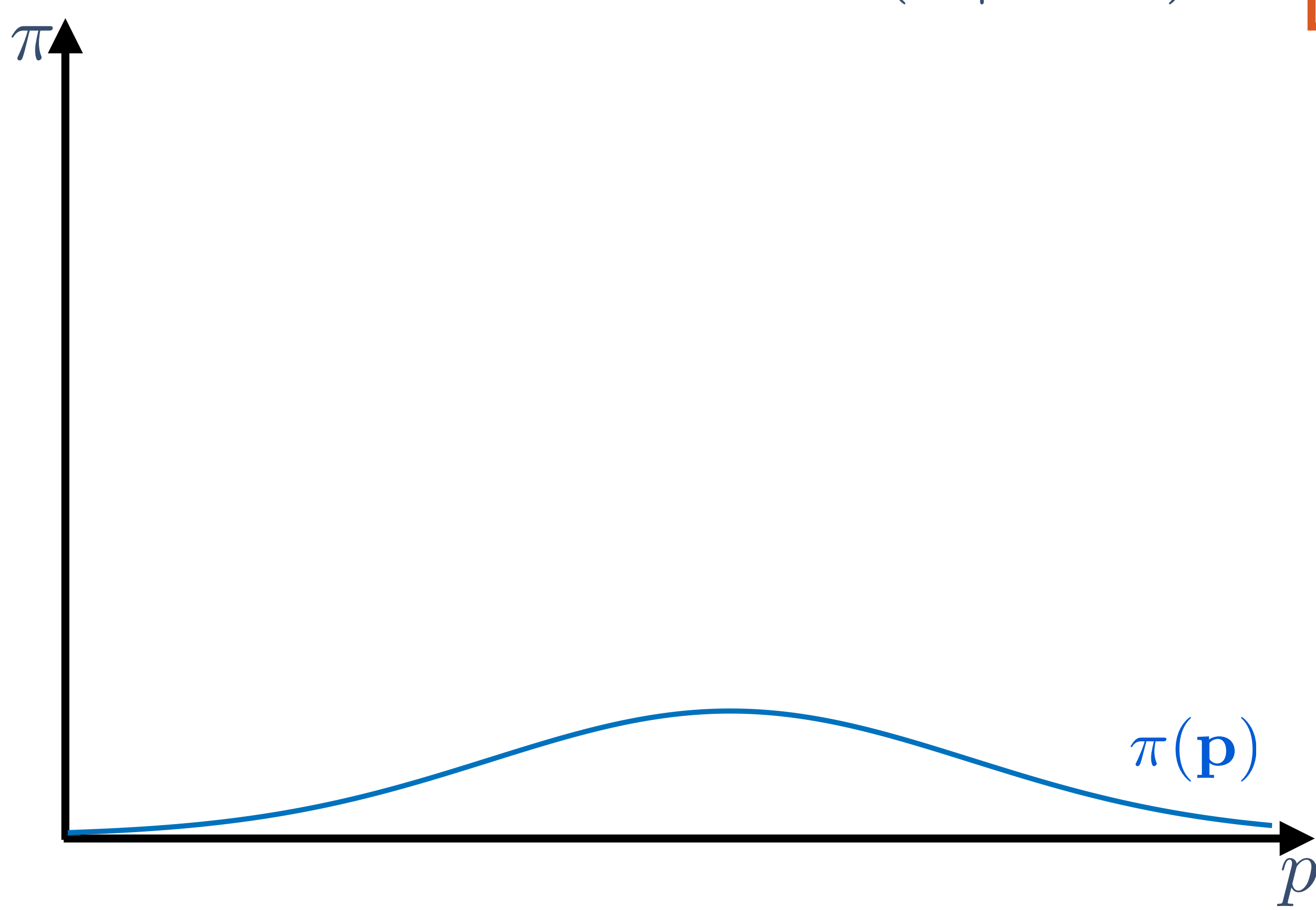

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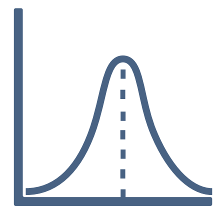

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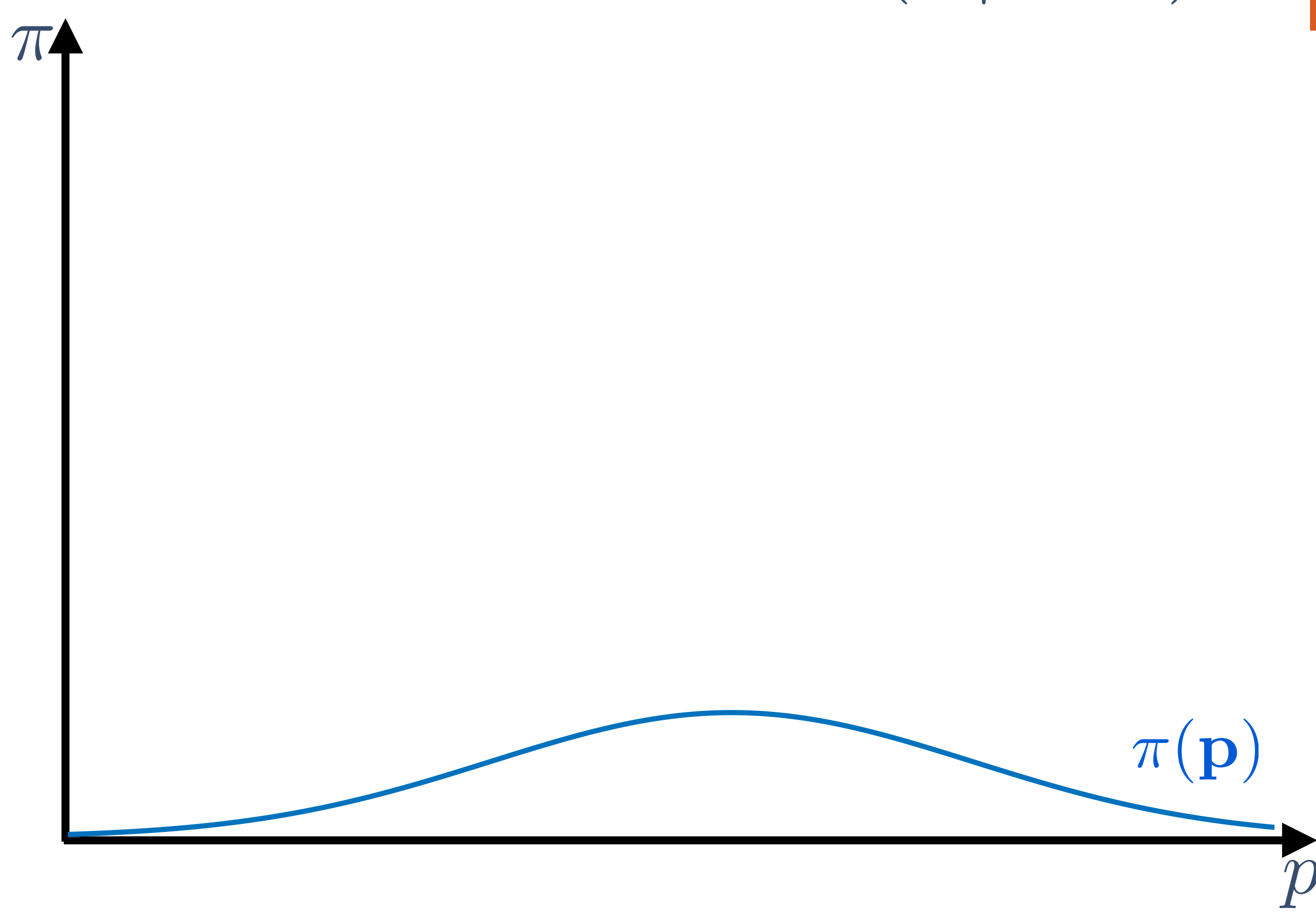
if additive measurement noise



costly multi-query process

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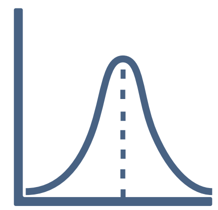

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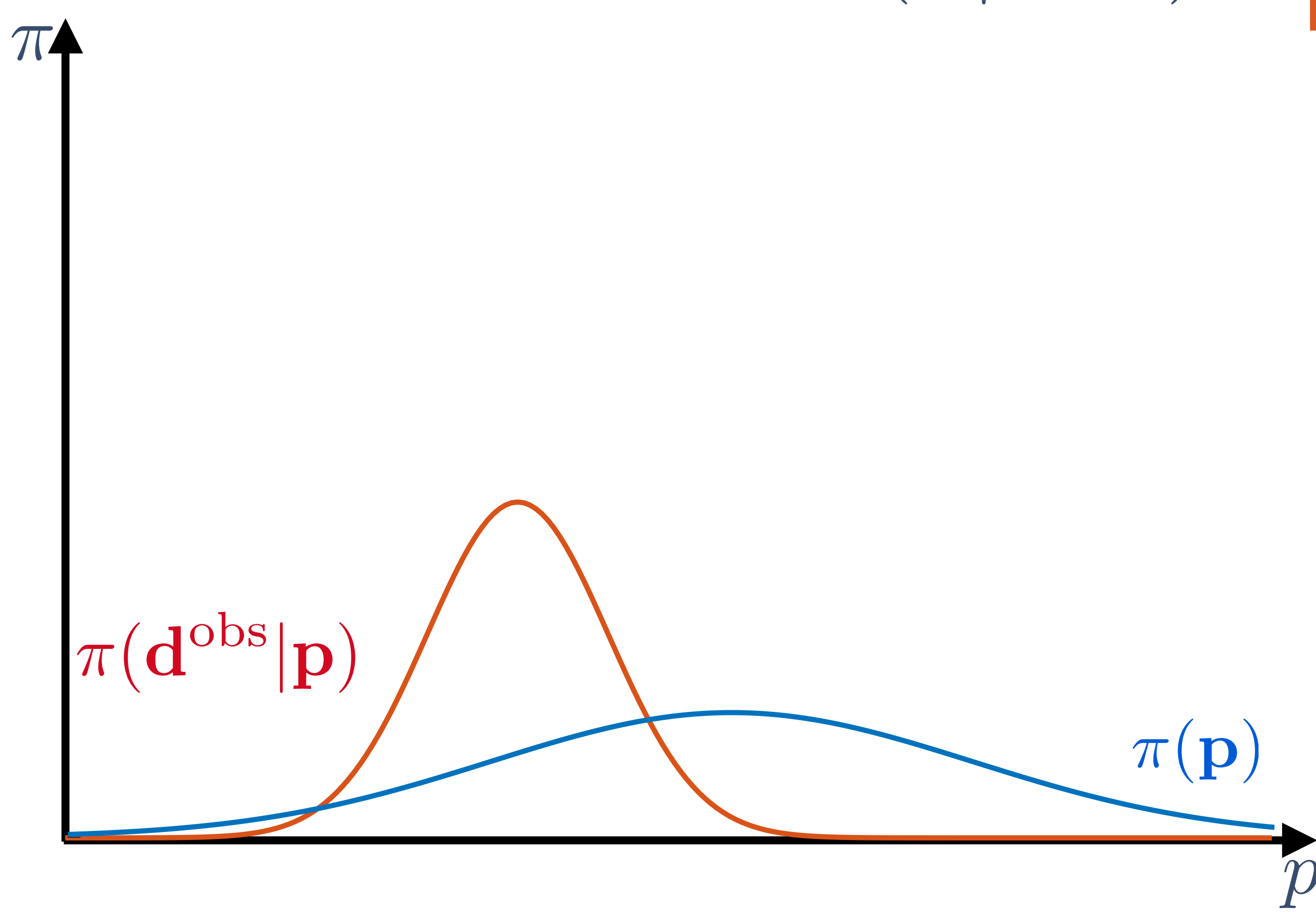

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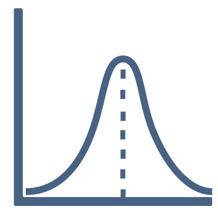

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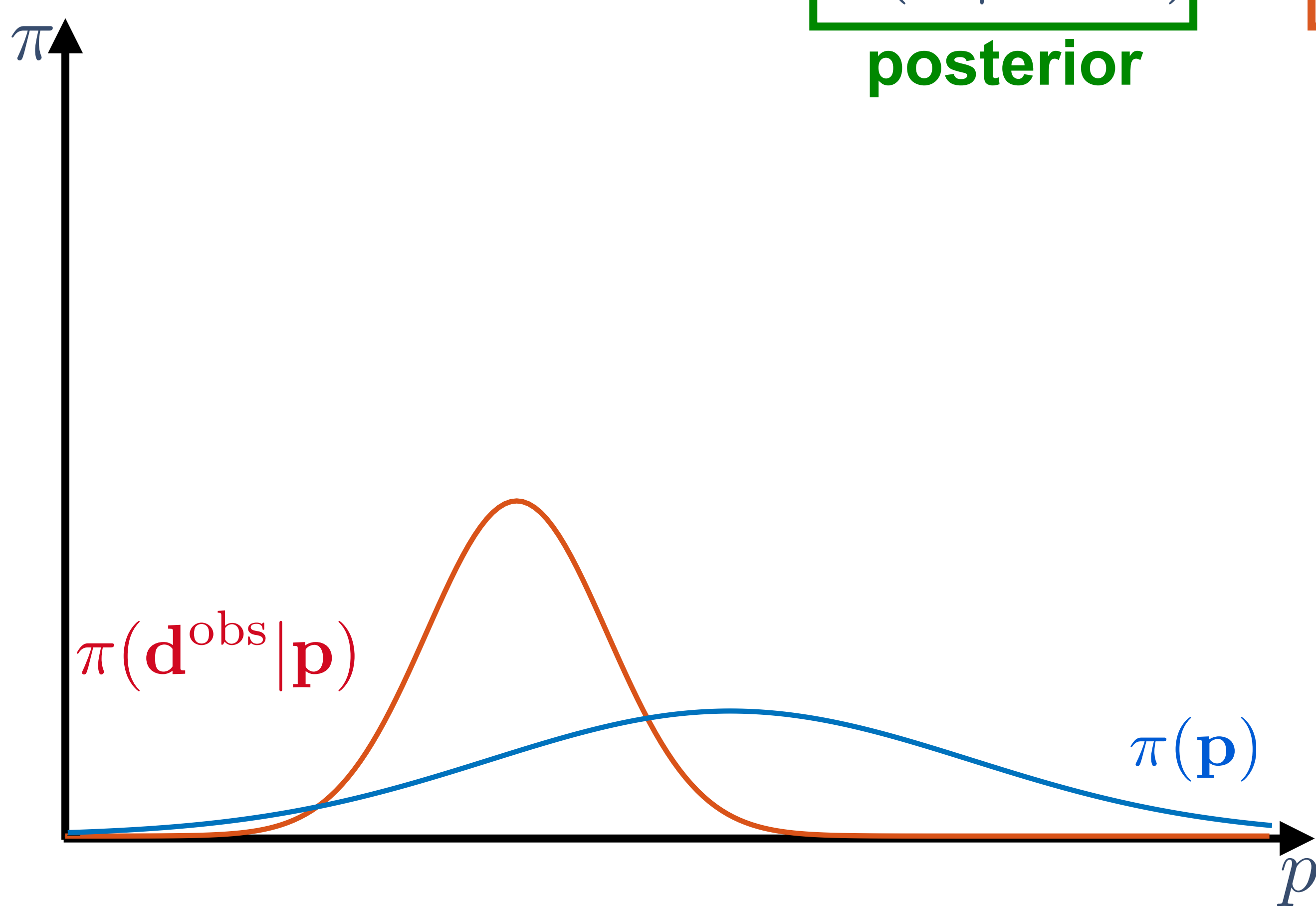

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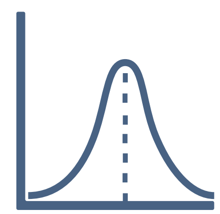

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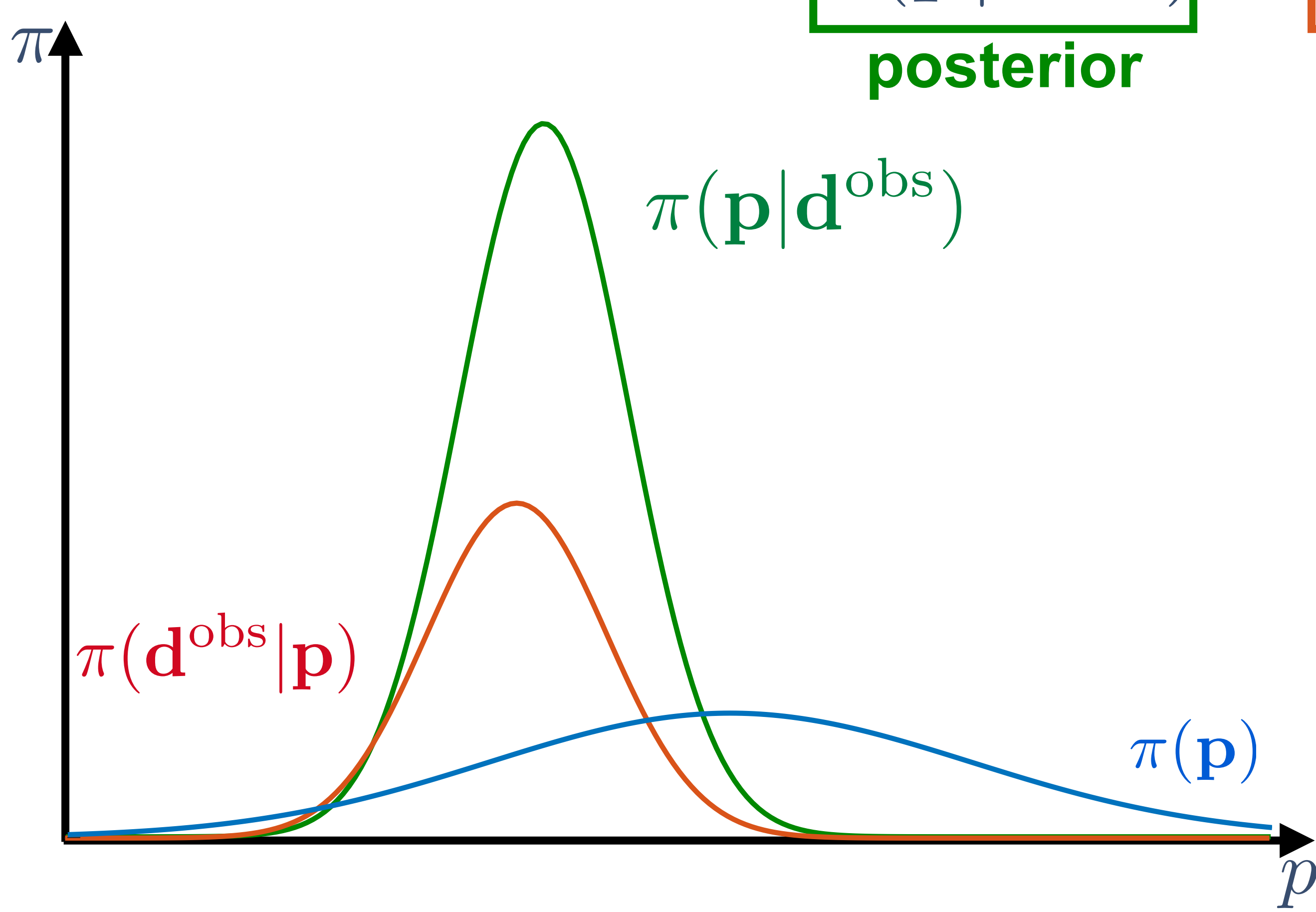

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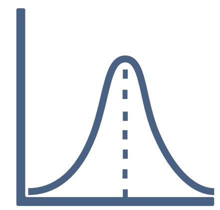

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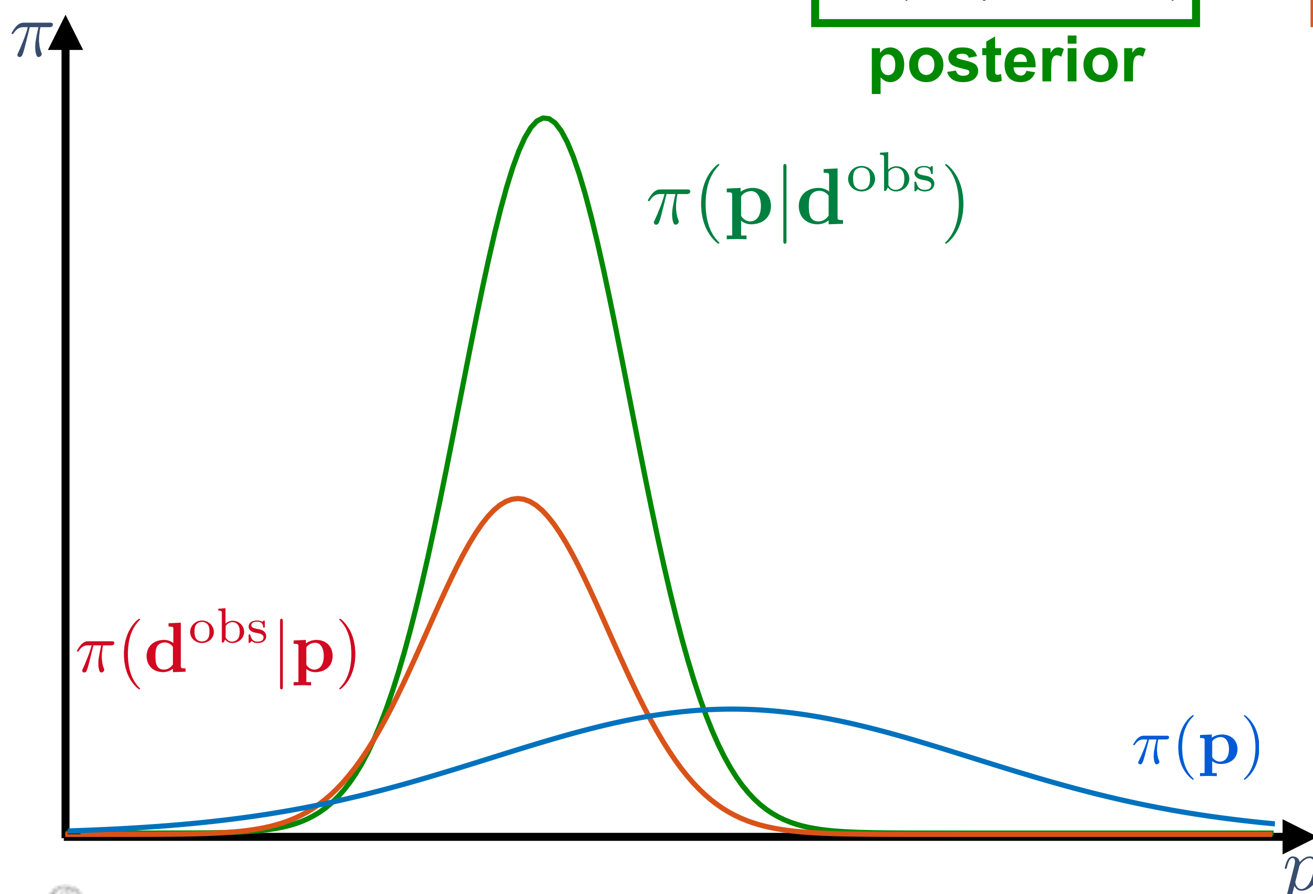

- **Sequential assimilation**

$$\pi(\mathbf{p}|\mathbf{d}_1^{\mathrm{obs}}, ..., \mathbf{d}_i^{\mathrm{obs}}) \propto \left(\prod_{j=1}^{i} \pi_{t_j}(\mathbf{d}_j^{\mathrm{obs}}|\mathbf{p}) \right) .\pi_0(\mathbf{p}) \qquad \pi_{t_j}(\mathbf{d}_j^{\mathrm{obs}}|\mathbf{p}) = \pi_{\mathrm{meas}}\left(\mathbf{d}_j^{\mathrm{obs}} - \mathbf{d}\left(\mathbf{p}, t_j\right)\right)$$

PGD Model Reduction

- modal description of multiparametric solution [Nouy 10, Chinesta *et al.* 14]
- low-rank canonical tensor format (separated variables)

$$\boxed{\mathbf{u}_m(\mathbf{x}, t, \mathbf{p}) = \sum_{k=1}^{m} \mathbf{\Lambda}_k(\mathbf{x}) \lambda_k(t) \prod_{i=1}^{d} \alpha_k^i(p_i)}$$

⟶ explicit dependency on parameters (extra-coordinates)
⟶ construction in the *offline* phase
⟶ use in the *online* phase of inference

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⟶ construction in the *offline* phase

⟶ use in the *online* phase of inference

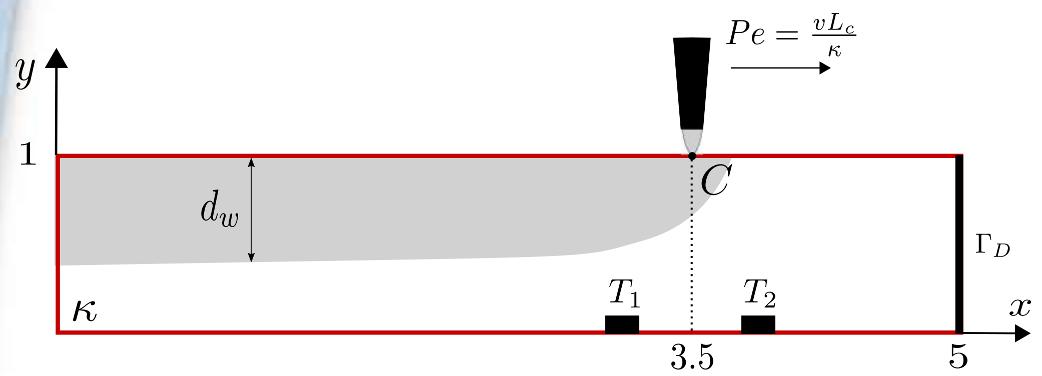
⟶ straightforward model evaluation in the likelihood function

$$\mathbf{d}(\mathbf{p},t) = \mathcal{O}\left(\mathbf{u}_m(\mathbf{x},t,\mathbf{p})\right)$$ [Berger *et al.* 17, Rubio *et al.* 18]

⟶ may provide an explicit formulation of the posterior density

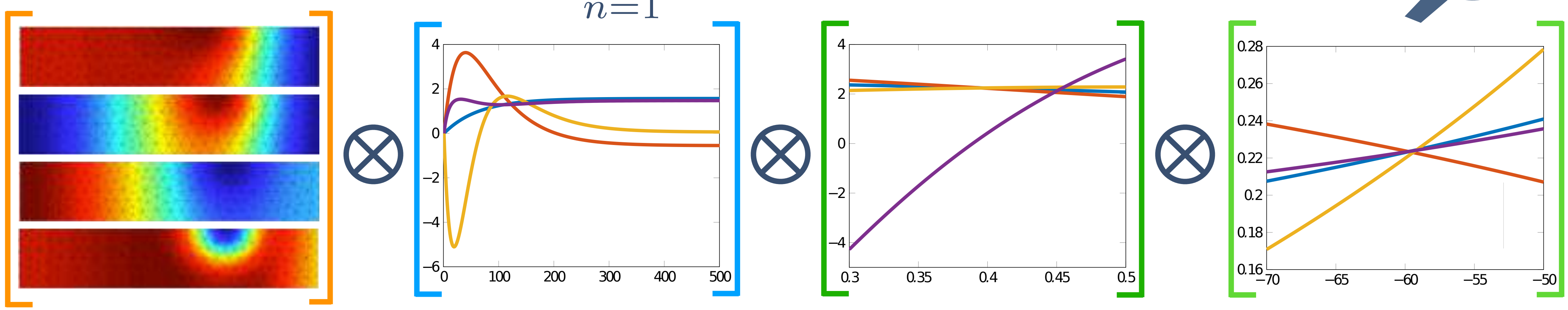
⟶ fast UQ on outputs of interest $q(\mathbf{p}) \approx \mathcal{Q}\left(\mathbf{u}_m(\mathbf{x},t,\mathbf{p})\right)$

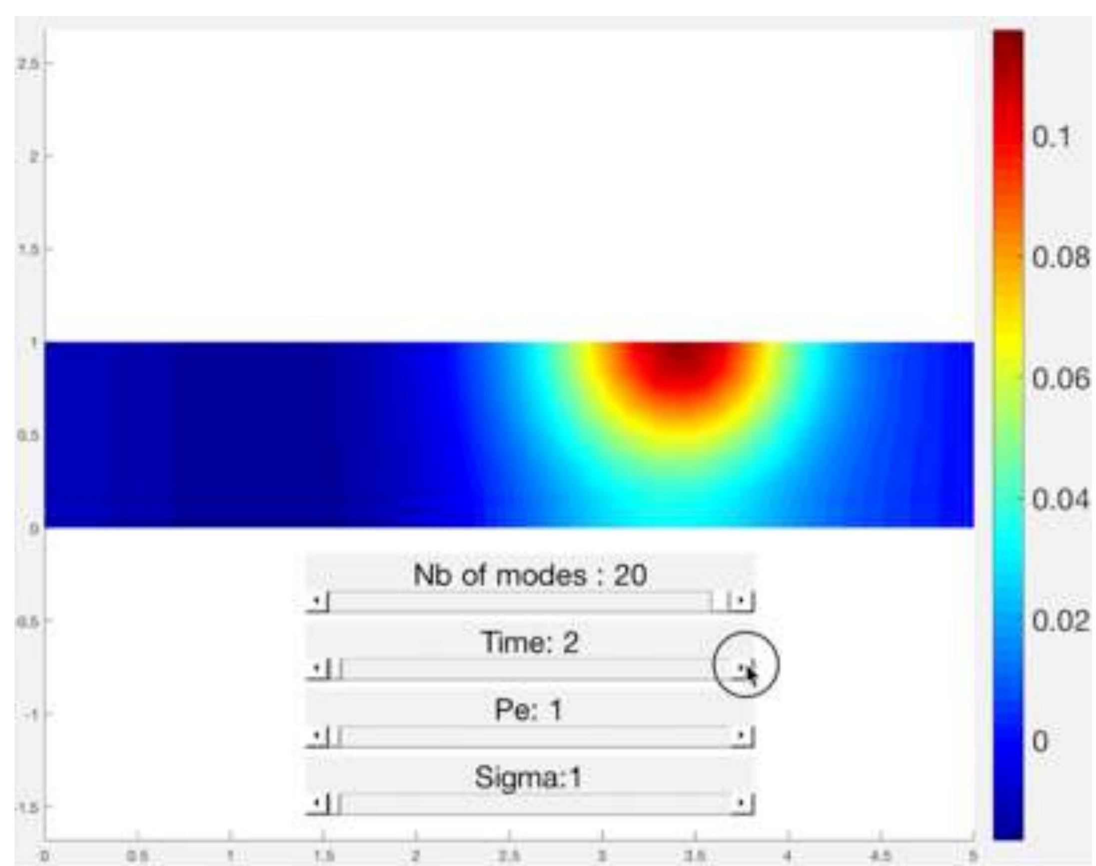
⟶ samples $q_k = q(\mathbf{p}_k)$



Application

$$T(x, y, t, \sigma, Pe) = \sum_{n=1}^{m} \Lambda_n(x, y)\lambda_n(t)\alpha_n^1(\sigma)\alpha_n^2(Pe)$$





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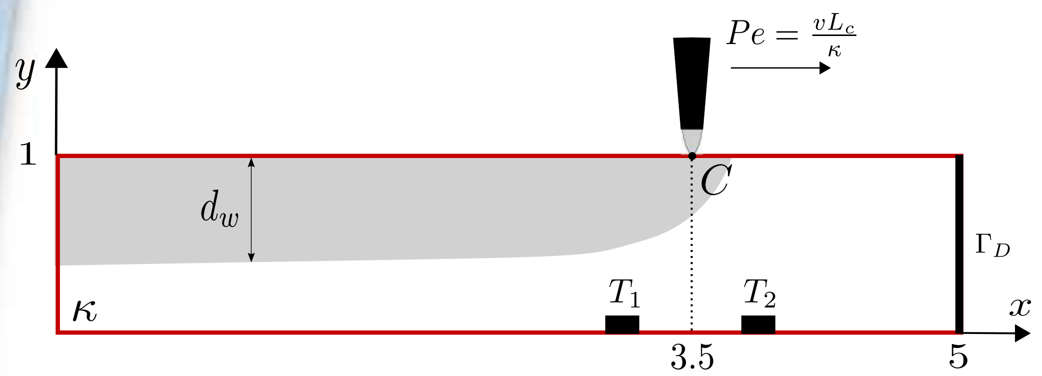

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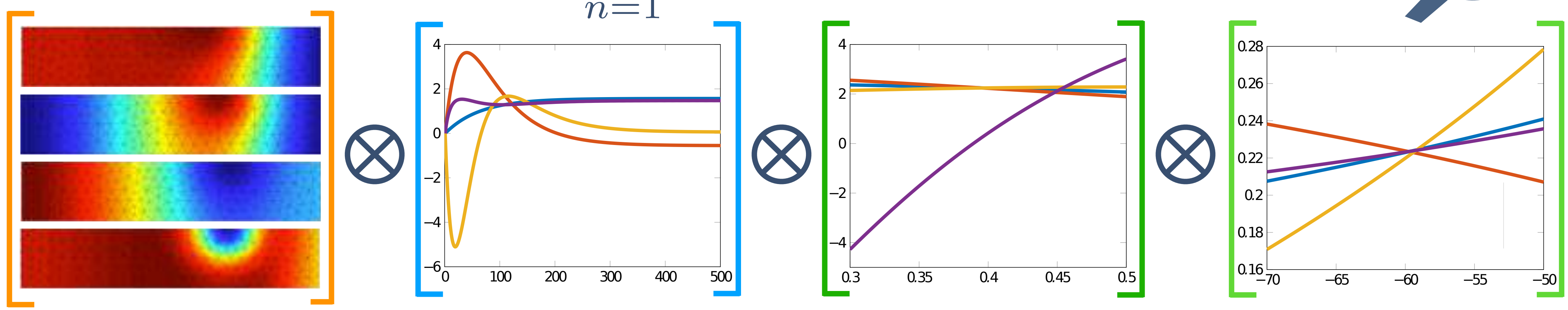
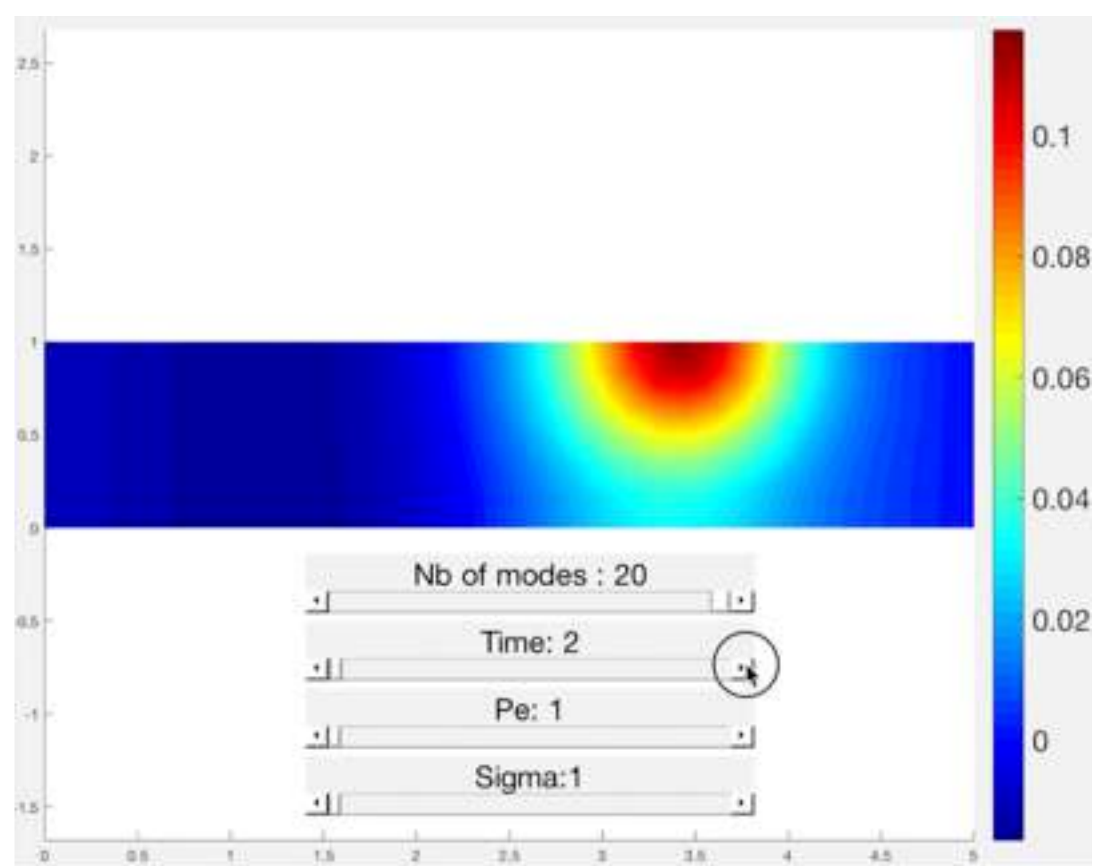
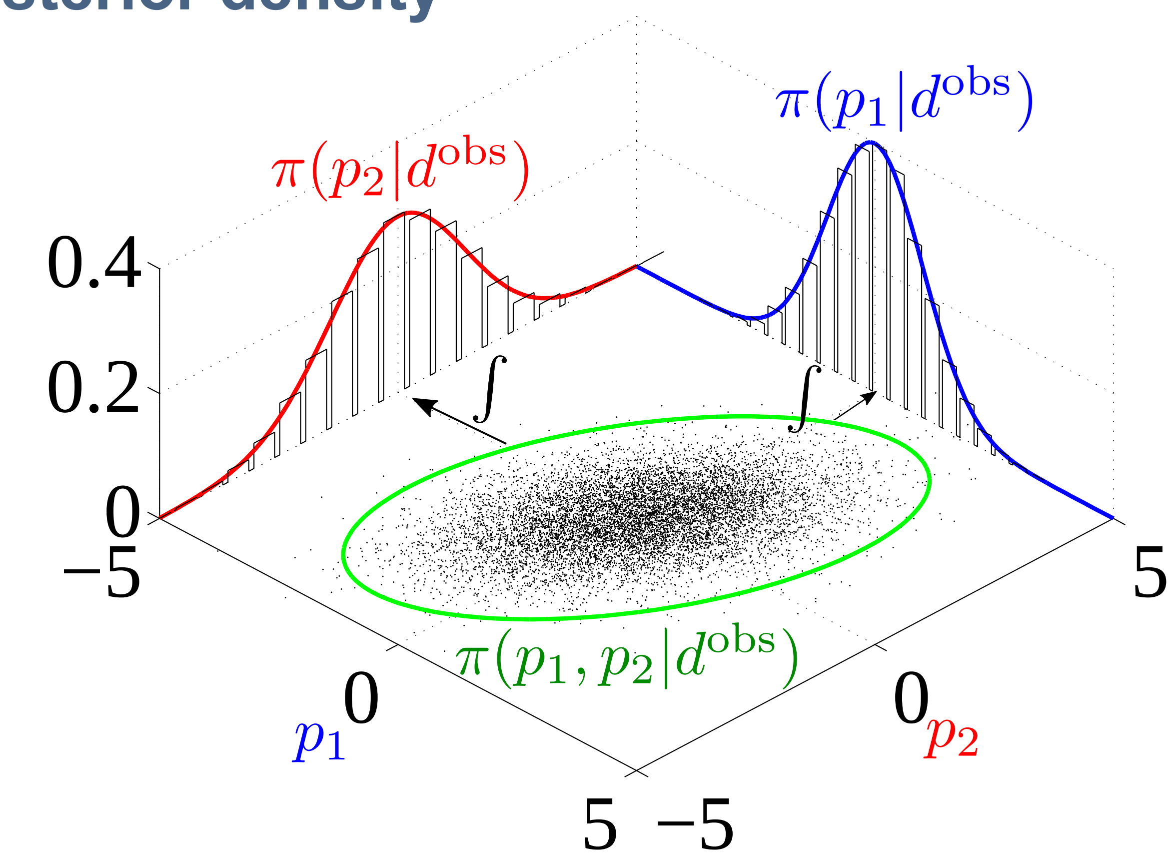




Sampling Posterior PDFs

- **Characterization/exploration of the posterior density**
  - Mean *a posteriori*
  - Maximum *a posteriori*
  - 1D Marginals
  - Uncertainty propagation



Sampling Posterior PDFs

Characterization/exploration of the posterior density

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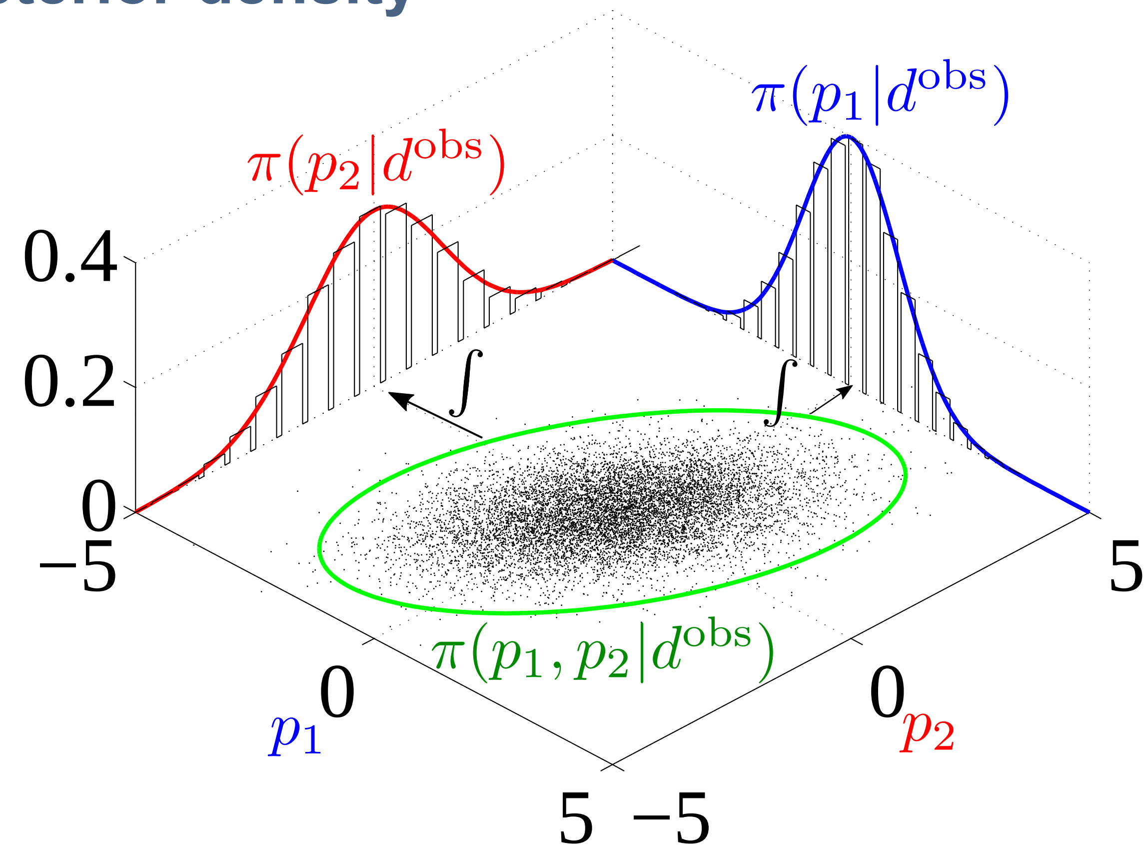
➡ Need multi-dimensional integration



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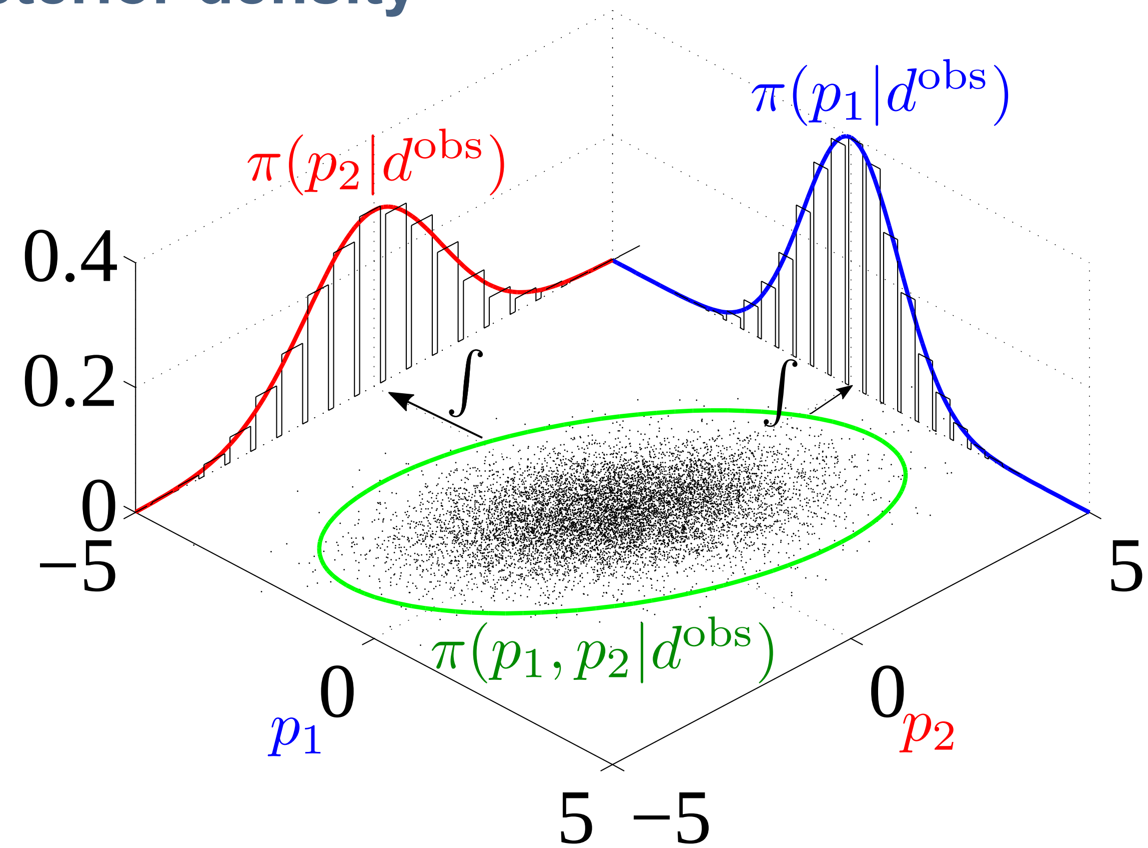

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- **Monte-Carlo integration:**
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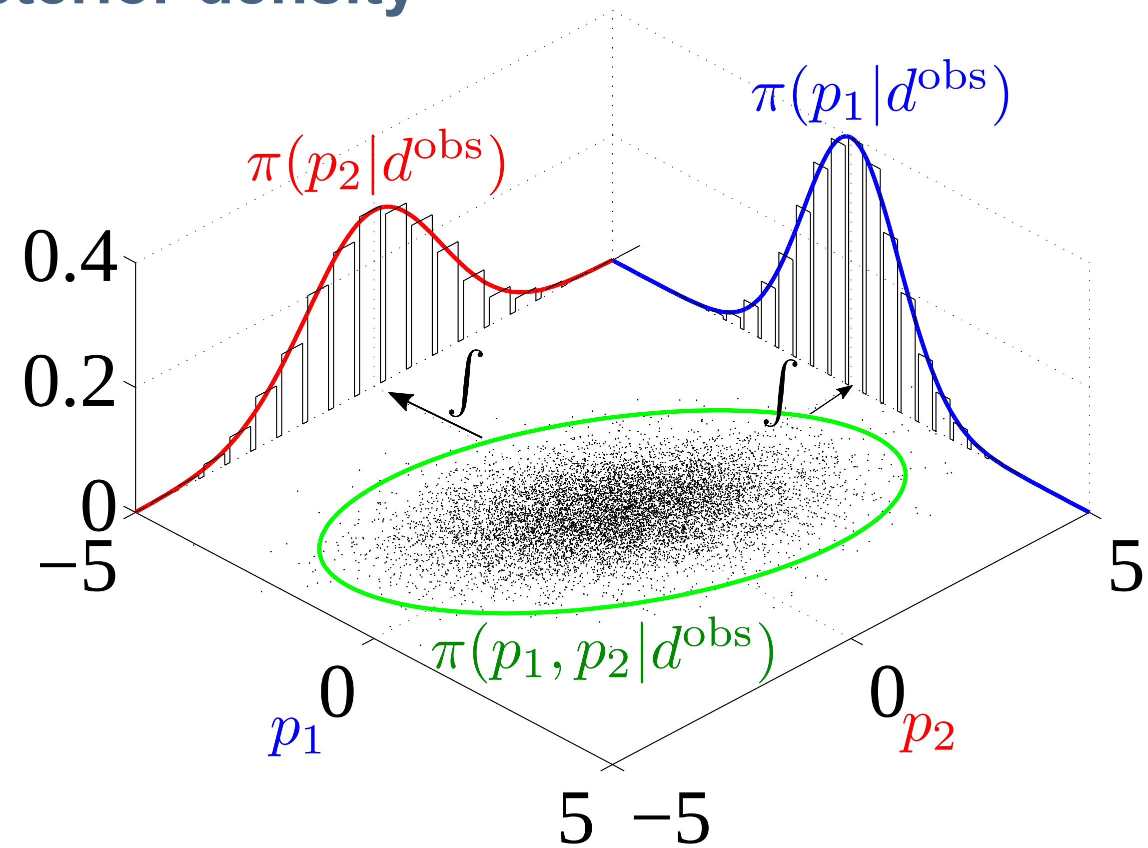

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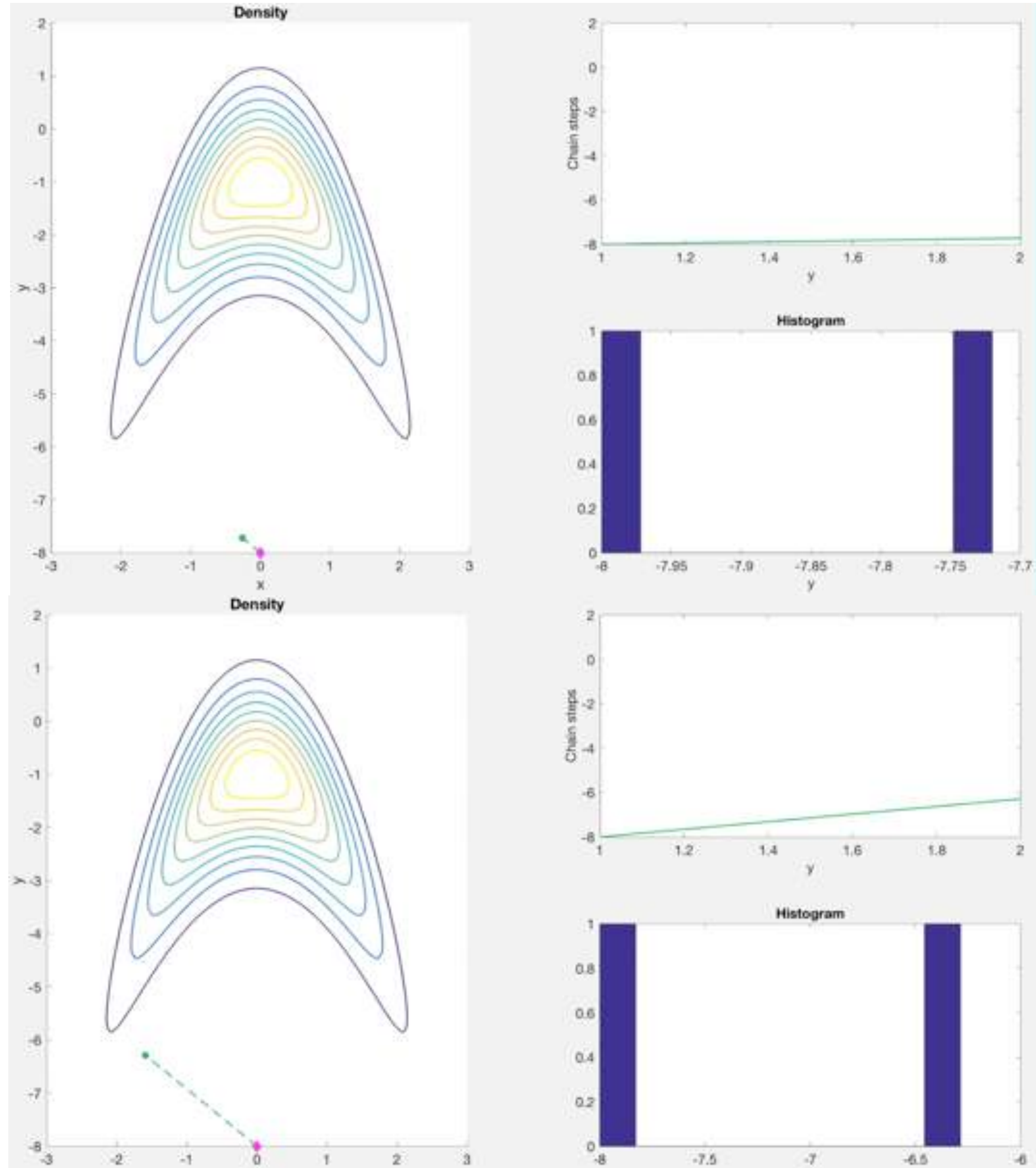
➡ Need samples from posterior density

Sampling Posterior PDFs

Markov Chain Monte-Carlo (MCMC) method

- **small** proposal

- **large** proposal



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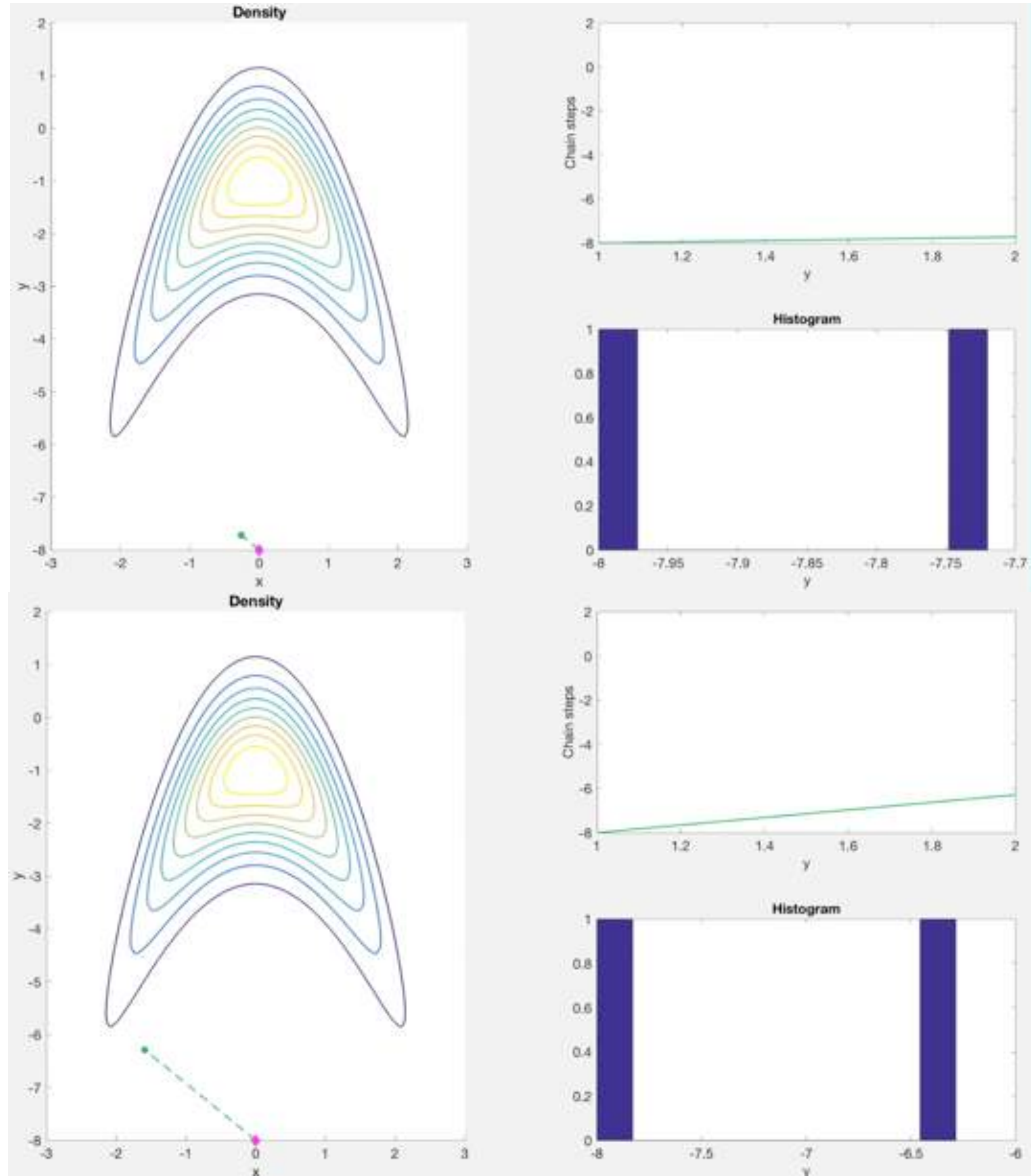

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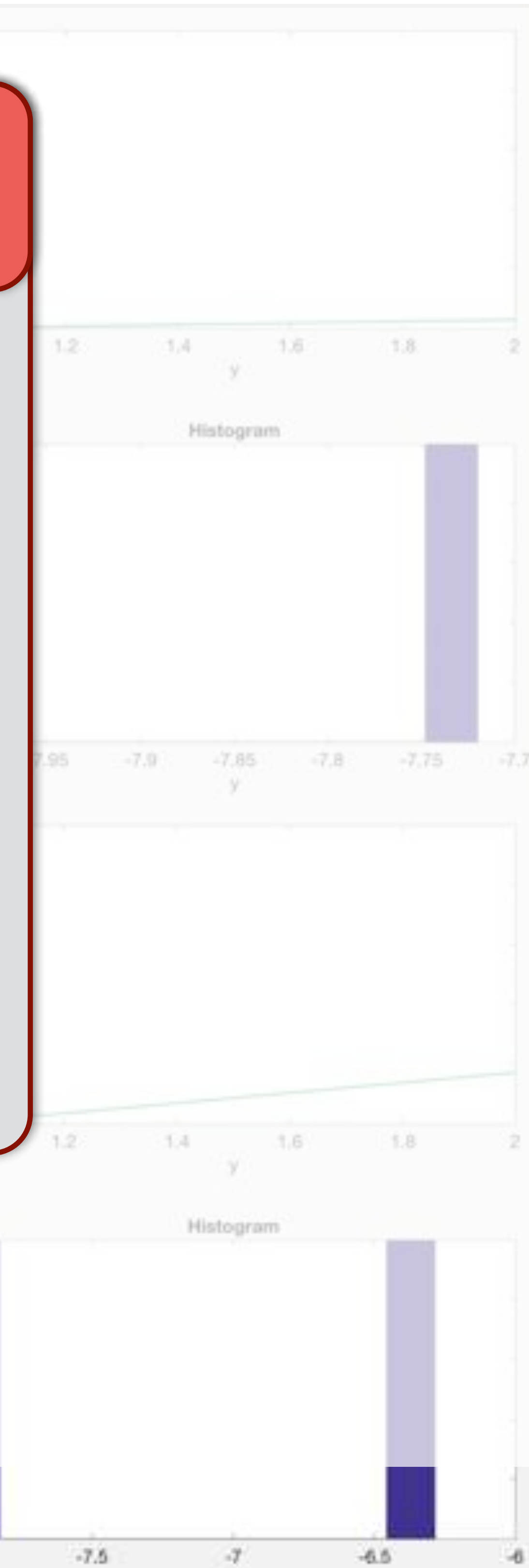


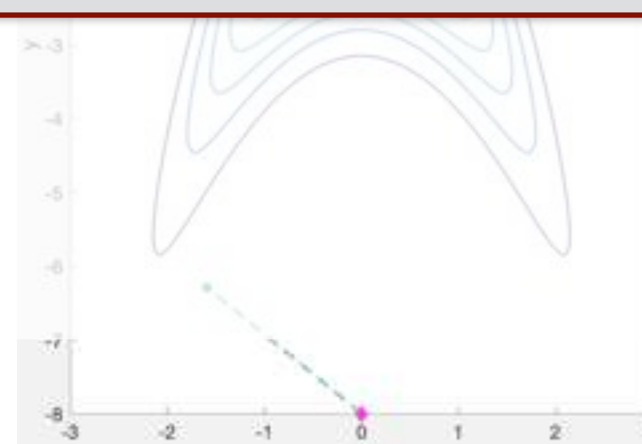
CONS

Set proposal density

Remove/reduce burn-in

Convergence/stopping criterion

Reuse of chains (sequential)



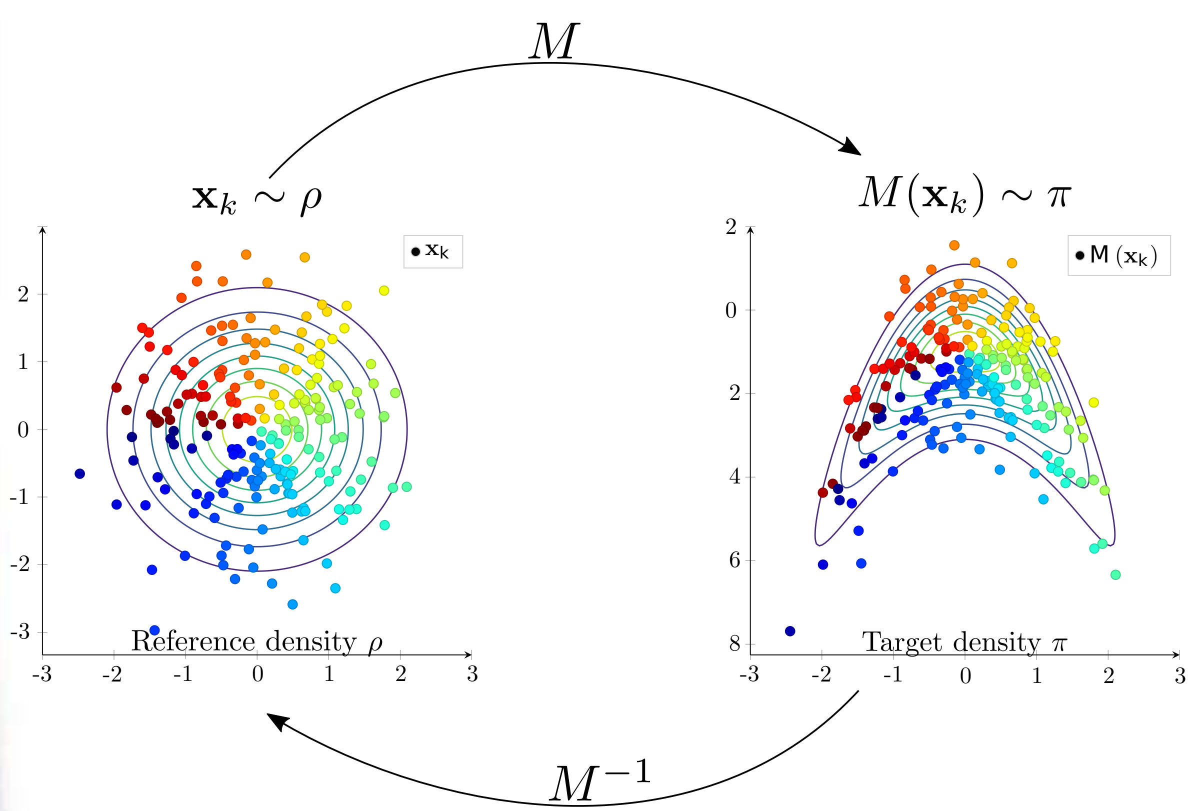
Transport Map Sampling

- **Transport Maps method** [Villani 07, El Moselhy & Marzouk 12]

Transport integrals over the **target** density to integrals over a **reference** density

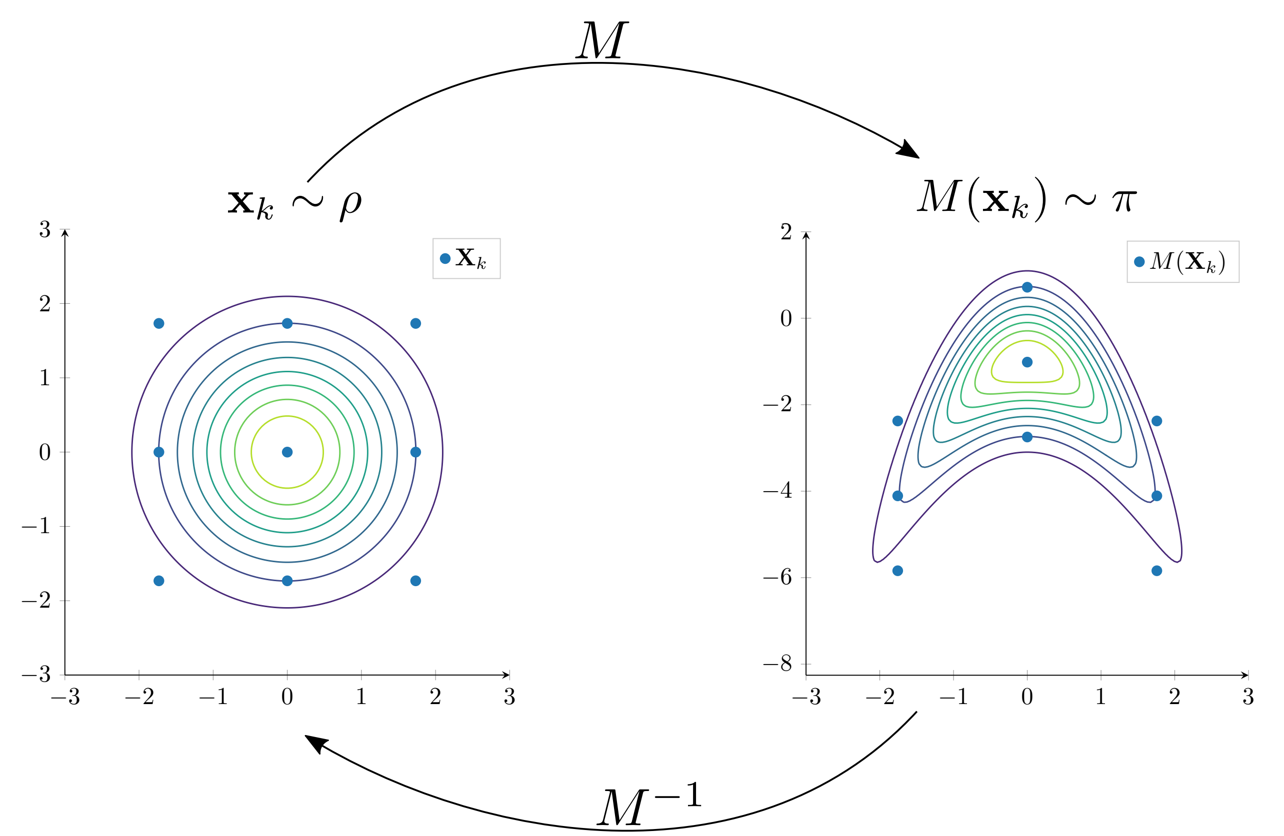
$$\boxed{\int g \mathrm{d}\nu_\pi = \int g \circ M \mathrm{d}\nu_\rho}$$

deterministic application



transport of samples

(pushing Gaussian samples through the map)



transport of quadrature rules

⟶ computations (sampling, integration) performed in the reference space [Marzouk 16]

Transport Map Sampling

Parametrization of Transport Maps

- **Structure:**

$$M(\mathbf{p}) = \begin{bmatrix} M^1(\mathbf{a}_c^1, \mathbf{a}_e^1, p_1) \\ M^2(\mathbf{a}_c^2, \mathbf{a}_e^2, p_1, p_2) \\ \vdots \\ M^d(\mathbf{a}_c^d, \mathbf{a}_e^d, p_1, p_2, ..., p_d) \end{bmatrix}$$

Knothe-Rosenblatt rearrangements (lower triangular monotonic maps)

- unique minimizer
- computational feasibility (inversion)
- optimality for a weighted metric
  [El Moselhy & Marzouk 12]
  [Papamakarios *et al.* 19]

- **Parametrization:**

$$M^k(\mathbf{a}_c^k, \mathbf{a}_e^k, \mathbf{p}) = \Phi_c(\mathbf{p})\mathbf{a}_c^k + \int_0^{p_k} (\Phi_e(p_1, ..., p_{k-1}, \theta)\mathbf{a}_e^k)^2 \mathrm{d}\theta$$

Φ_c, Φ_e : Hermite polynomials which given order

$\mathbf{a}_c, \ \mathbf{a}_e$: parameters

→ obtained from miminization of Kullback-Liebler divergence

$$\boxed{\mathcal{D}_{KL}(M_\sharp \nu_\rho || \nu_\pi) = \mathbb{E}_\rho \left[\log \frac{\nu_\rho}{M_\sharp^{-1} \nu_\pi} \right]}$$

push forward operator

requires unnormalized pdfs alone

Transport Map Sampling

- **Minimization problem**

$$\min_{\mathbf{a}_c^{1,\dots,d},\mathbf{a}_e^{1,\dots,d}} \sum_{i=1}^{N} \omega_i \left[-\log(\tilde{\pi} \circ M(\mathbf{a}_c^{1,\dots,d}, \mathbf{a}_e^{1,\dots,d}, \mathbf{p}_i) - \log(|\det \nabla M(\mathbf{a}_c^{1,\dots,d}, \mathbf{a}_e^{1,\dots,d}, \mathbf{p}_i))|)\right]$$

$\tilde{\pi}(\mathbf{p}|\mathbf{d}^{\text{obs}}) = \pi_{\text{meas}}(\mathbf{d}^{\text{obs}} - u_m(\mathbf{p}))\pi(\mathbf{p})$ (non-normalized pdf)

→ solved with gradient/Hessian information (BFGS,…)

→ partial derivatives explicitly recovered and stored in the **offline phase**

$$\frac{\partial^n \mathbf{u}_m}{\partial p_j^n}(\mathbf{x}, t, \mathbf{p}) = \sum_{k=1}^{m} \mathbf{\Lambda}_k(\mathbf{x})\lambda_k(t)\frac{\partial^n \alpha_k^j}{\partial p_j^n}(p_j)\prod_{\substack{i=1\\i\neq j}}^{d}\alpha_k^i(p_i)$$

→ large speed-up for the computation of maps!!! maps [Rubio *et al.* 19]

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- **Variance diagnostic** [Spantini *et al.* 18]

$$\epsilon_\sigma = \frac{1}{2}\mathbb{V}\text{ar}_\rho\left[\ln \frac{\nu_\rho}{M_\sharp^{-1}\nu_\pi}\right]$$

- sampling error estimate
- clear convergence criterion
- adaptive strategy on map order

Transport Map Sampling

- **Sequential updating (composition of maps)**

Transport Map Sampling

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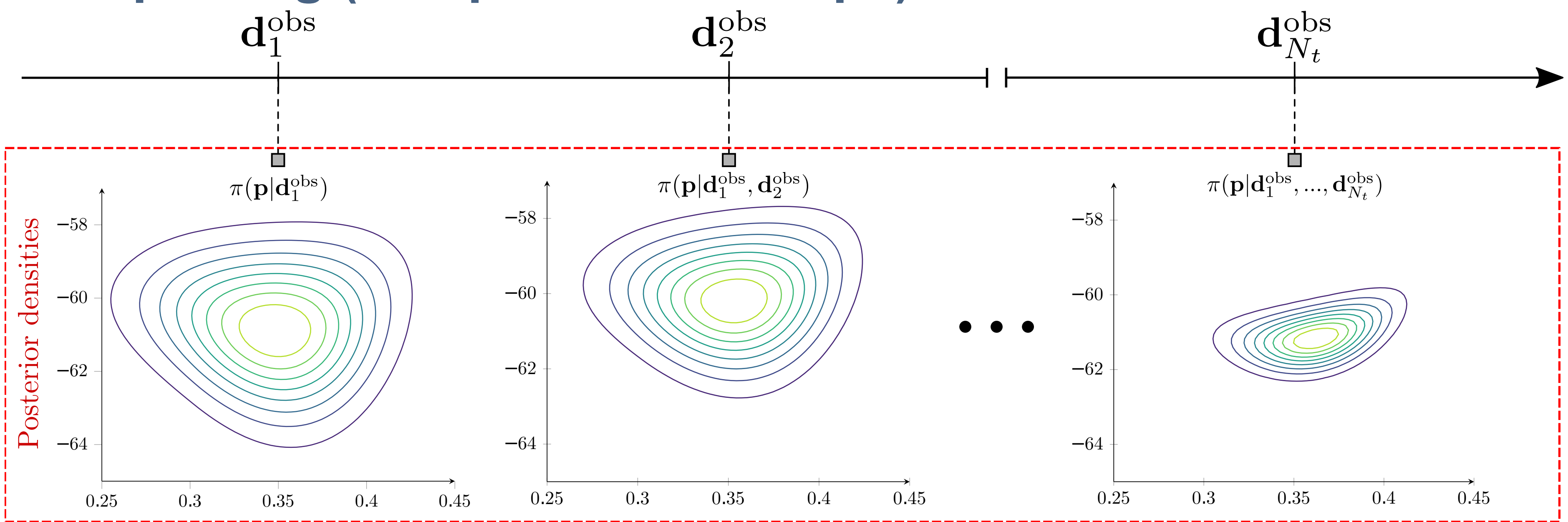

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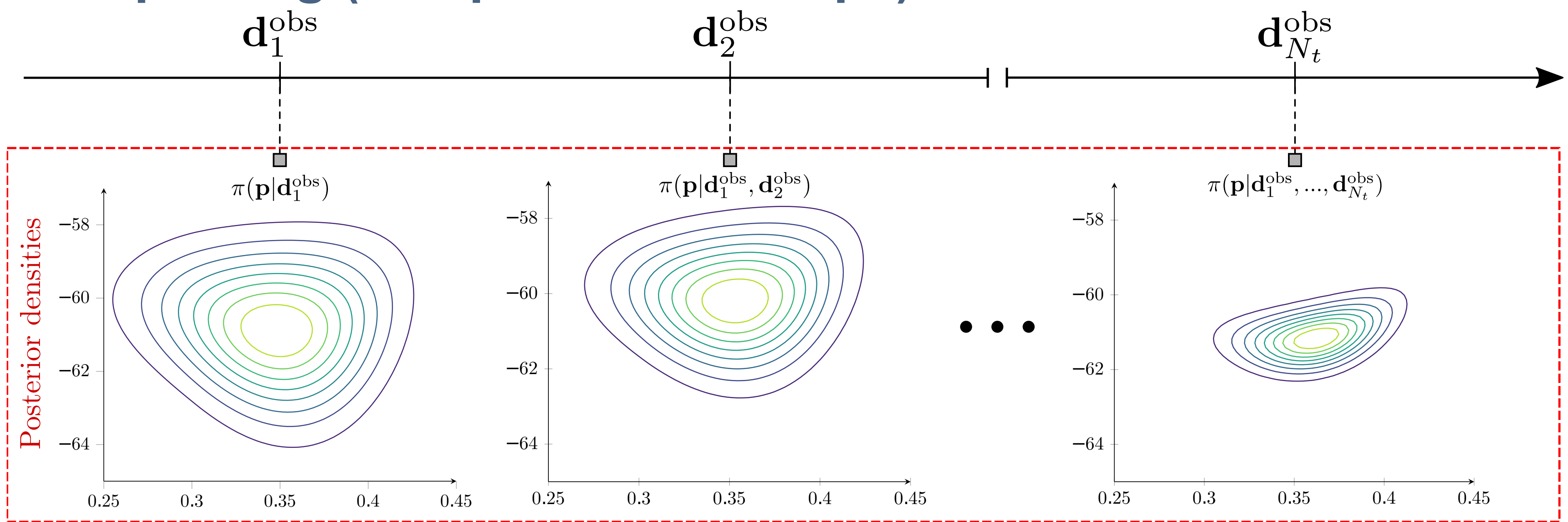
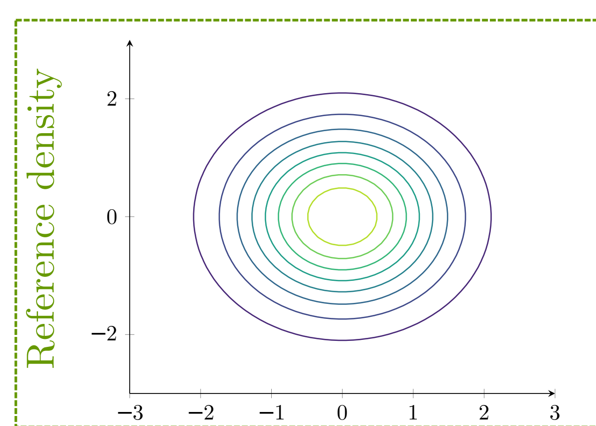
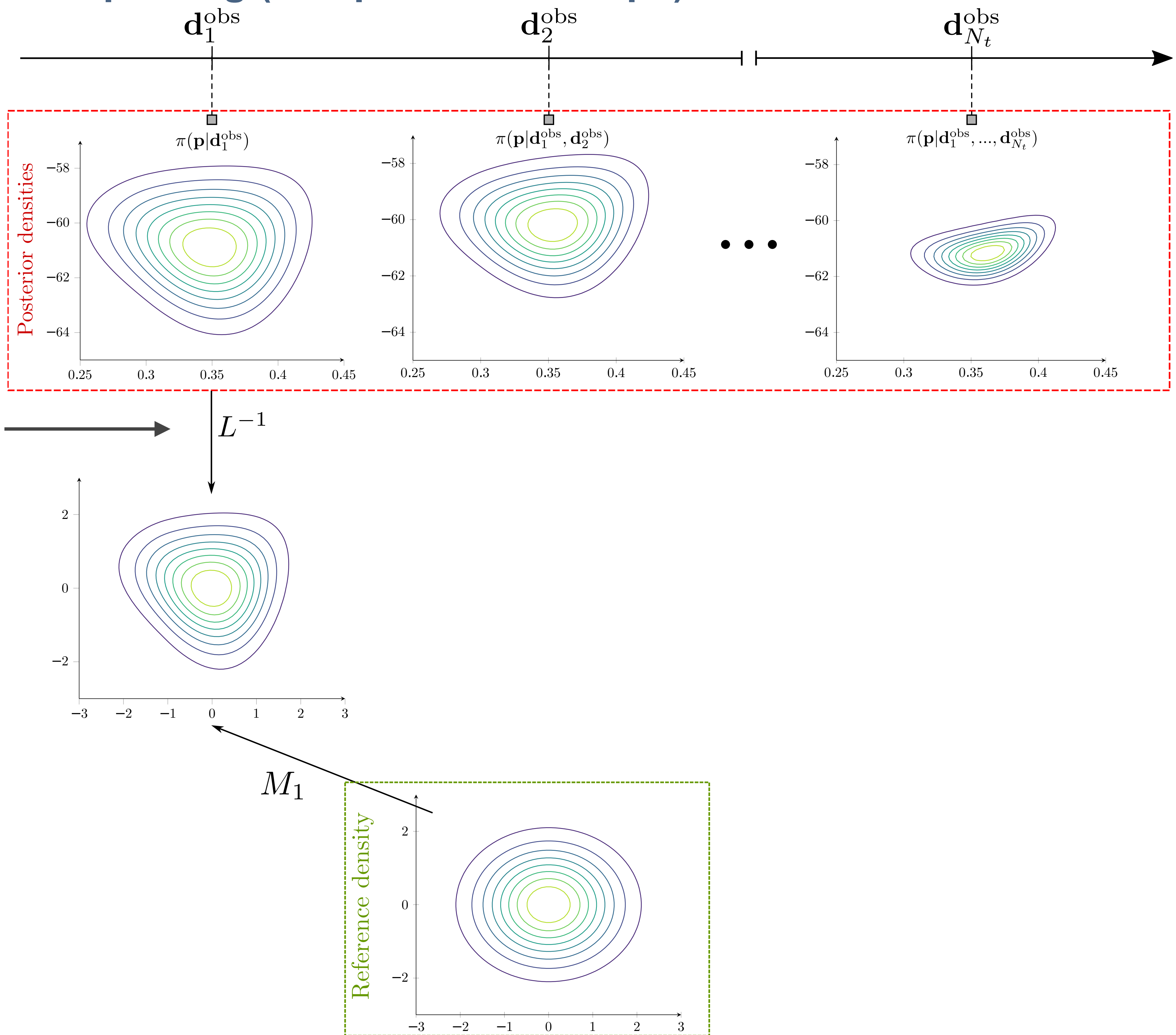




Transport Map Sampling

- **Sequential updating (composition of maps)**

Laplace approximation *(linear map for normalization & local Gaussian approximation from gradient and Hessian)*



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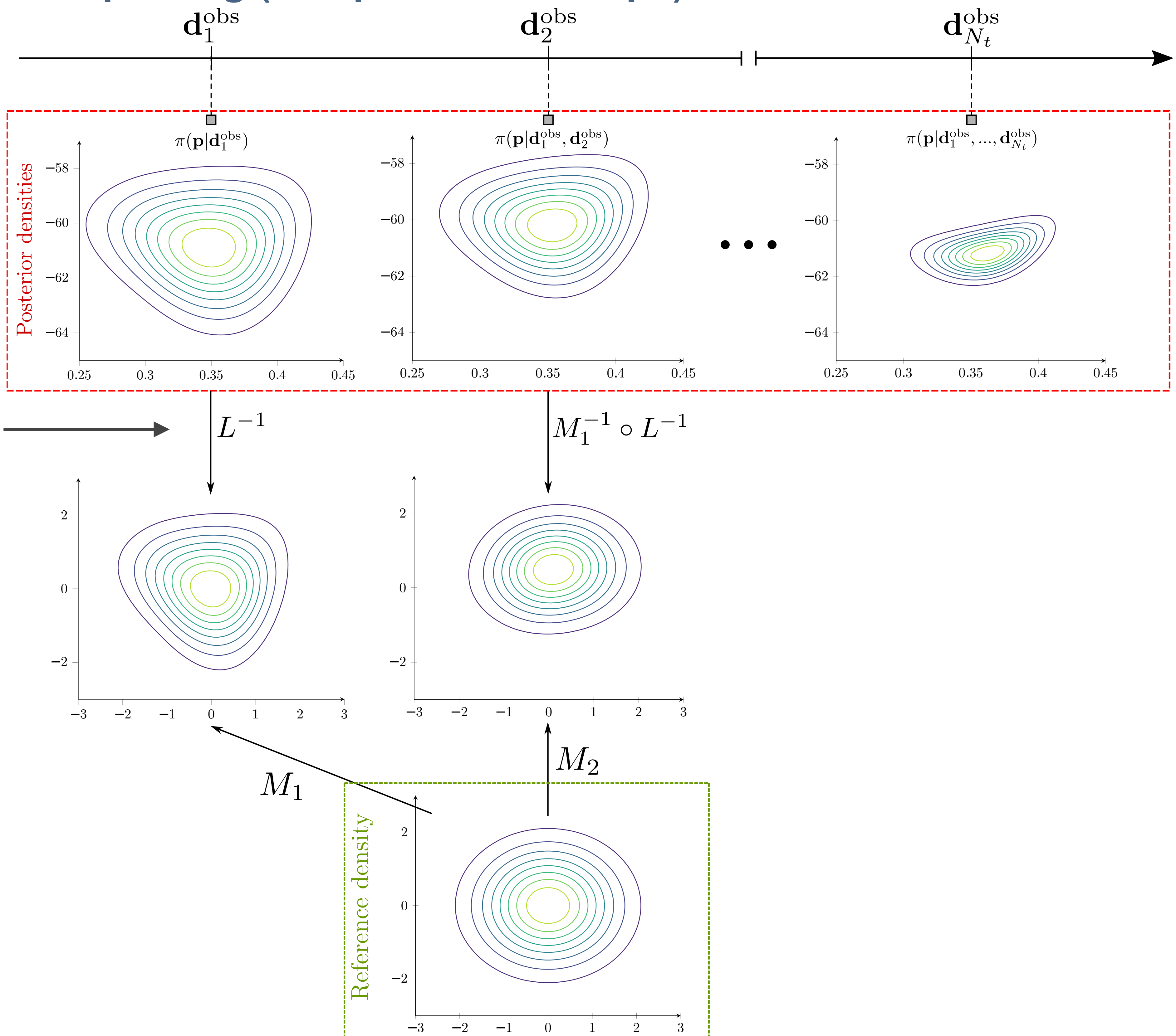

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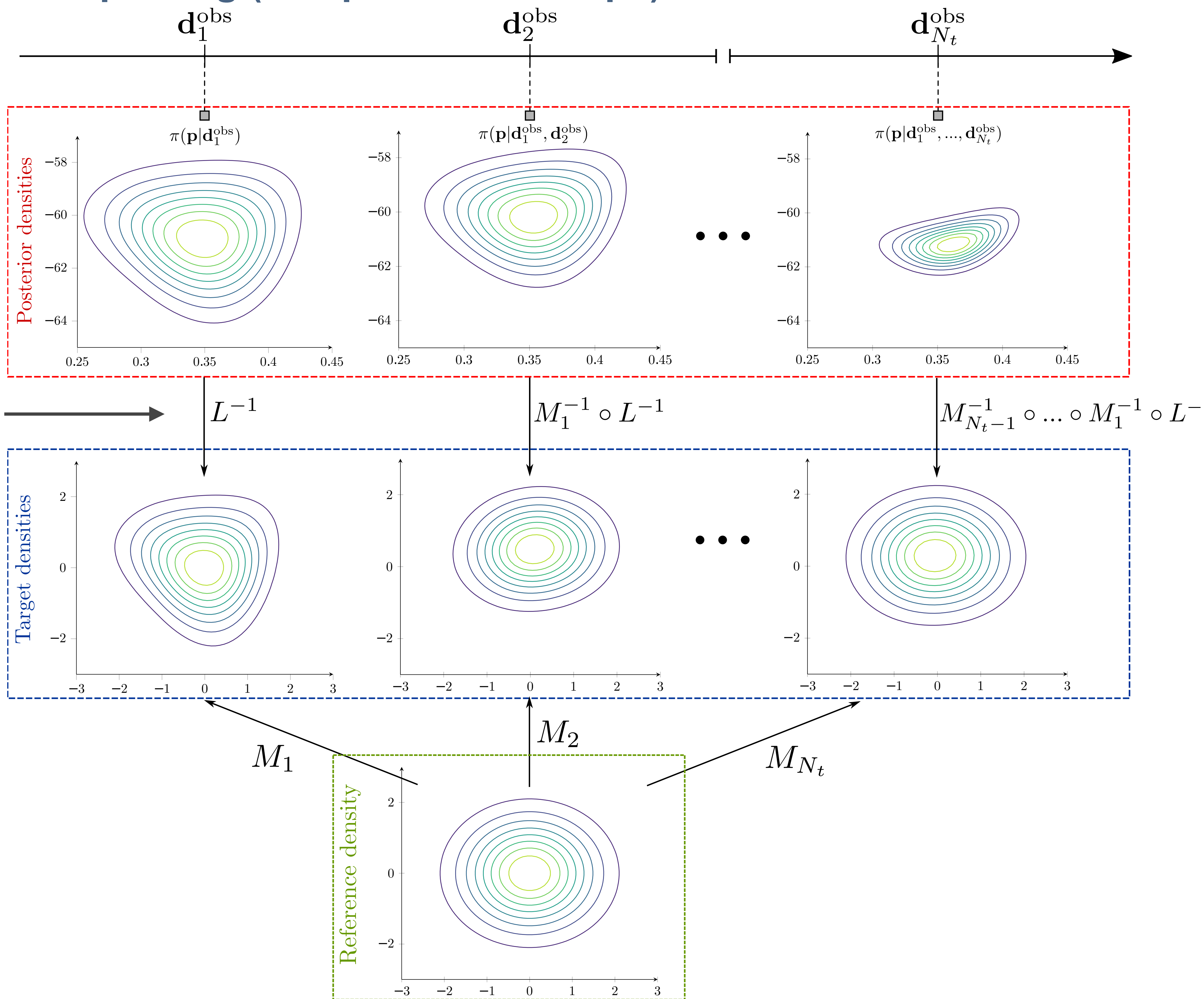

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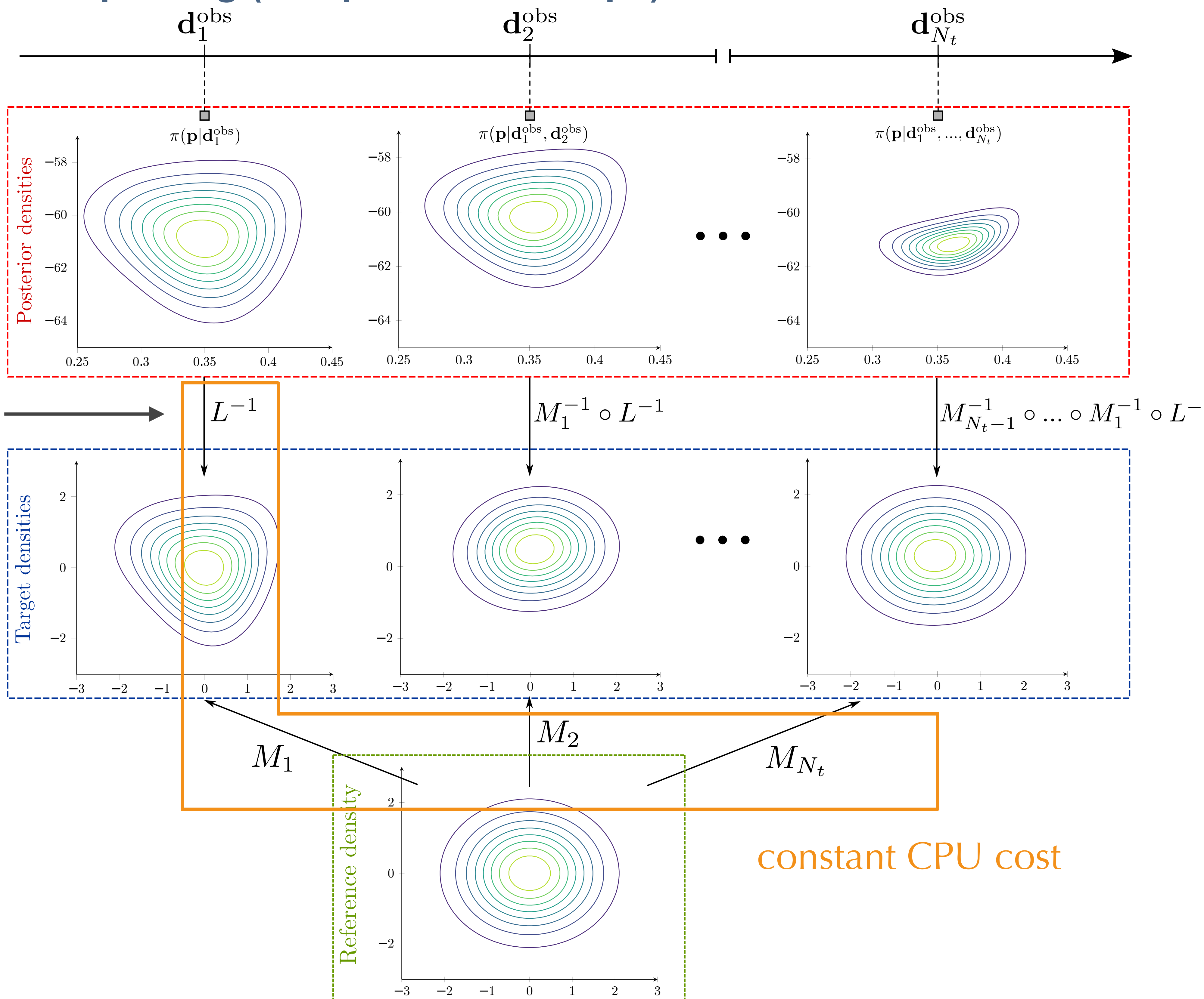

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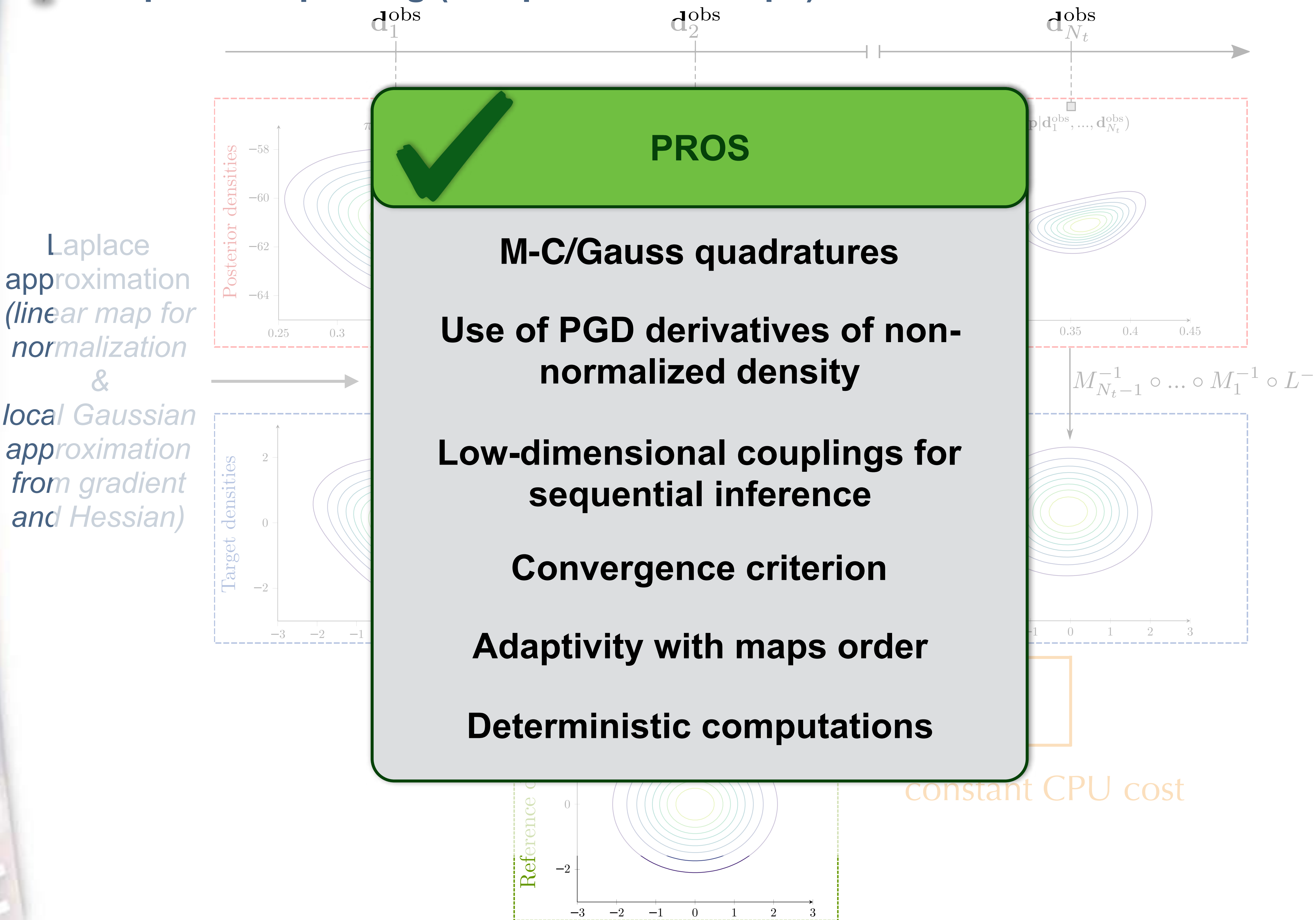


PROS

M-C/Gauss quadratures

Use of PGD derivatives of non-normalized density

Low-dimensional couplings for sequential inference

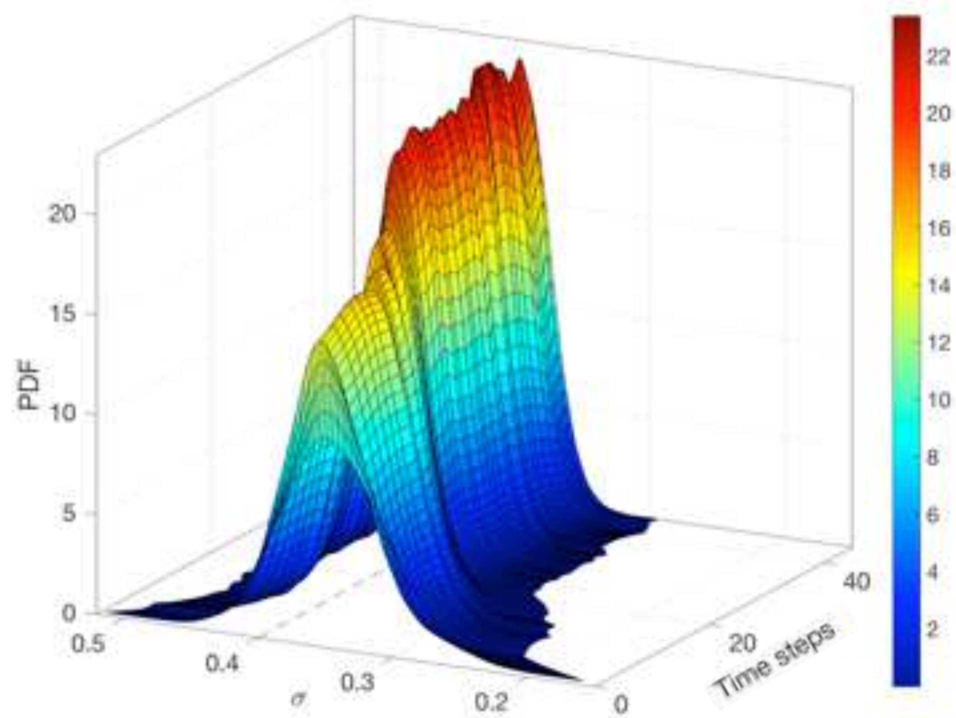
Convergence criterion

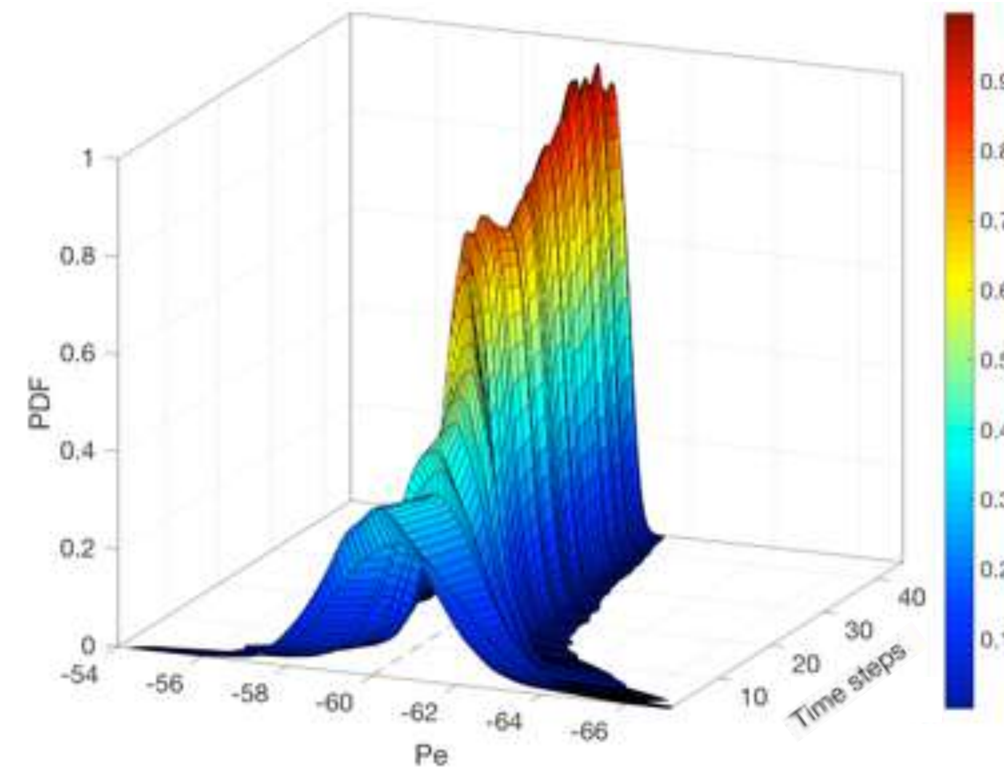
Adaptivity with maps order

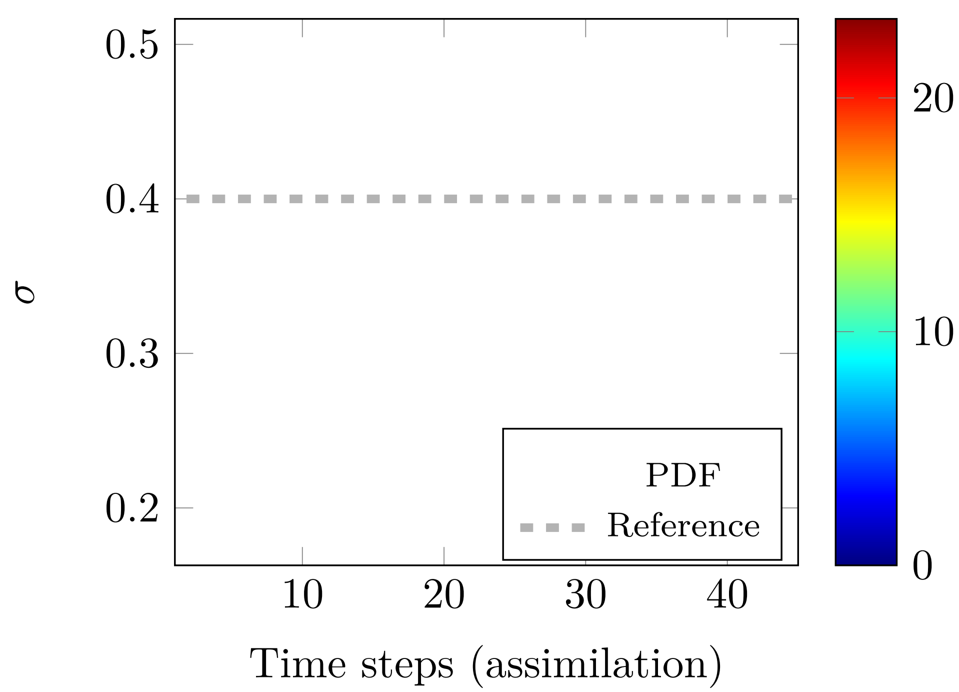
Deterministic computations

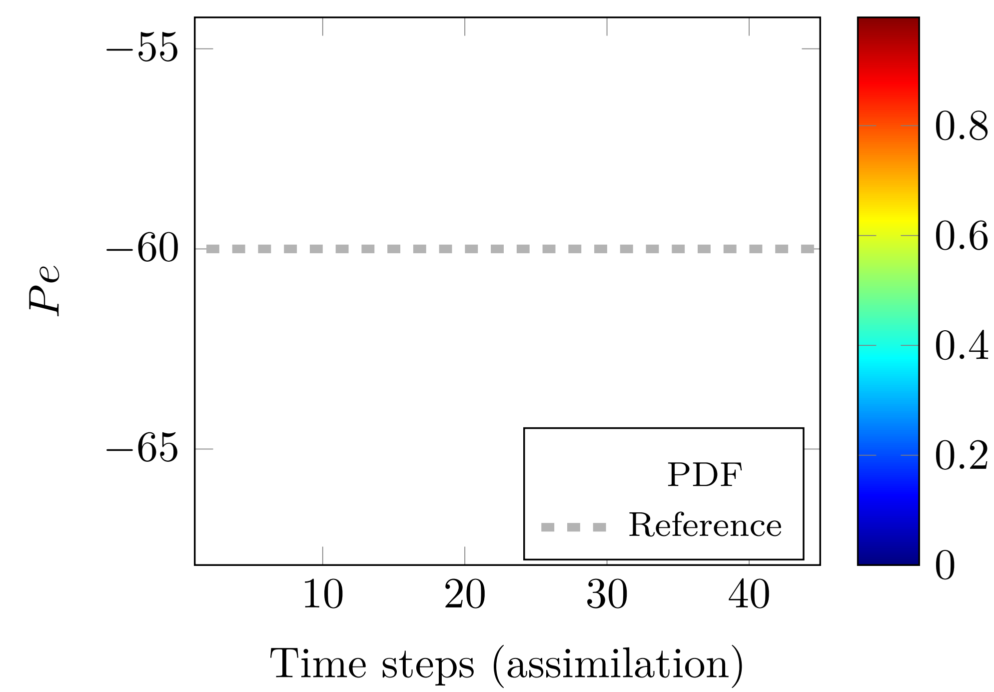
Application

- Post-processing: marginals densities



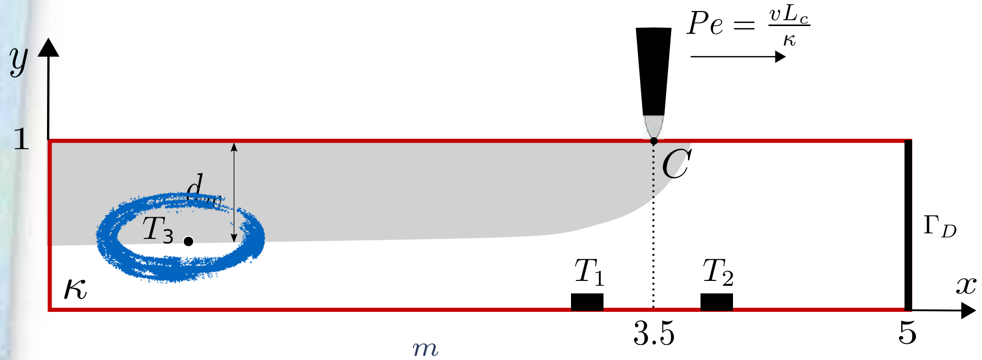






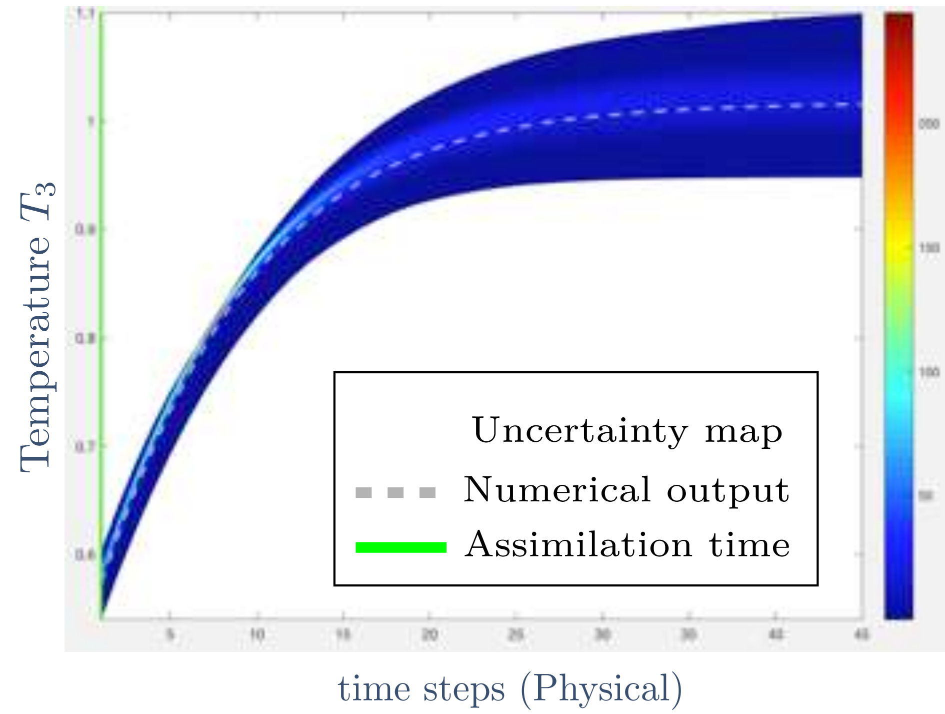
Application

Post-processing: prediction with uncertainty propagation



$$T_3(x_3, y_3, t, \sigma^j, Pe^j) = \sum_{n=1}^{m} \Lambda_n(x_3, y_3)\lambda_n(t)\alpha_n^1(\sigma^j)\alpha_n^2(Pe^j)$$

(good weld indicator)



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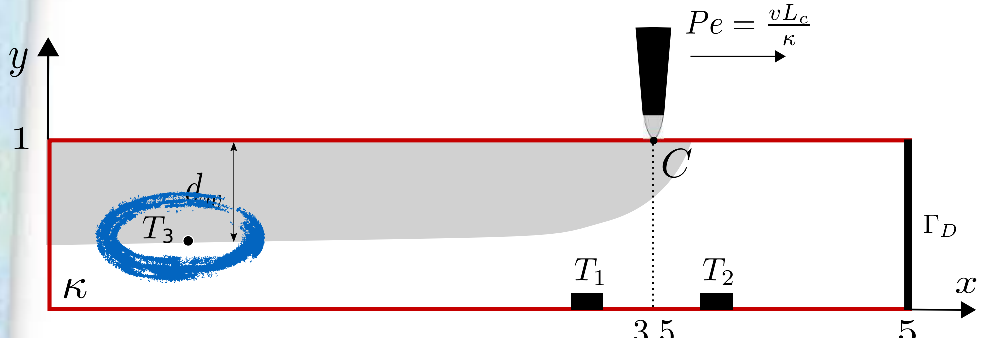

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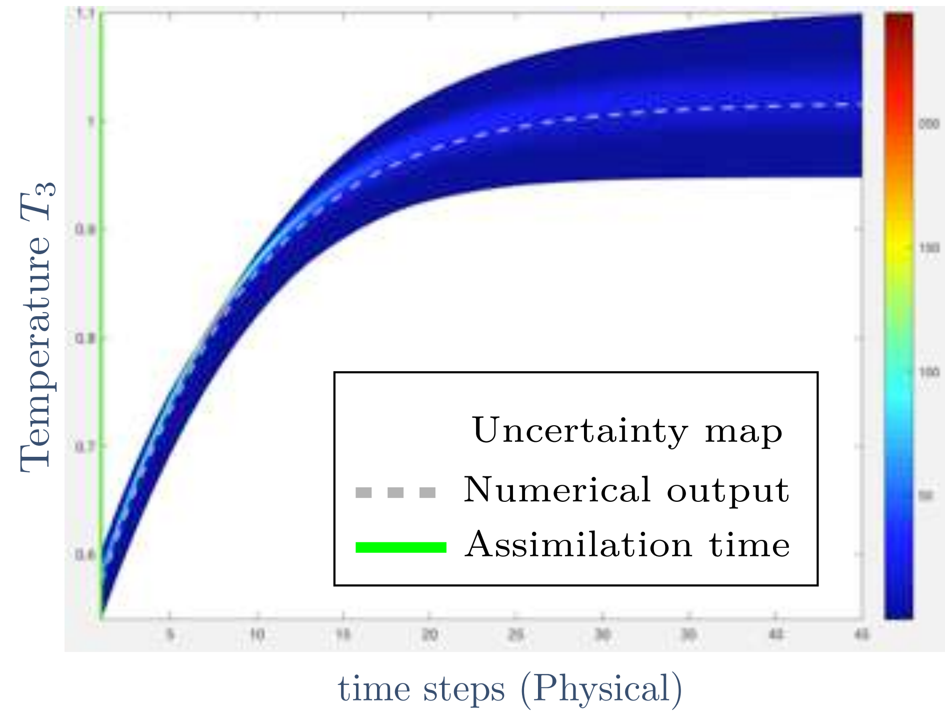
assimilation+prediction CPU time $\sim 0.1s$



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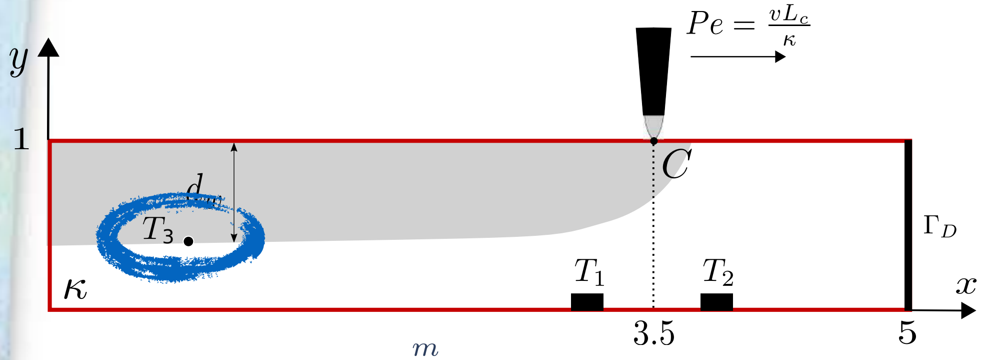

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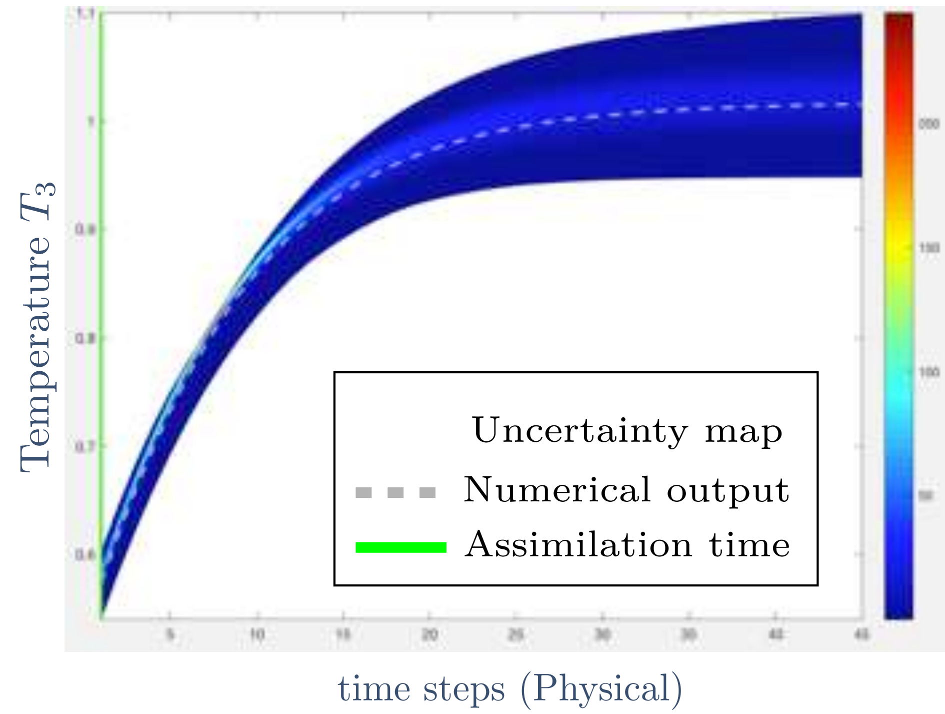

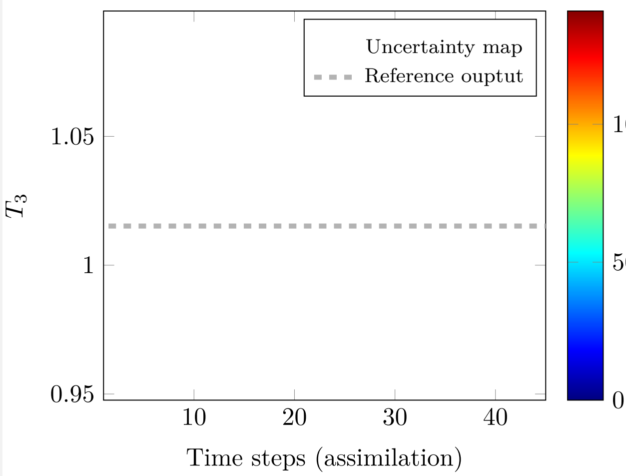

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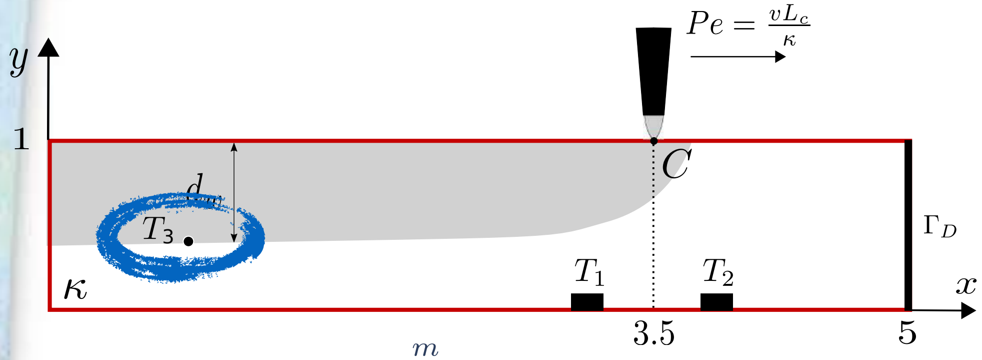

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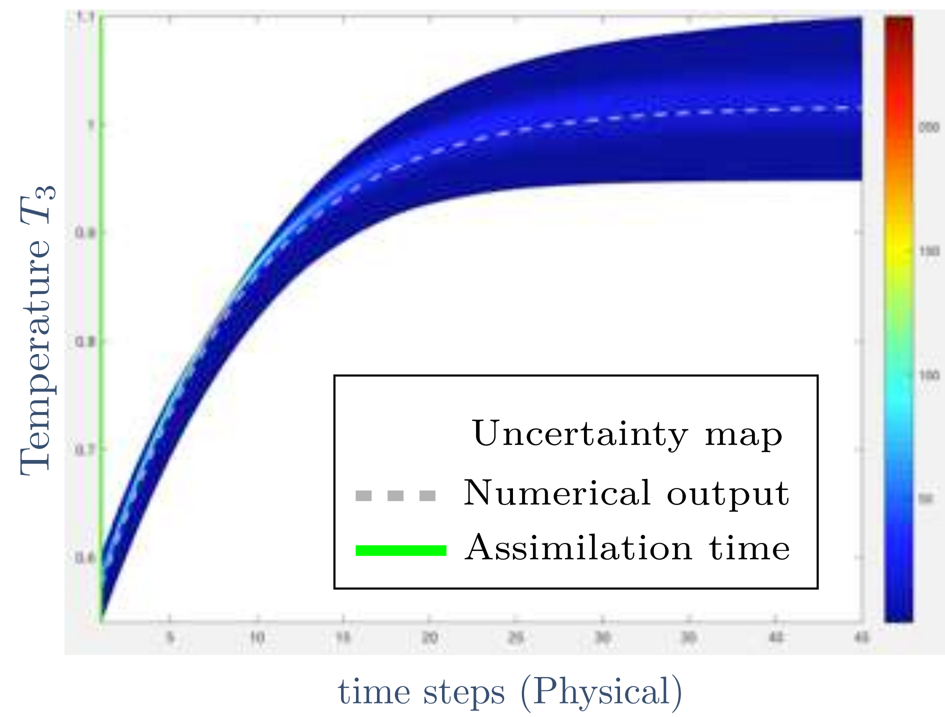
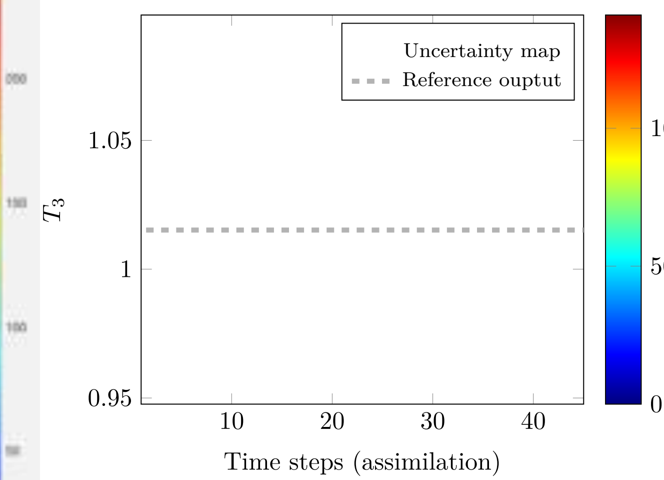
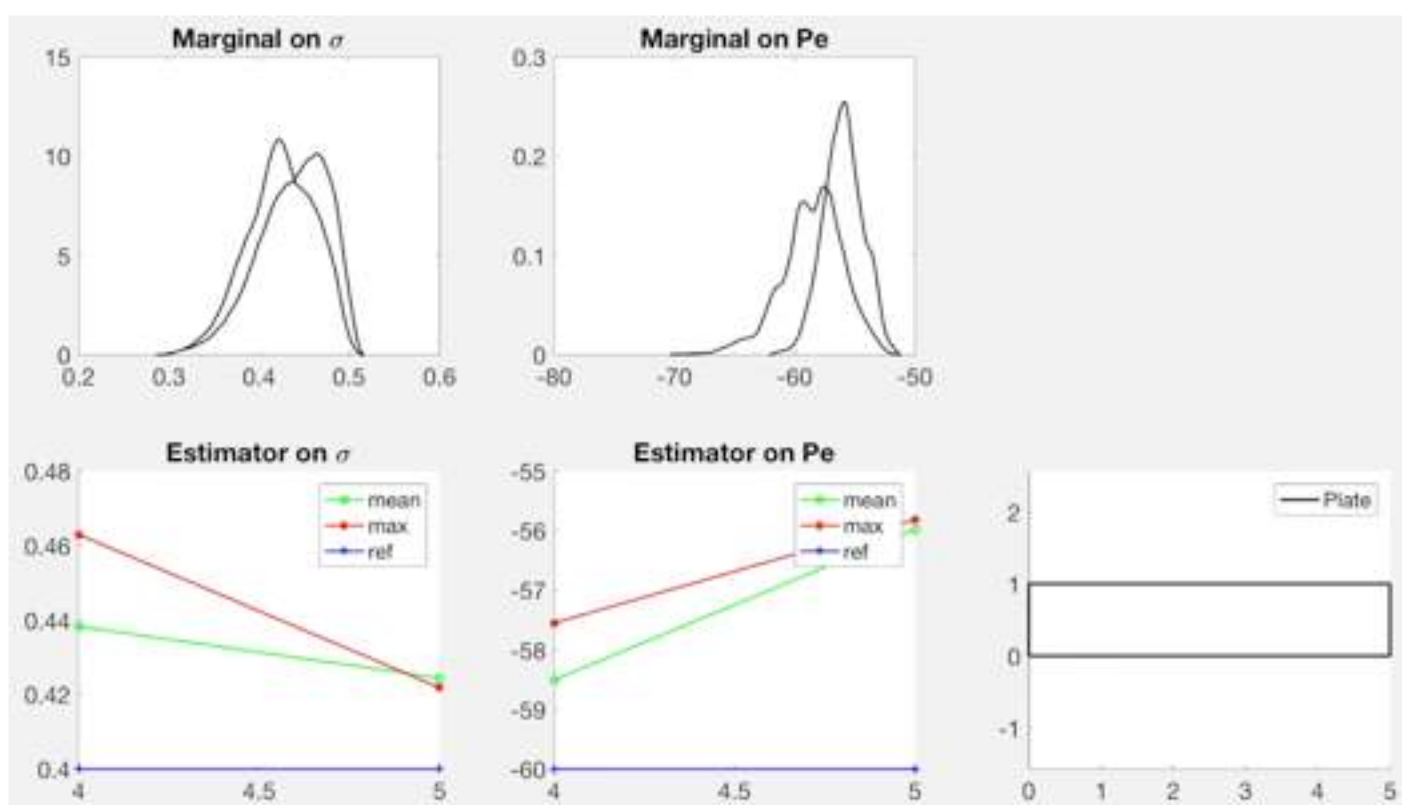


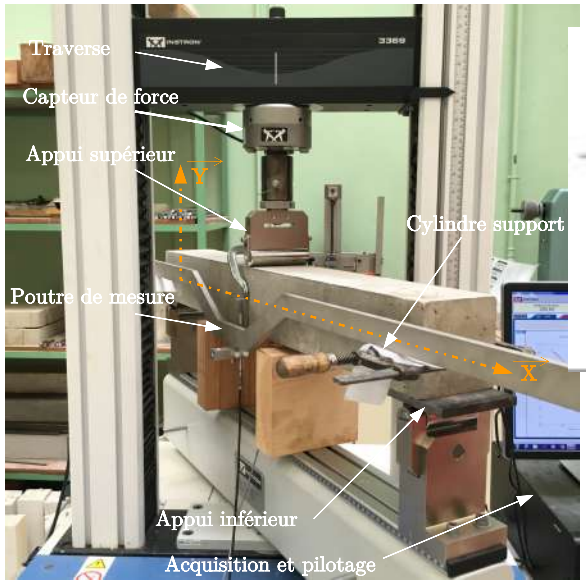


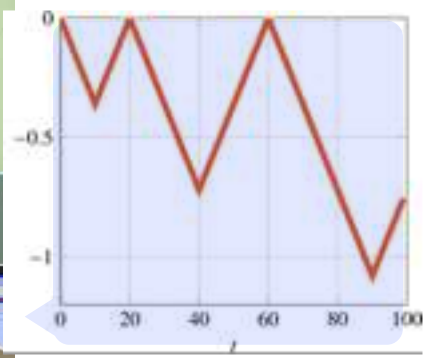
sequential estimation of the welding zone depth from the updated numerical model



Real-case Illustration

Structural integrity on a large-scale damageable concrete structure



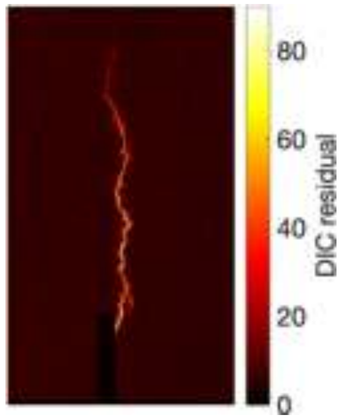


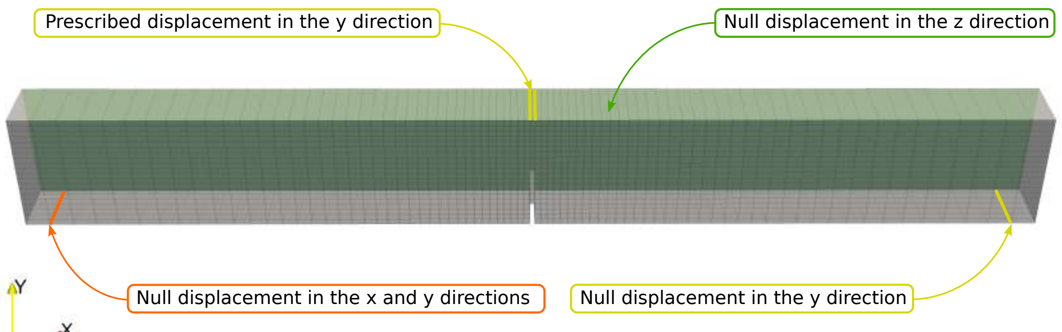
Ditical Image Correlation (DIC)

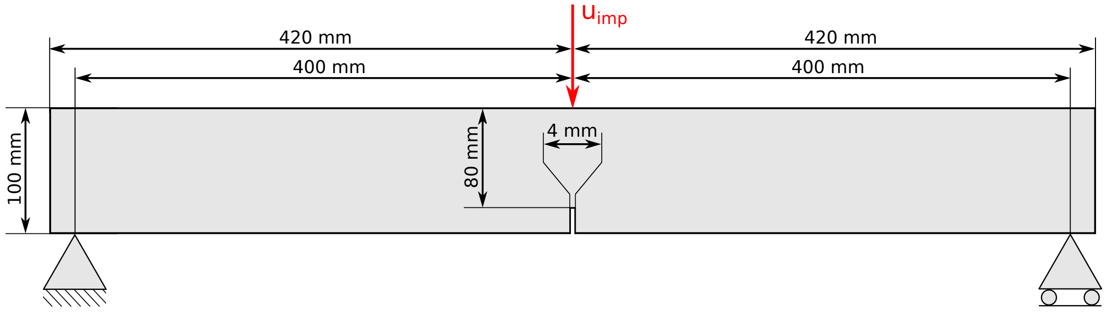
→ DIC pictures taken every 5s, and post-processed with Corelli [Leclerc *et al.* 15]

→ prediction of crack propagation & failure (before the physics!!)

→ simulator using an isotropic damage model







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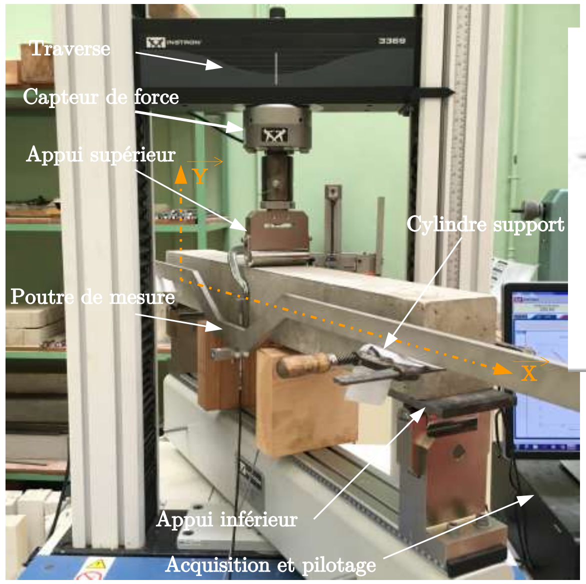

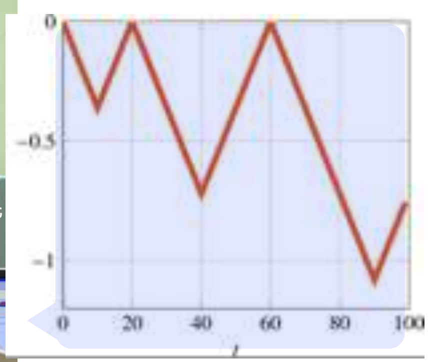


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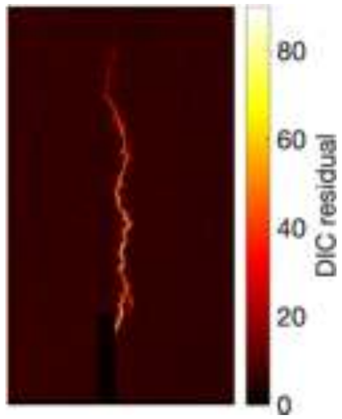
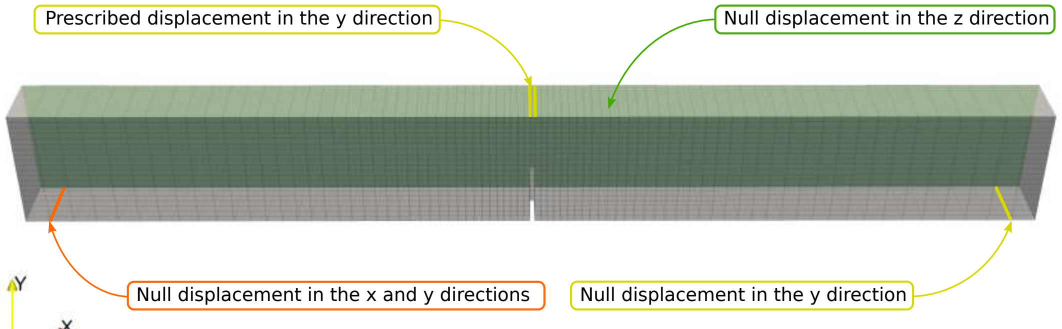
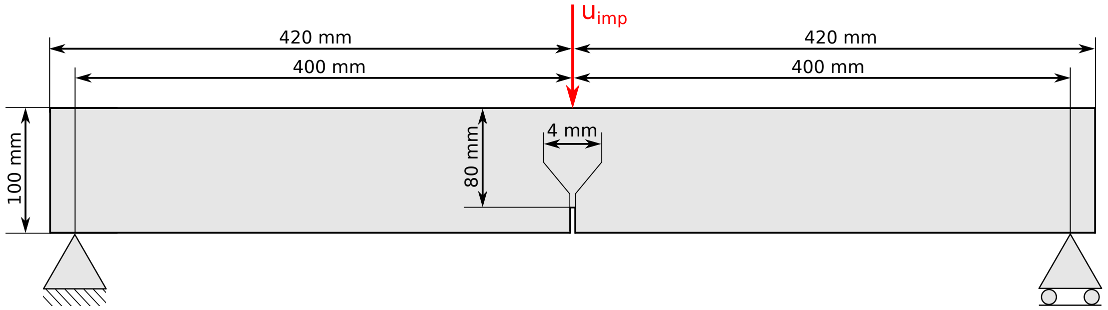






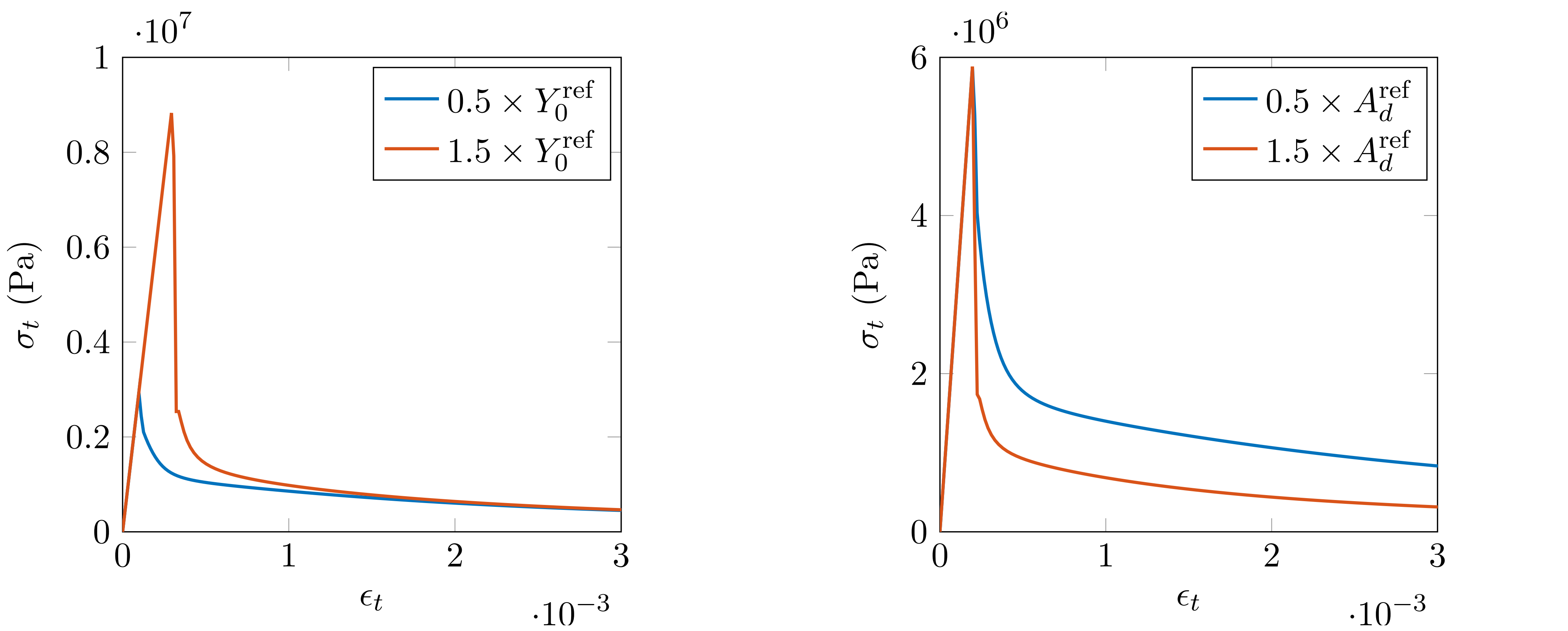
Objectives

$$\boxed{\sigma = (1-d)\mathbf{C}\epsilon \quad ; \quad d(Y, A_d, Y_0) = 1 - \frac{1}{1 + A_d(Y - Y_0)}}$$

$Y = \frac{1}{2}\langle\epsilon\rangle_+ : \mathbf{C} : \langle\epsilon\rangle_+$ released energy

Y_0 initial threshold for damage initiation

A_d scalar brittleness (post-peak behavior)



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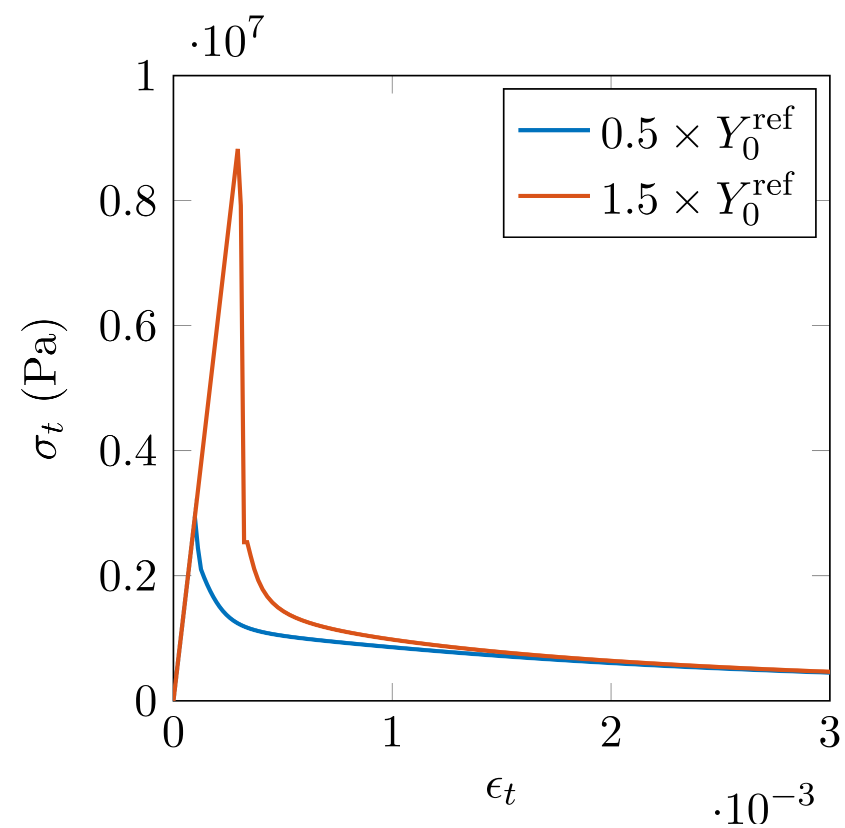
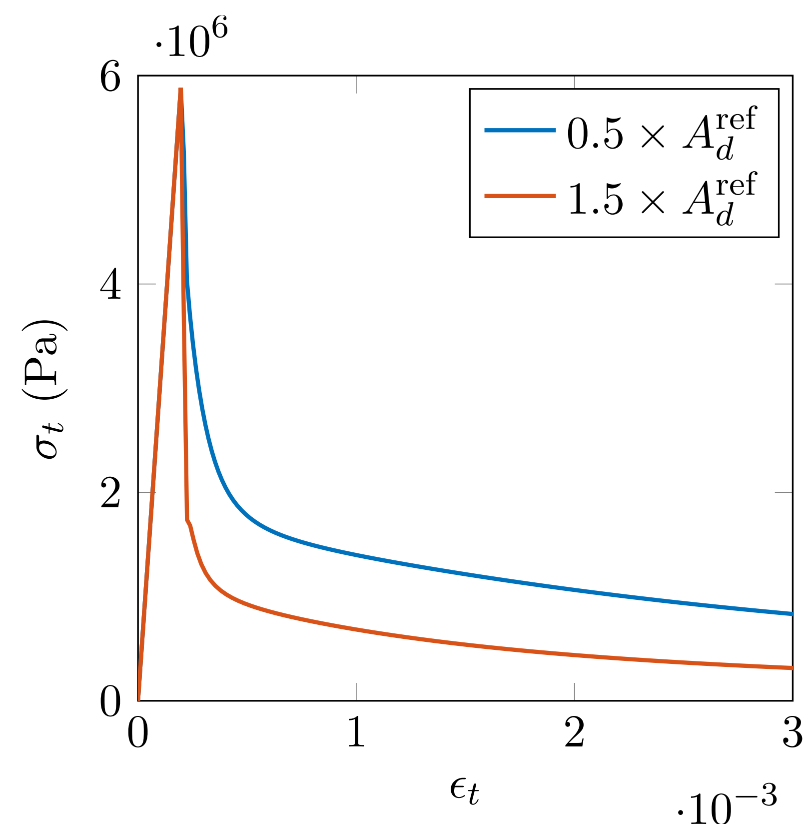




→ **updating of parameters (Y_0, A_d) from data**

PGD Modes

$$\boxed{\mathbf{u}_m(\mathbf{x}, t, Y_0, A_d) = \sum_{k=1}^{m} \mathbf{\Lambda}_k(\mathbf{x})\lambda_k(t)\alpha_k^1(Y_0)\alpha_k^2(A_d)}$$

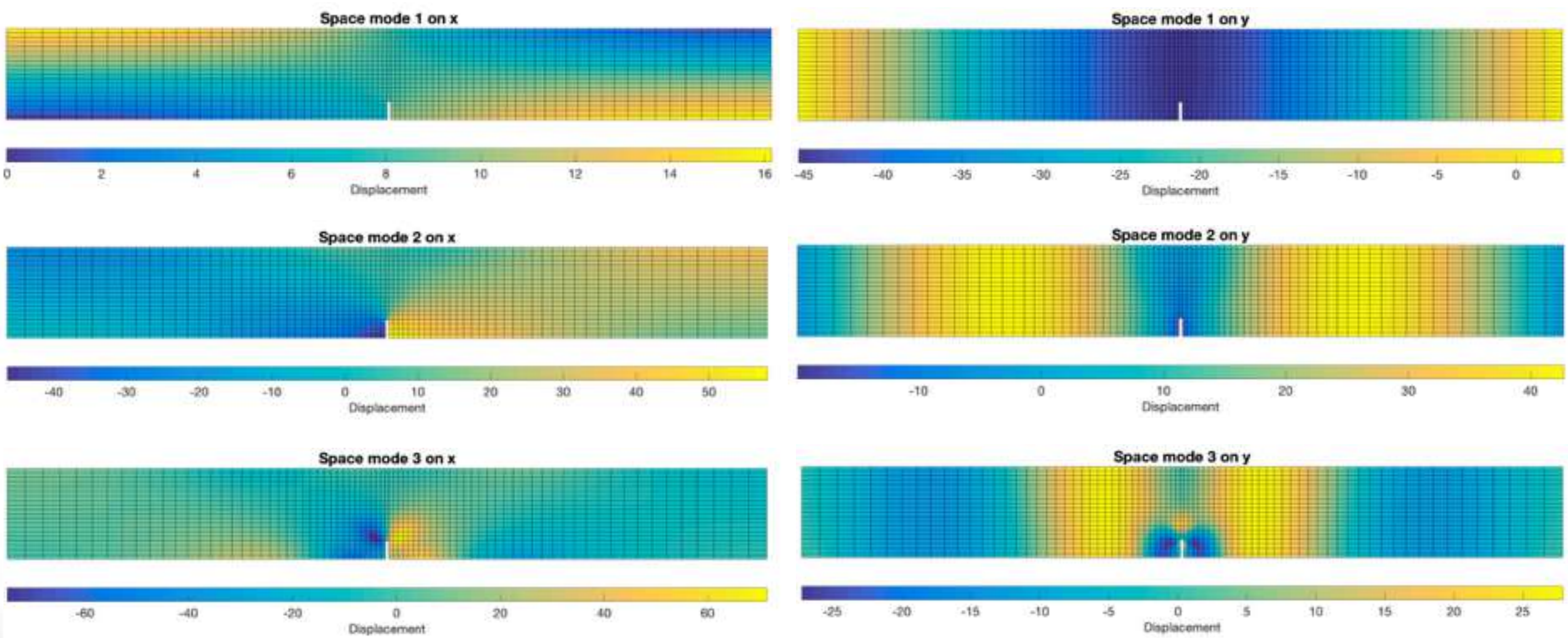
100 loading steps

$E = 30\ GPa \qquad \nu = 0.23$

$Y_0^{\text{ref}} = 216\ J.m^{-3}$

$A_d^{\text{ref}} = 2.25\ J^{-1}.m^3$

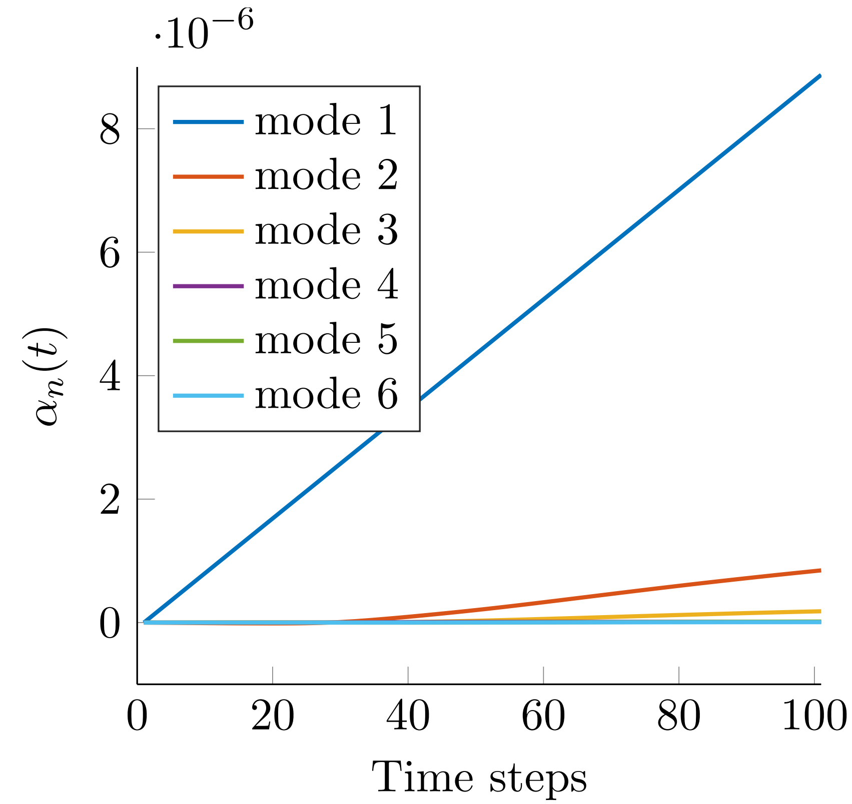
- Space modes $\mathbf{\Lambda}_k$



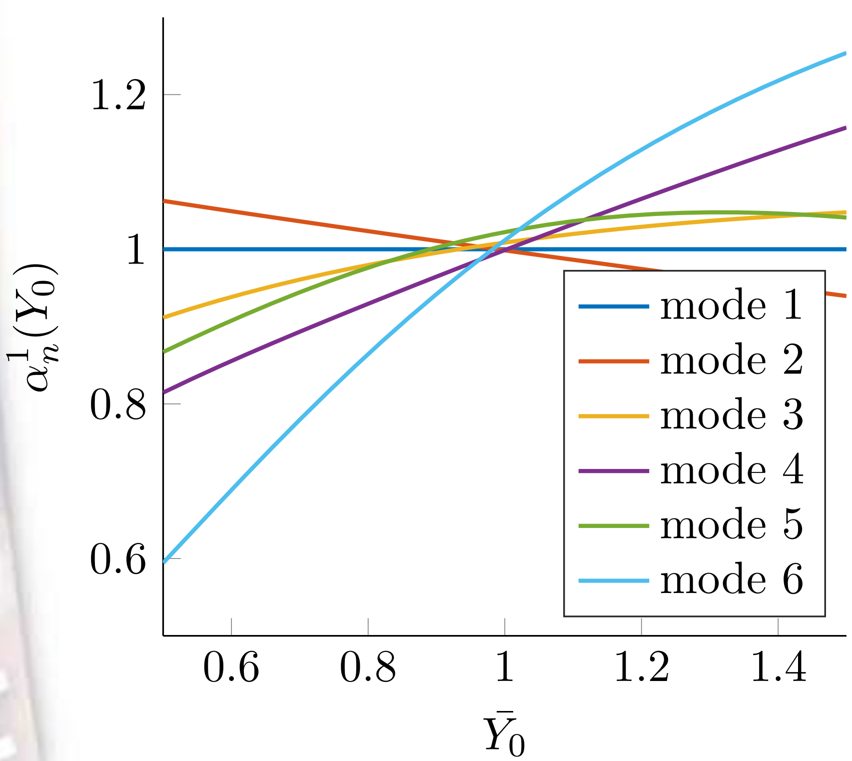
$\longrightarrow$ computed using the LATIN-PGD algorithm for nonlinear models [Ladevèze *et al.* 10, Vitse *et al.* 19]

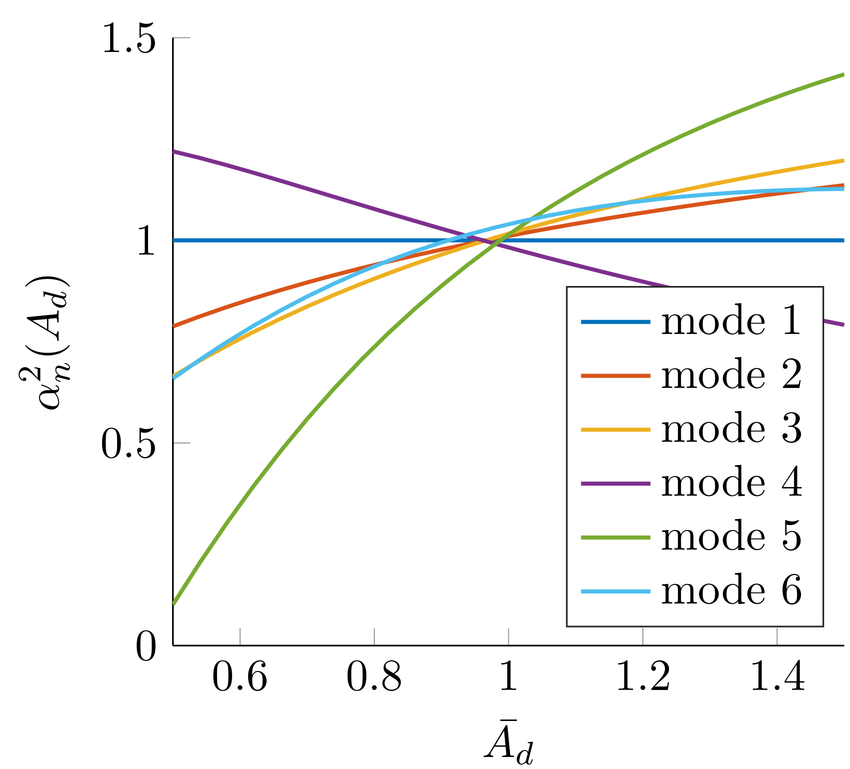
PGD Modes

- Time modes λ_k



- Parameter modes (α_k^1, α_k^2)(with $\bar{Y}_0 = Y_0/Y_0^{\text{ref}}$ $\bar{A}_d = A_d/A_d^{\text{ref}}$





→ **1st mode = full elasticity mode**

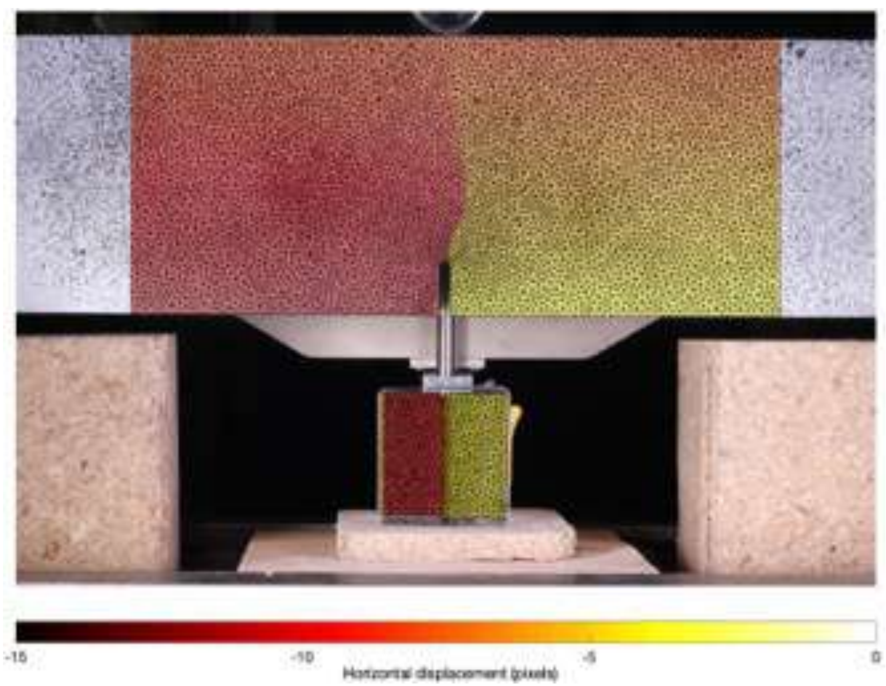
Results

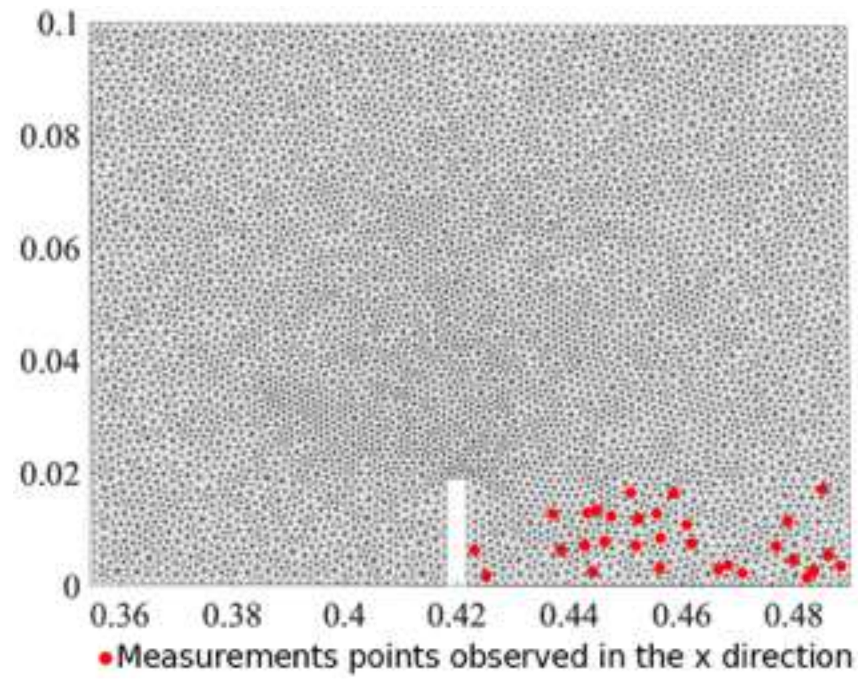
Assimilation with Transport Maps & PGD

$$\pi(\bar{Y}_0, \bar{A}_d | \mathbf{d}_1^{\text{obs}}, \ldots, \mathbf{d}_i^{\text{obs}}) \propto \prod_{j=1}^{i} \pi(\mathbf{d}_j^{\text{obs}} | \bar{Y}_0, \bar{A}_d).\pi_0(\bar{Y}_0, \bar{A}_d)$$

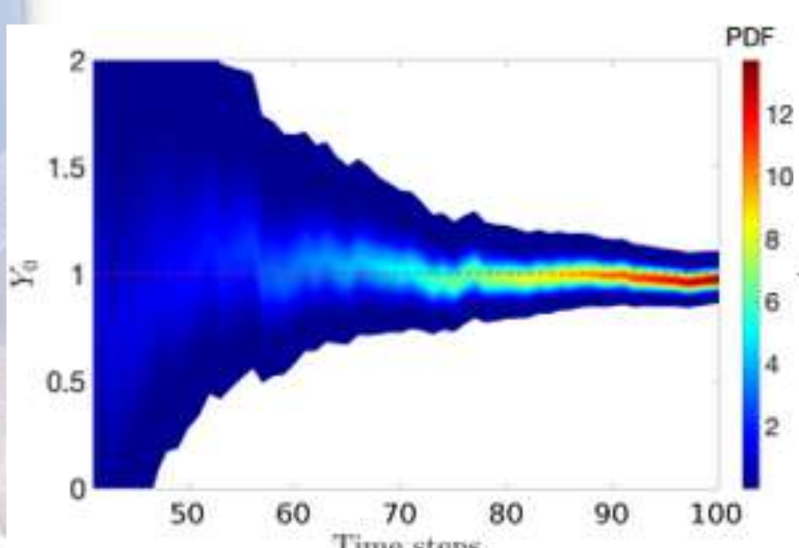
$$\epsilon_\sigma = 10^{-3}$$

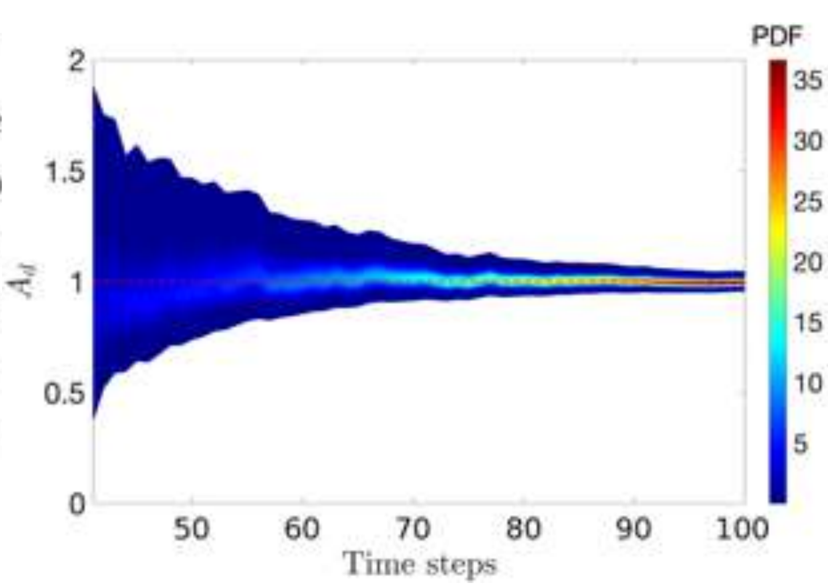
PGD approximation with $m = 6$

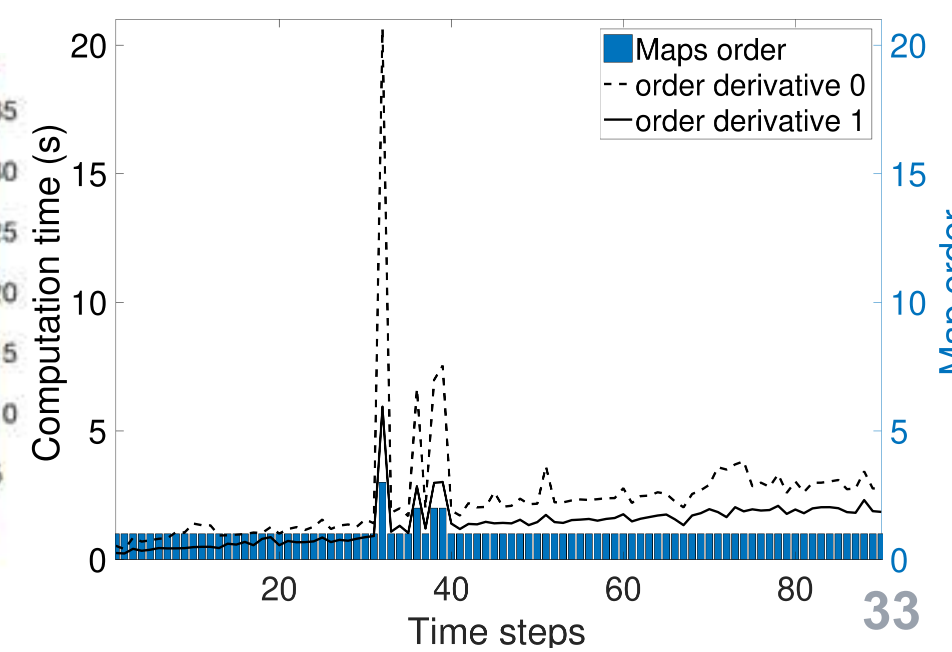




selection of most relevant DIC data
(sensitivity analysis)



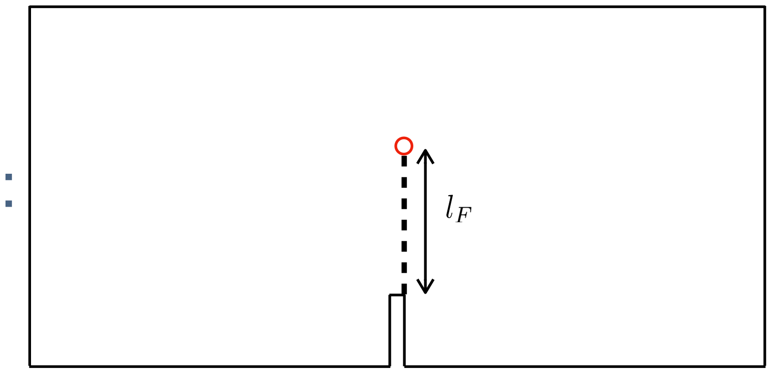


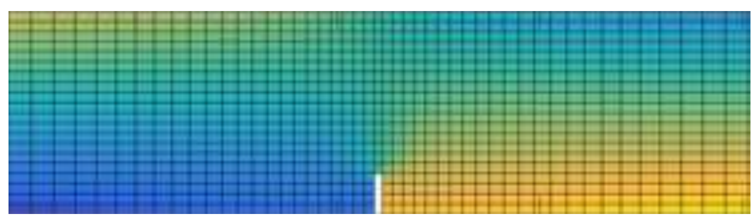


Post-processing

- **Kinematic bridge between damage & fracture mechanics**

Goal: Predict the final crack propagation:

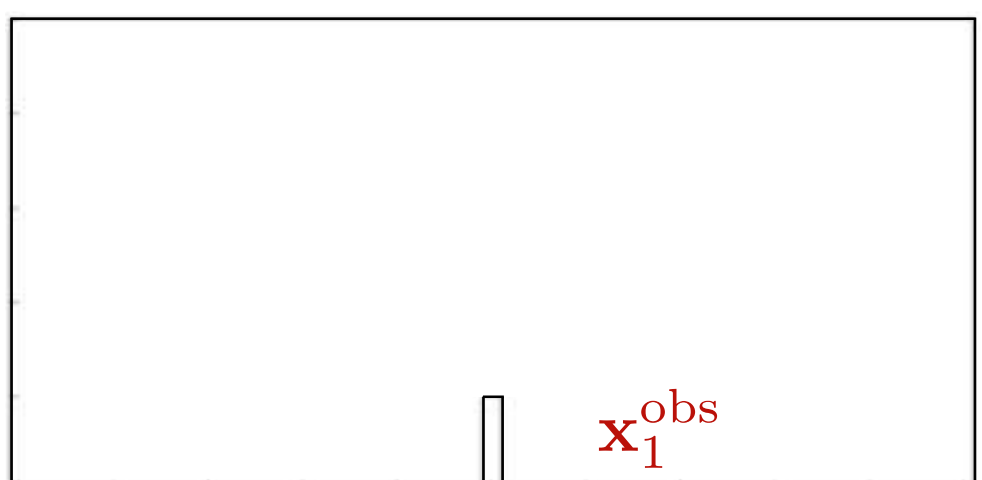


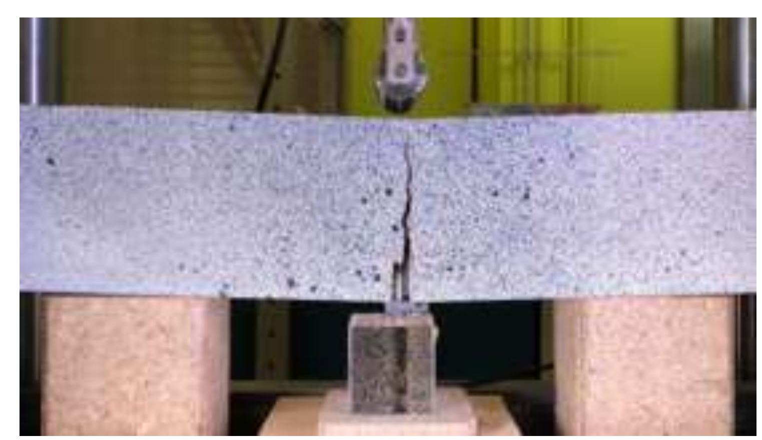


$\mathbf{u}_{\mathrm{PGD}}(\mathbf{x}, t_F, Y_0, A_d)$

$\mathbf{u}_{\mathrm{SVD}}(\mathbf{x}, l)$

$$\mathbf{x} = \mathbf{x}_1^{\mathrm{obs}}$$



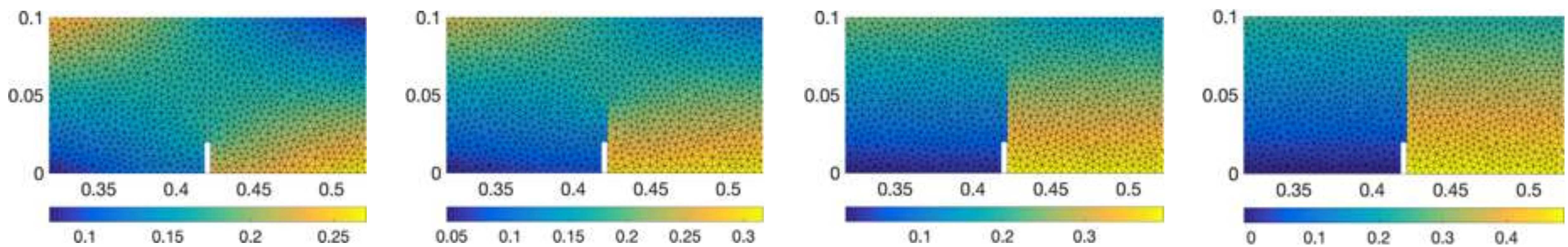


- Bayesian bridge: $\pi(l) = \pi_u(u^{\mathrm{SVD}}(l)).\pi_{\mathrm{pr}}(l)$

Post-processing

- **Kinematic bridge between damage and fracture mechanics**

$\longrightarrow$ post-process of elastic solutions with varying crack length l (unit loading)



$$l \in \{0, 1, 2, \ldots, 79\} \longrightarrow \mathbb{Y} = \{\mathbf{Y}_1, ..., \mathbf{Y}_{80}\} \longrightarrow \mathbb{Y} = \mathbb{U}\mathbb{D}\mathbb{V}^T$$

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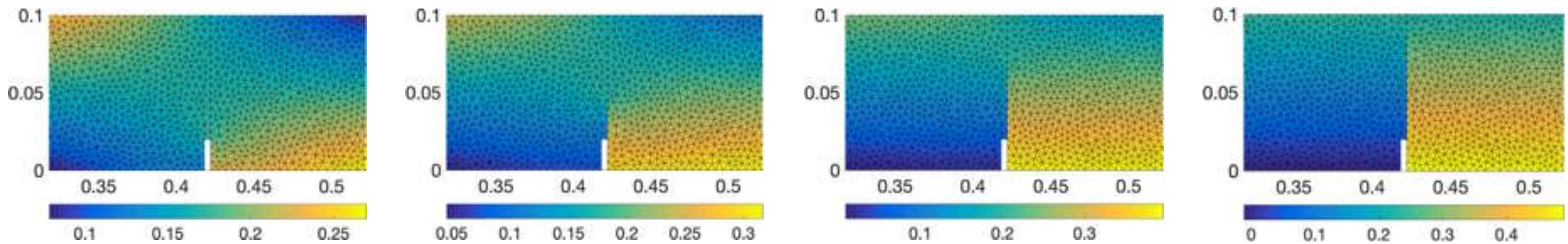

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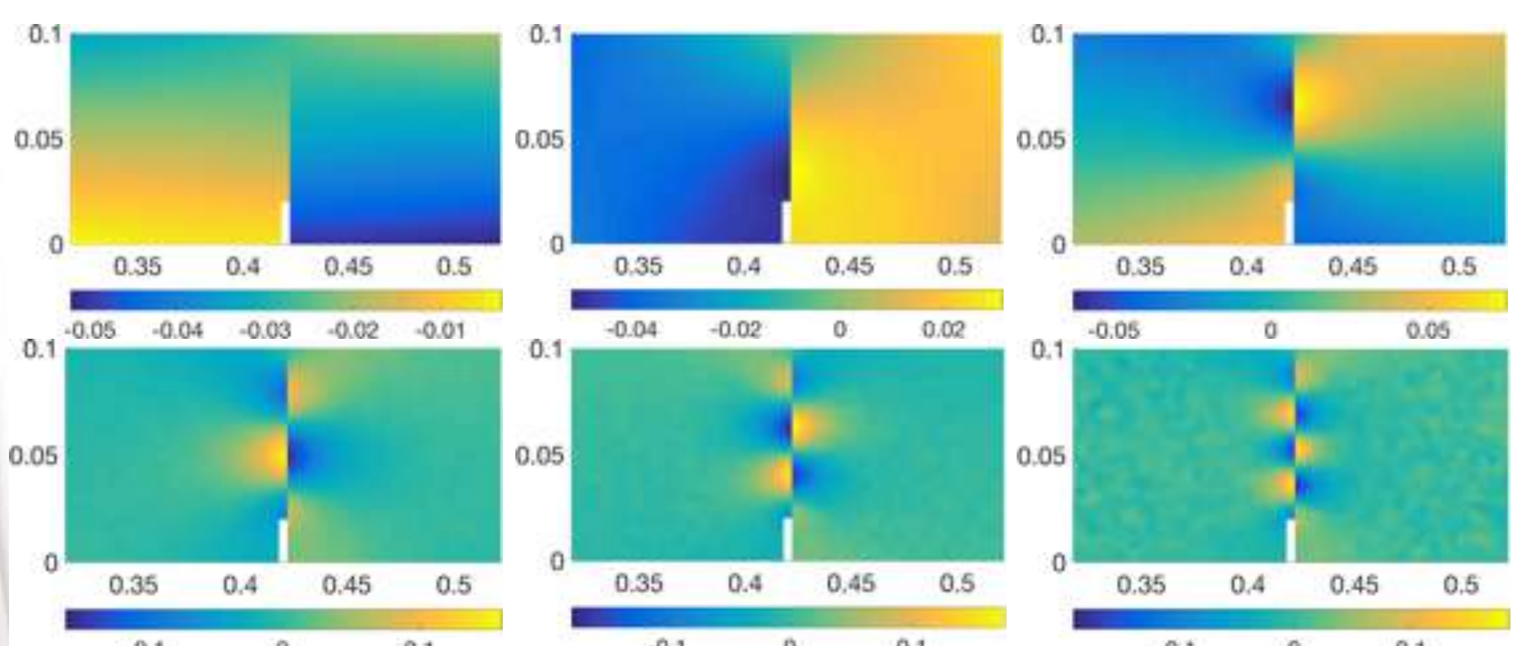
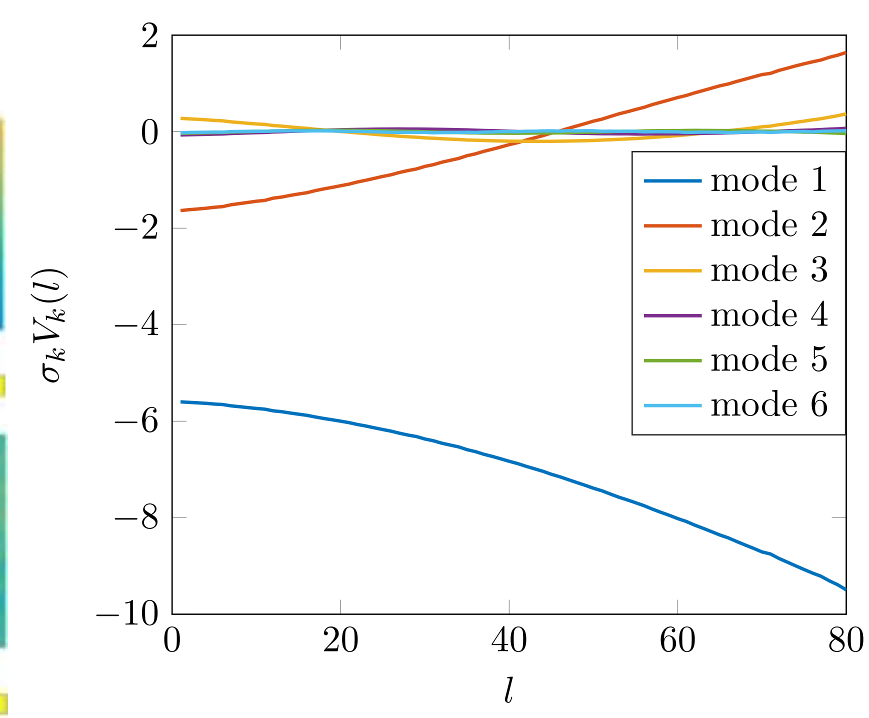
$$\boxed{\mathbf{u}_{\mathrm{SVD}}(\mathbf{x}, l) = \sum_{k=1}^{N_{\mathrm{SVD}}} \sigma_k \mathbf{u}_k(\mathbf{x}) v_k(l)}$$ (meta-model constructed in the offline phase)

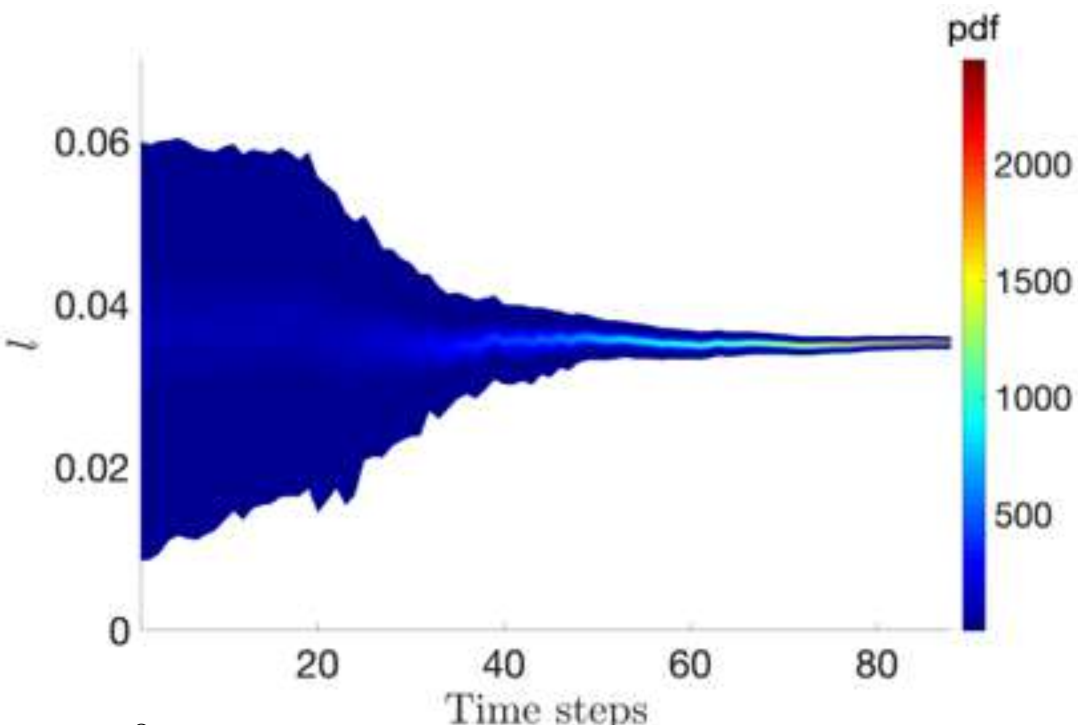


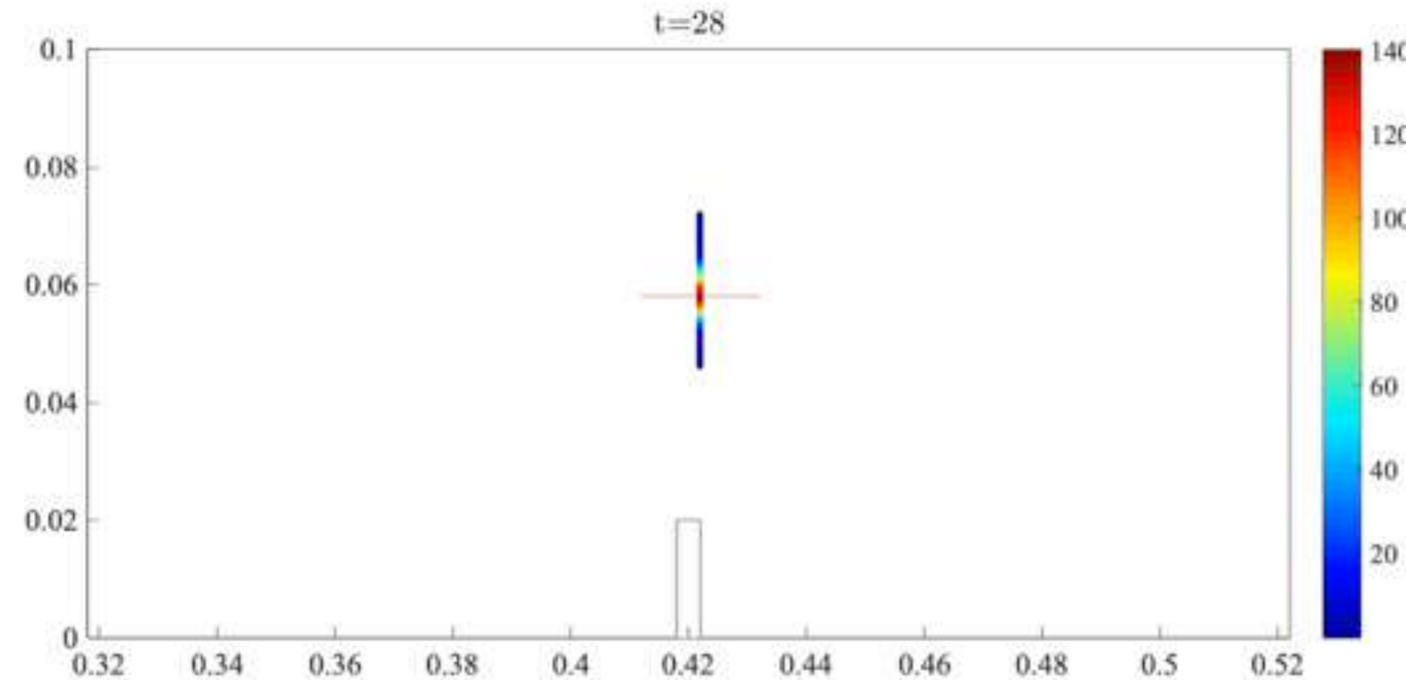


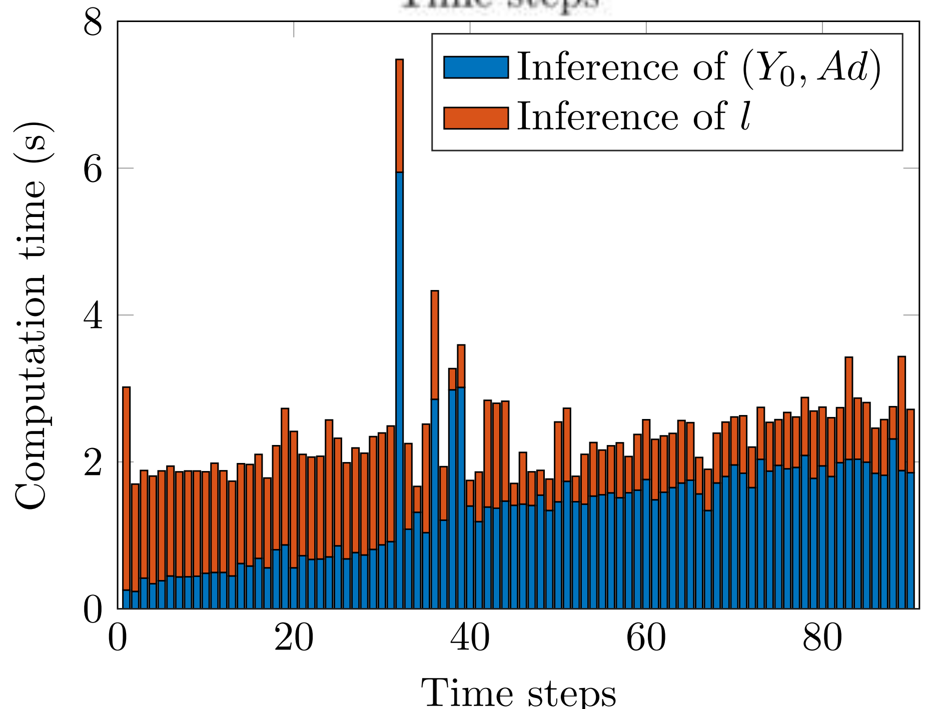
Post-processing

- On-the-fly prediction of the final crack length l_T

$$\boxed{\pi(l_T) = \pi_{\mathrm{u}}(\mathbf{u}^{\mathrm{SVD}}(l_T)).\pi_0(l_T)}$$







→ can be considered as real-time

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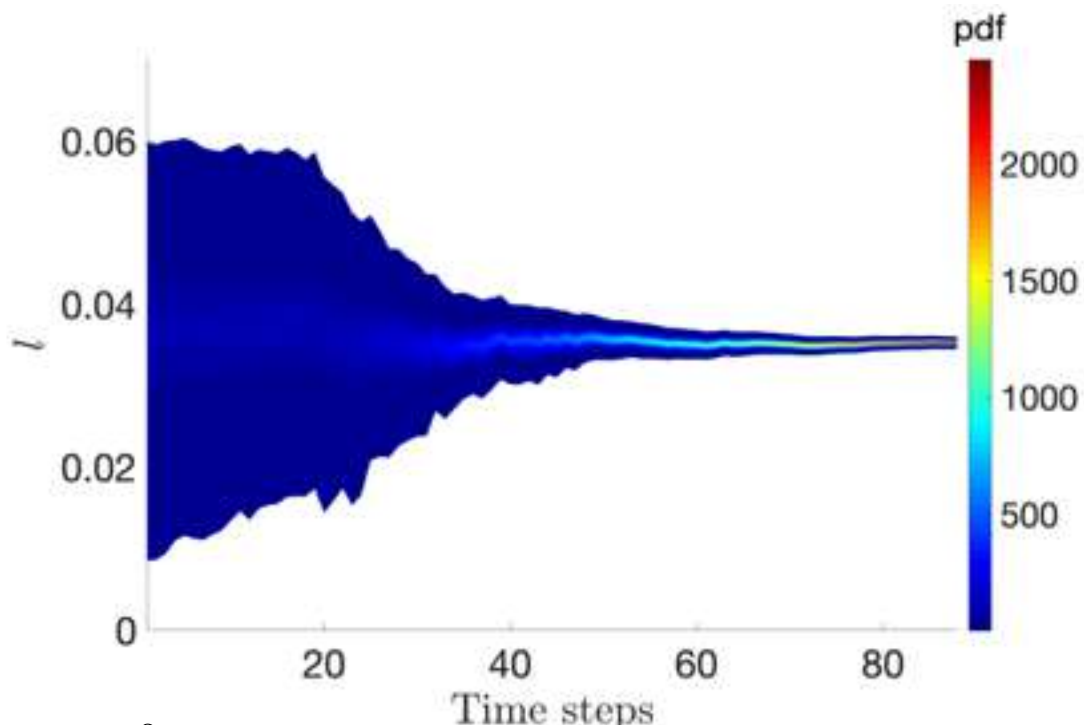
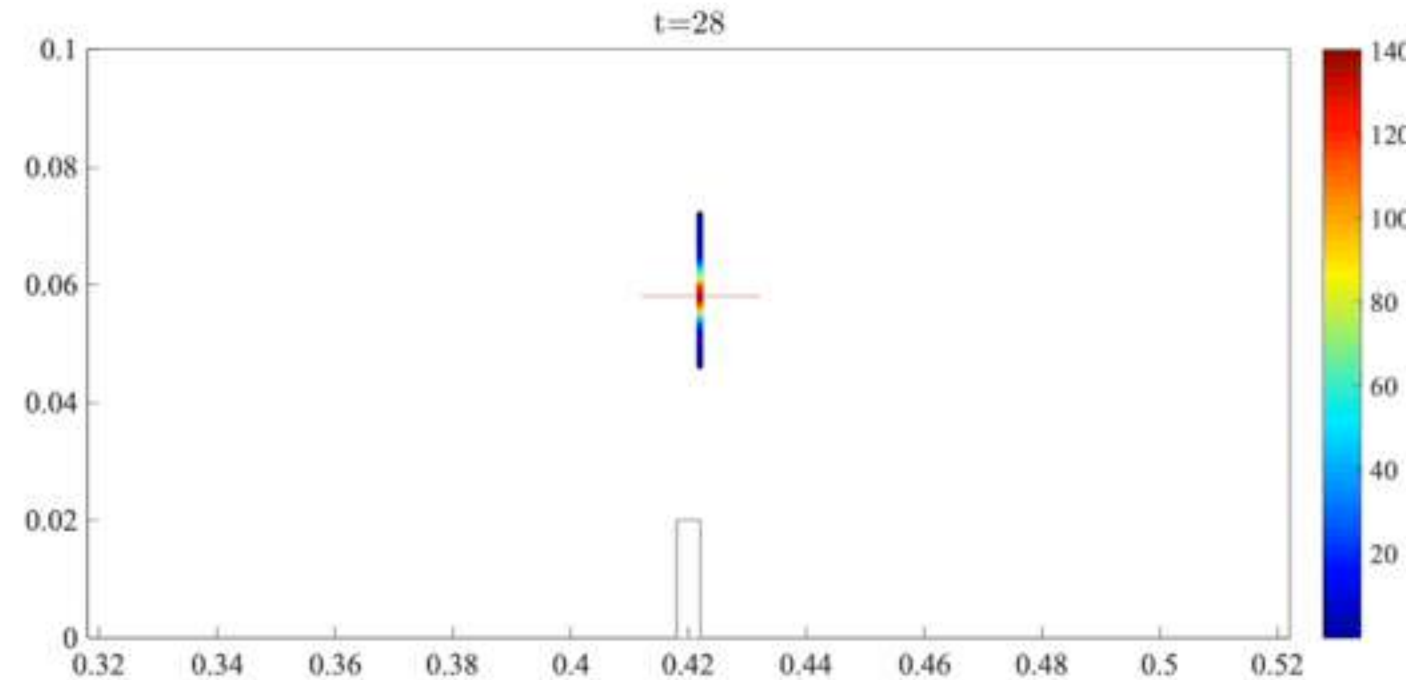
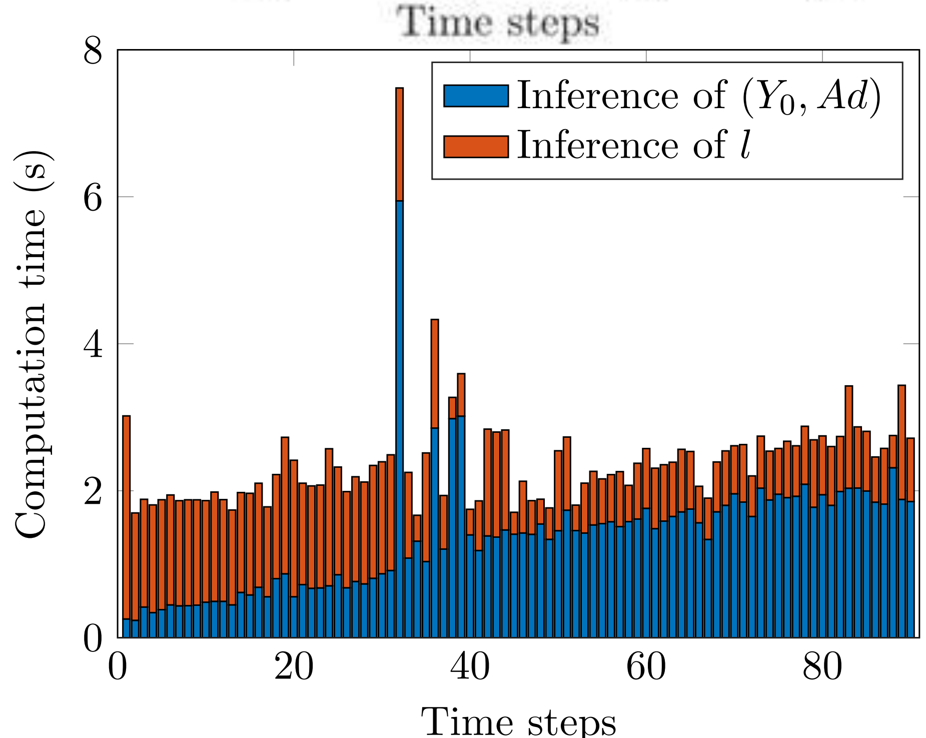






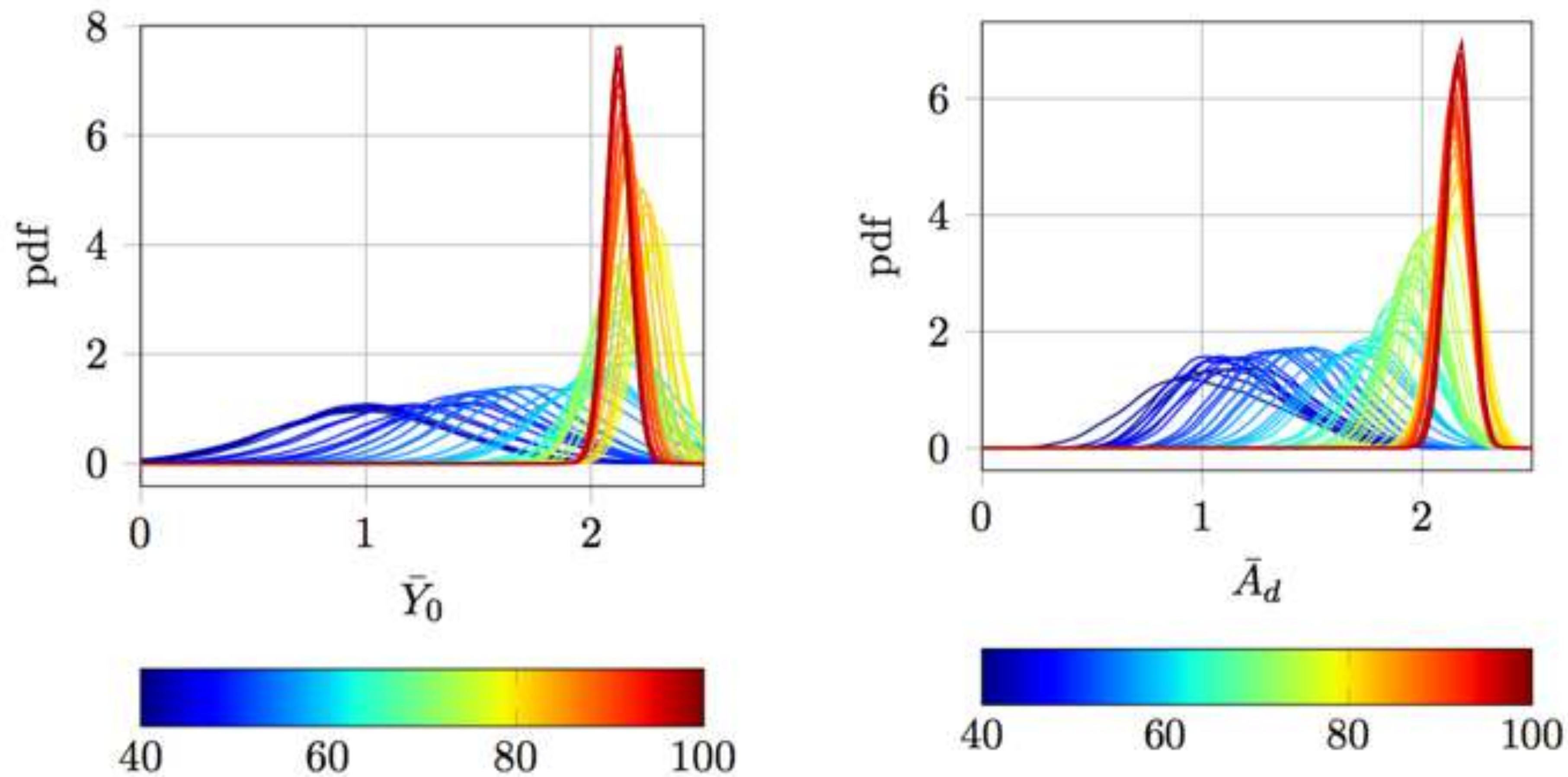
→ can be considered as real-time

Outline

1. Reduced order modeling
2. Fully stochastic approach for sequential data assimilation
3. Online model bias correction & feedback control
4. Alternative coupled deterministic-stochastic approach
5. Ongoing projects & new challenges

Model Bias Effects

- use of a surrogate PGD model with $m = 3$



⟶ influence limited during first time steps (elasticity with 1st mode alone)

⟶ divergence of the sequential data assimilation procedure (shifted marginals)

⟶ use of error estimates or high-fidelity models often not possible [Calvetti *et al.* 18]

Model Bias Correction

→ **data-based enrichment**, comparing predicted outputs and actual data

→ defined dynamically and in a stochastic setting

extension of PBDW/hybrid twins [Maday *et al.* 15, Chinesta *et al.* 18]

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- **Stochastic residual (computable)**

$$\mathbf{B}(\mathbf{x}^{\mathrm{obs}}, t_i) = \mathbf{d}_i^{\mathrm{obs}} - \mathbf{e}_{\mathrm{meas}} - \mathcal{M}(\mathbf{x}^{\mathrm{obs}}, t_i, \mathbf{p})$$

spatial coordinates of measurement

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spatial coordinates of measurement

- **Corrected model**

$$\mathcal{M}^{\mathrm{corr}}(\mathbf{x}^{\mathrm{obs}}, \mathbf{p}, t_{i+1}) = \mathcal{M}(\mathbf{x}^{\mathrm{obs}}, \mathbf{p}, t_{i+1}) + \boxed{\hat{\mathbf{B}}_{i\to i+1}(\mathbf{x}^{\mathrm{obs}})}$$

extrapolated model bias
(Gaussian pdf)

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extrapolated model bias
(Gaussian pdf)

$$\boxed{\pi(\mathbf{d}_{i+1}^{\mathrm{obs}}|\mathbf{p}) = \pi_{\hat{B}}(\mathbf{d}_{i+1}^{\mathrm{obs}} - \mathcal{M}(\mathbf{x}^{\mathrm{obs}}, \mathbf{p}, t_{i+1}))}$$

Model Bias Correction

Extrapolation procedure

- linear independent extrapolation of mean and standard deviation of $\mathbf{B}(\mathbf{x}^{\mathrm{obs}}, t_i)$

$\longrightarrow$ no physics consideration / inconsistent results (noise extrapolation)

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⟶ no physics consideration / inconsistent results (noise extrapolation)

- global linear extrapolation involving physics (filtering noise)

$$\mathbb{B}_{\text{mean}} = \left[\text{mean}\left(\mathbf{B}(\mathbf{x}^{\text{obs}}, t_1)\right), ..., \text{mean}\left(\mathbf{B}(\mathbf{x}^{\text{obs}}, t_i)\right)\right] \qquad \mathbb{B}_{\text{std}} = \left[\text{std}\left(\mathbf{B}(\mathbf{x}^{\text{obs}}, t_1)\right), ..., \text{std}\left(\mathbf{B}(\mathbf{x}^{\text{obs}}, t_i)\right)\right]$$

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SVD decomposition (+truncation)

$$\mathbb{B}_{\mathrm{mean}} = \mathbb{U}_{\mathrm{mean}}\mathbb{D}_{\mathrm{mean}}\mathbb{V}^T_{\mathrm{mean}} \qquad \mathbb{B}_{\mathrm{std}} = \mathbb{U}_{\mathrm{std}}\mathbb{D}_{\mathrm{std}}\mathbb{V}^T_{\mathrm{std}}$$

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$\longrightarrow$ linear extrapolations of time SVD modes: $\hat{\mathbf{V}}_{\text{mean}}$ $\hat{\mathbf{V}}_{\text{std}}$

$\hookrightarrow$ *use of the Sequential-Karhunen-Loeve (SKL) method* [Ross *et al.* 08]
fast SVD decomposition of $[\mathbb{M}_{[1,i-1]}\ \mathbb{M}_i]$ *knowing that of* $\mathbb{M}_{[1,i-1]}$

$\longrightarrow$ recovery of $\hat{\mathbf{B}}_{i\rightarrow i+1}(\mathbf{x}^{\text{obs}})$ with truncated SVD

Indication on Model Bias

$$\boxed{\pi(\mathbf{p}|\mathbf{d}^{\mathrm{obs}}) = \frac{1}{C}\pi(\mathbf{d}^{\mathrm{obs}}|\mathbf{p}).\pi_0(\mathbf{p})}$$

$$C = \int \pi(\mathbf{d}^{\mathrm{obs}}|\mathbf{p}).\pi_0(\mathbf{p})\mathrm{d}\mathbf{p} = \pi(\mathbf{d}^{\mathrm{obs}})$$ **: model evidence**

$$= \exp\left(\mathbb{E}_\rho\left[\log\left(M_\sharp^{-1}\pi\right) - \log(\rho)\right]\right)$$ [El Moselhy & Marzouk 12]

decreases when the model becomes inaccurate

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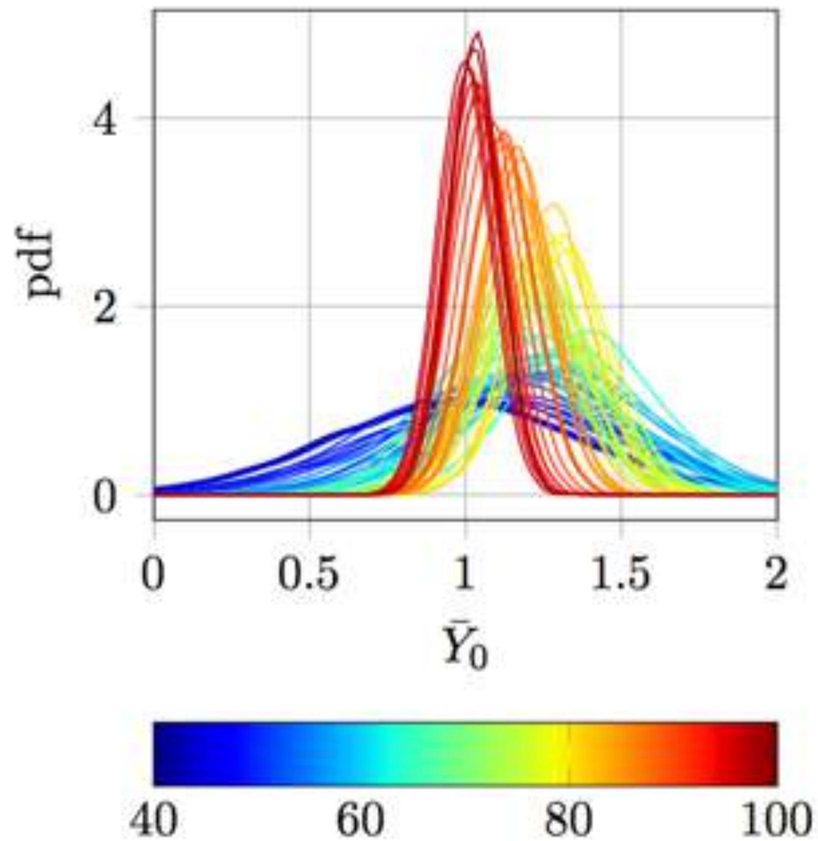
→ decreases when the model becomes inaccurate

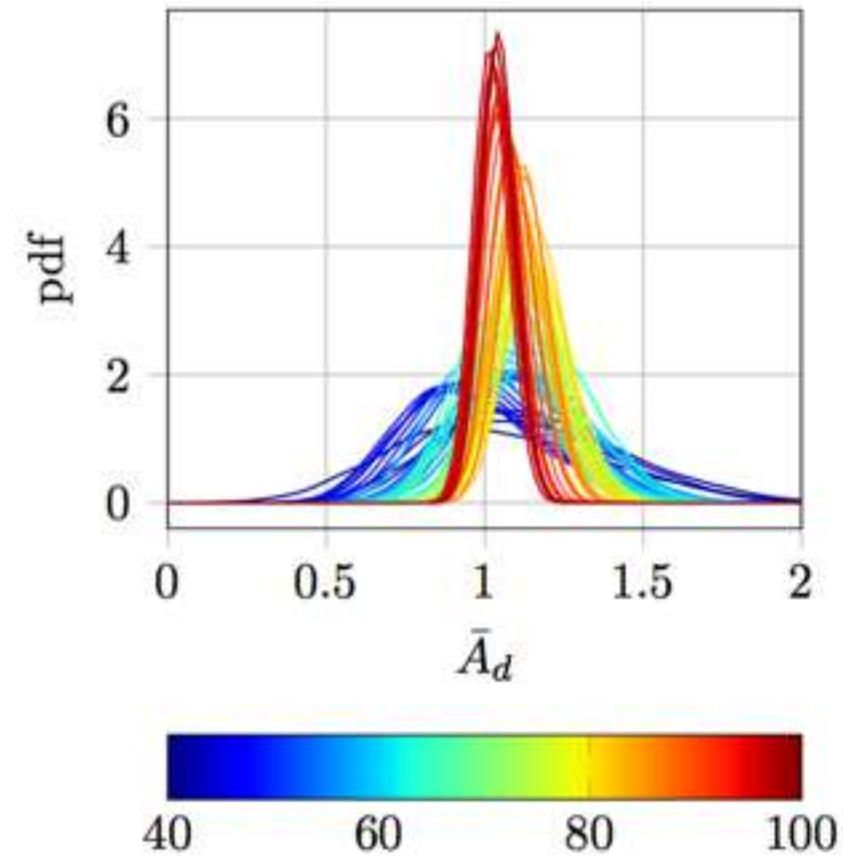
→ evolution of C monitored along the assimilation process

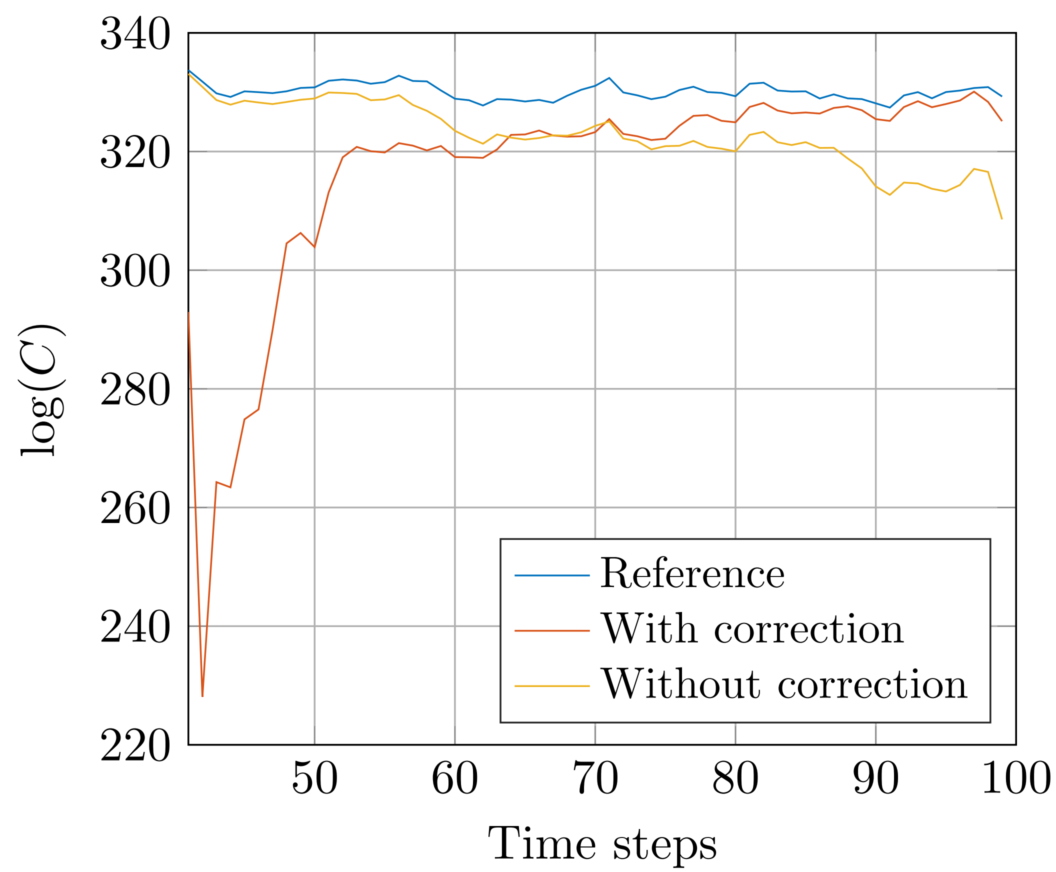
→ implementation of the correction **when** C **drops drastically**

→ if model inaccurate in a given time range, corresponding maps removed

Results

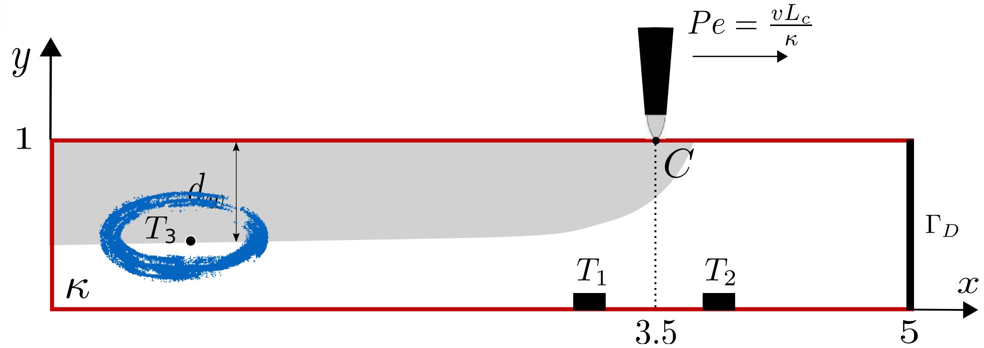

pdf
4
2
0
0
0.5
1
1.5
2
$\bar{Y}_0$
40
60
80
100


pdf
6
4
2
0
0
0.5
1
1.5
2
$\bar{A}_d$
40
60
80
100


log(C)
340
320
300
280
260
240
220
50
60
70
80
90
100
Time steps
Reference
With correction
Without correction

Command Synthesis

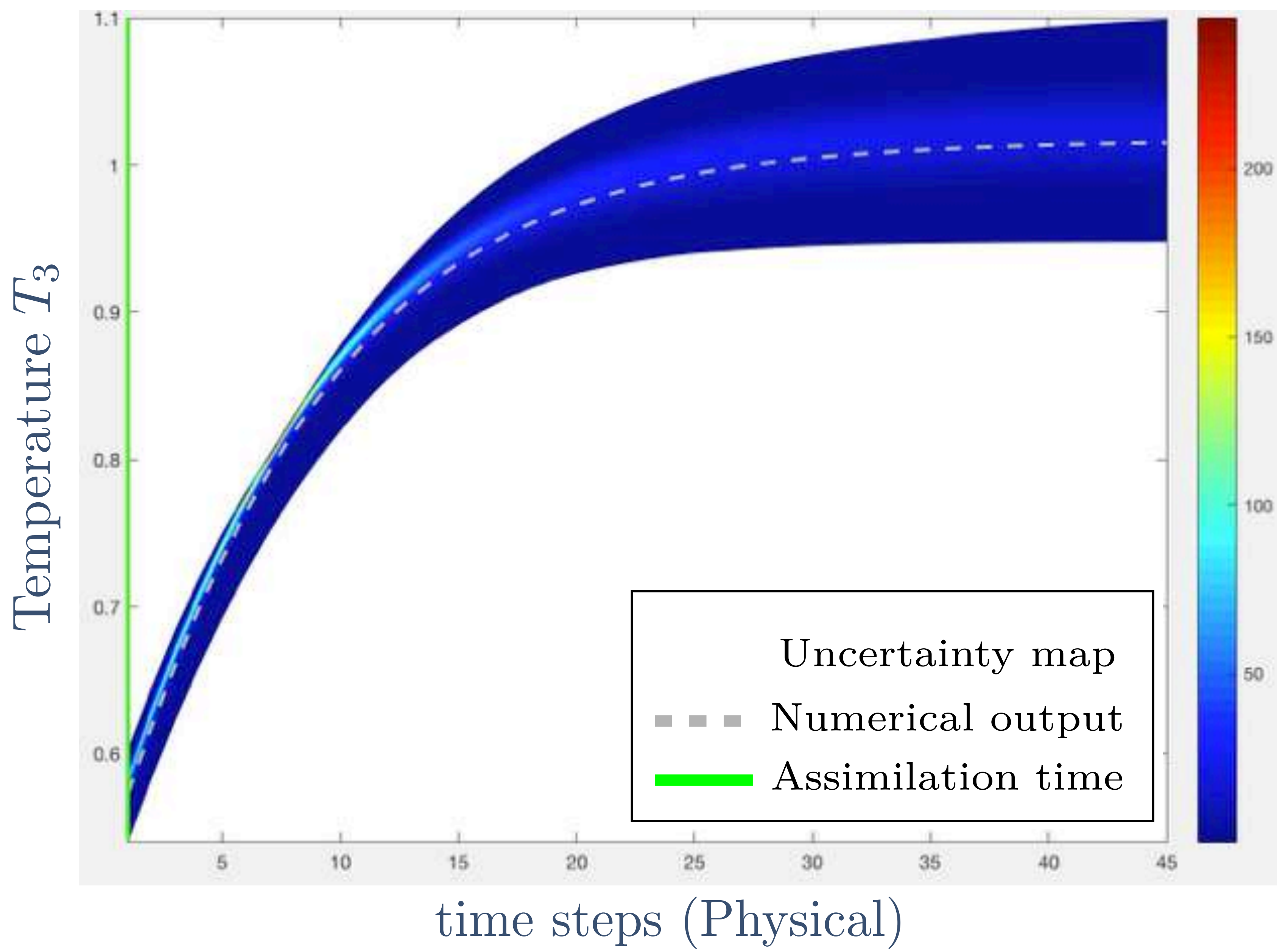
- **Post-processing: prediction with uncertainty propagation**



(good weld indicator)

$$(\sigma^j, Pe^j) \sim \pi(\sigma, Pe | T_1^{\text{obs},1:i}, T_2^{\text{obs},1:i})$$

$$T_3(x_3, y_3, t, \sigma^j, Pe^j) = \sum_{n=1}^{m} \Lambda_n(x_3, y_3)\lambda_n(t)\alpha_n^1(\sigma^j)\alpha_n^2(Pe^j)$$



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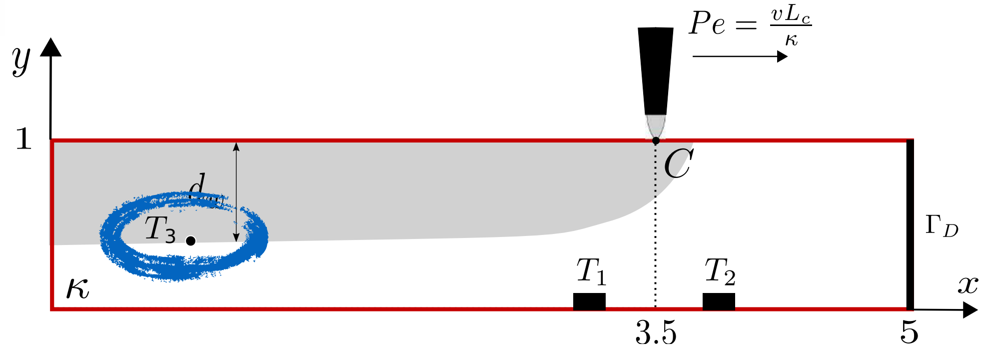

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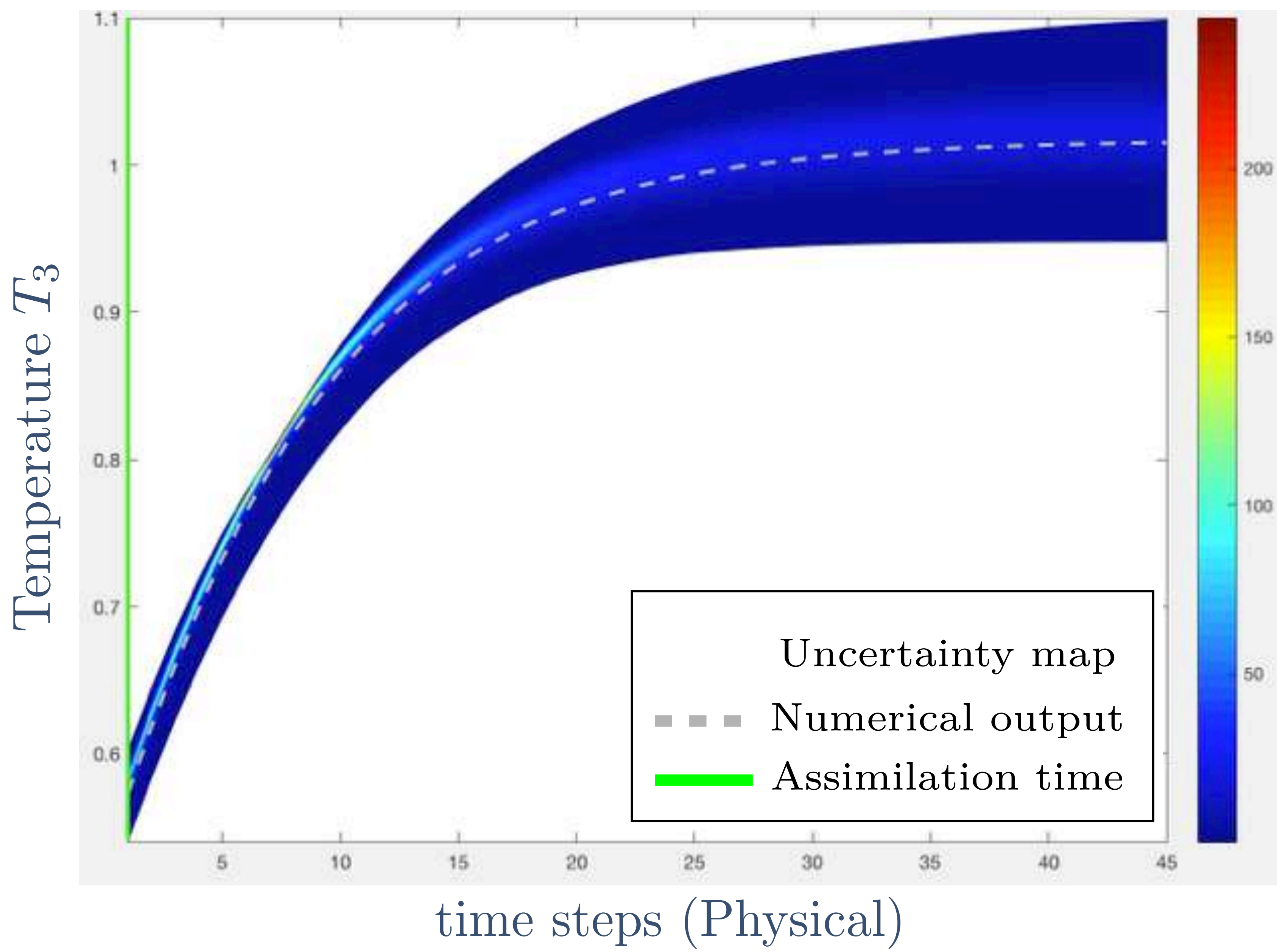

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- Objective: $\mathrm{q_{obj}} = 1$ (probability of effective welding = 0.99)

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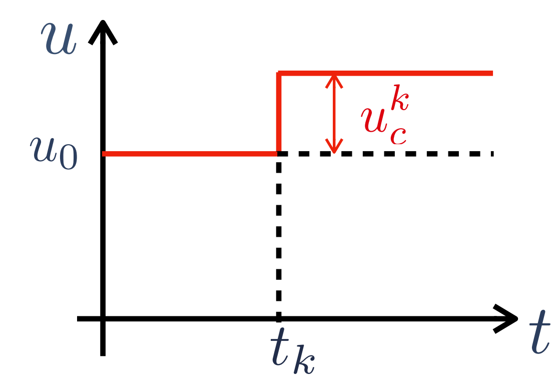

- Computation of the control variable: $u_c^k = \dfrac{q_{\mathrm{obj}} - q^k}{q^0}$

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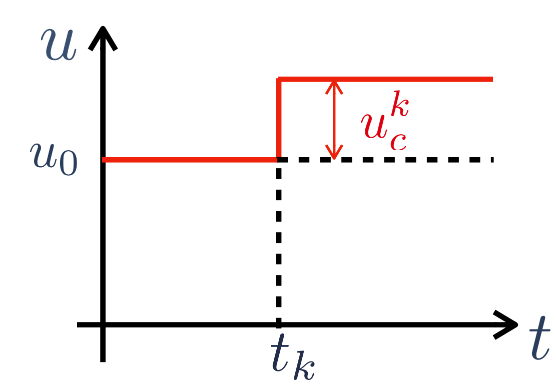


- Computation of the control variable: $u_c^k = \dfrac{q_{\mathrm{obj}} - q^k}{q^0}$

- Computation of the controlled model:

$$T_c(\mathbf{x}, t, \mathbf{p}) = u^{k-1}.T(\mathbf{x}, t, \mathbf{p}) + u_c^k.T(\mathbf{x}, t - t_k, \mathbf{p}) \quad \text{(linearity)}$$

$$= u^0. \sum_{n=1}^{m} \Lambda_n(\mathbf{x}) \lambda_n^k(t) \alpha_n(\mathbf{p}) \quad \text{(factorization)}$$

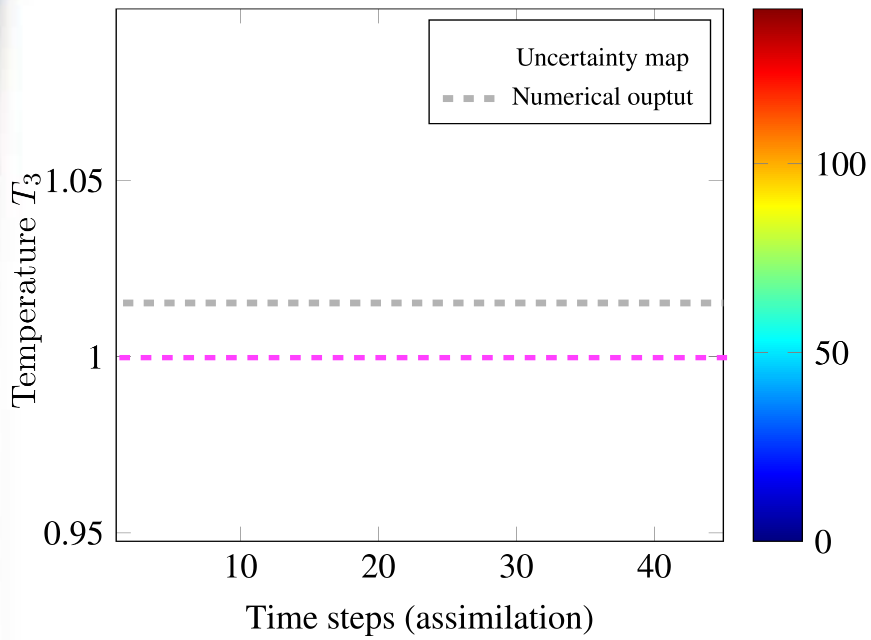
with: $\lambda_n^k(t) = \lambda_n^{k-1}(t) + u_c^k.\lambda_n^k(t - t_k)$

→ *propagating uncertainty on parameters through PGD model & TM sampling*

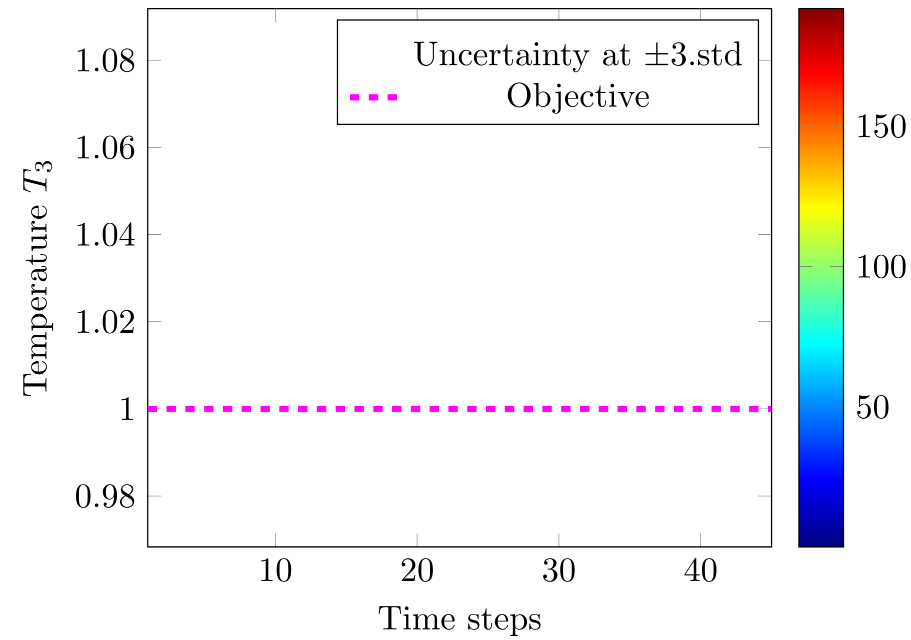
Command Synthesis

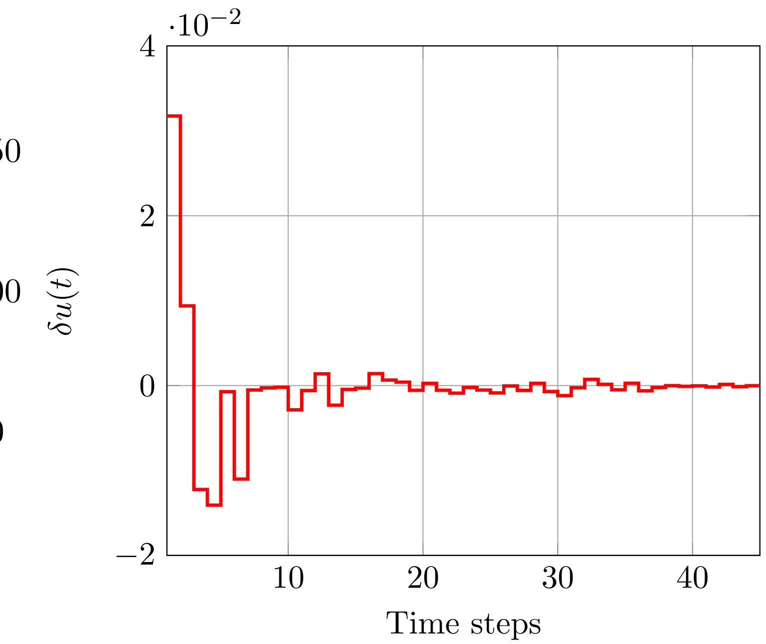
Results

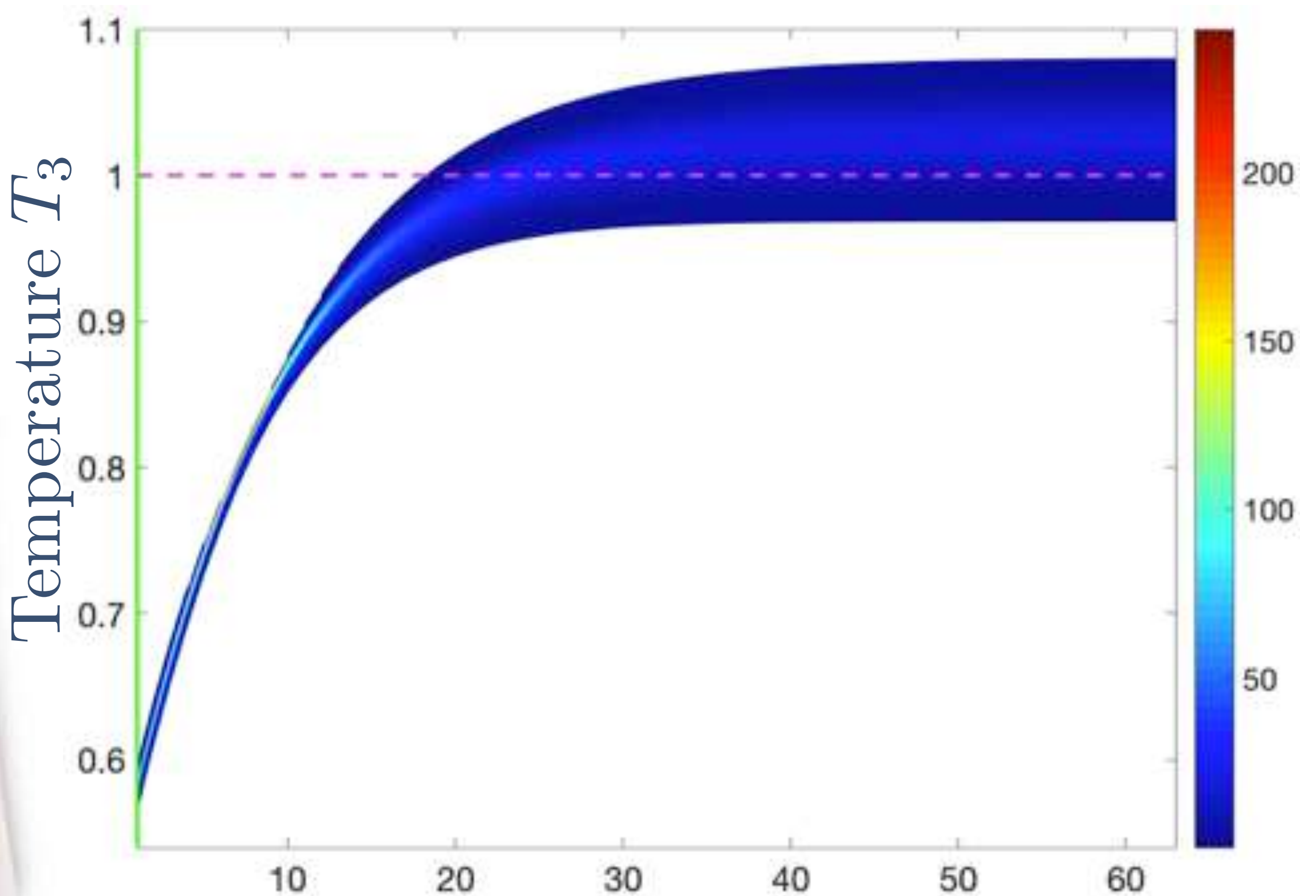
Without control:



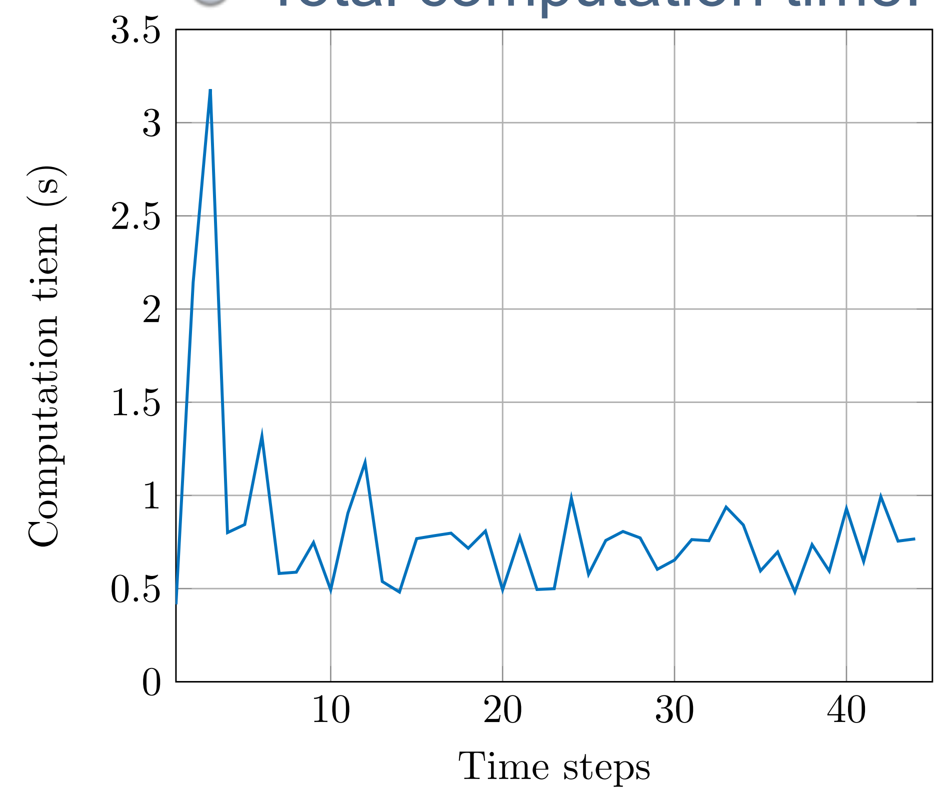
With control:







Total computation time:



Command Synthesis

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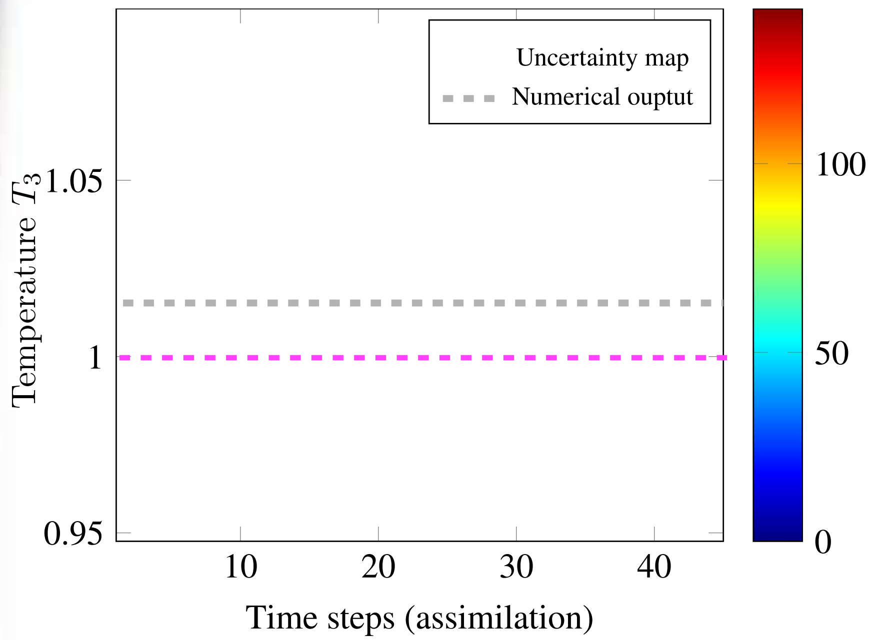
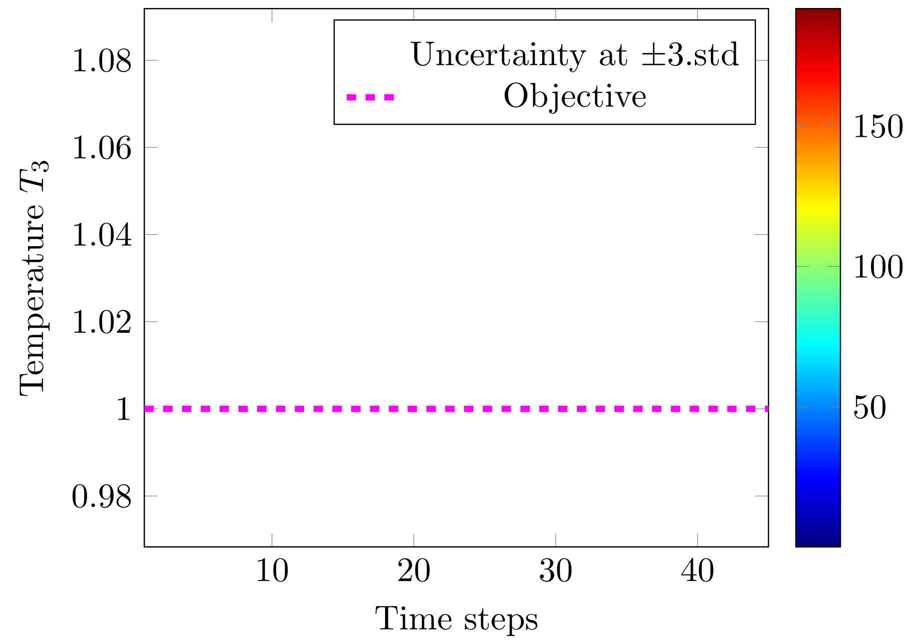
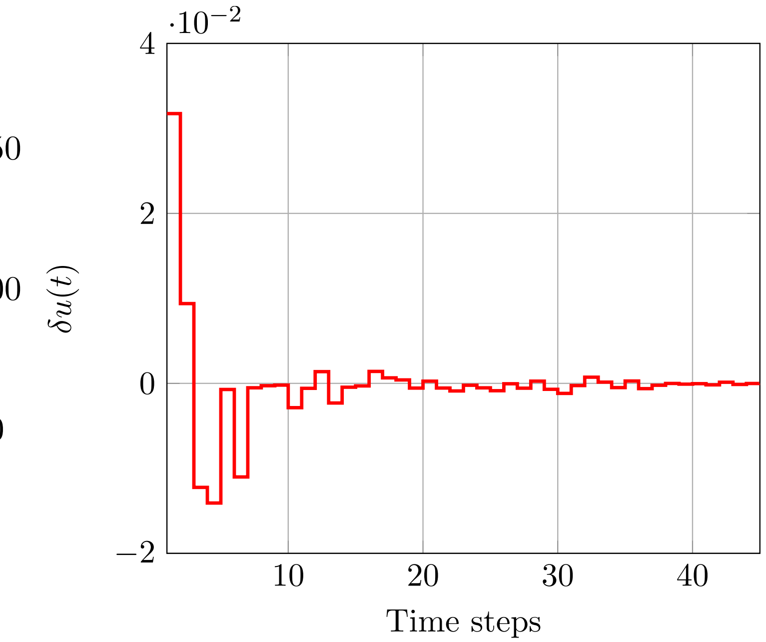
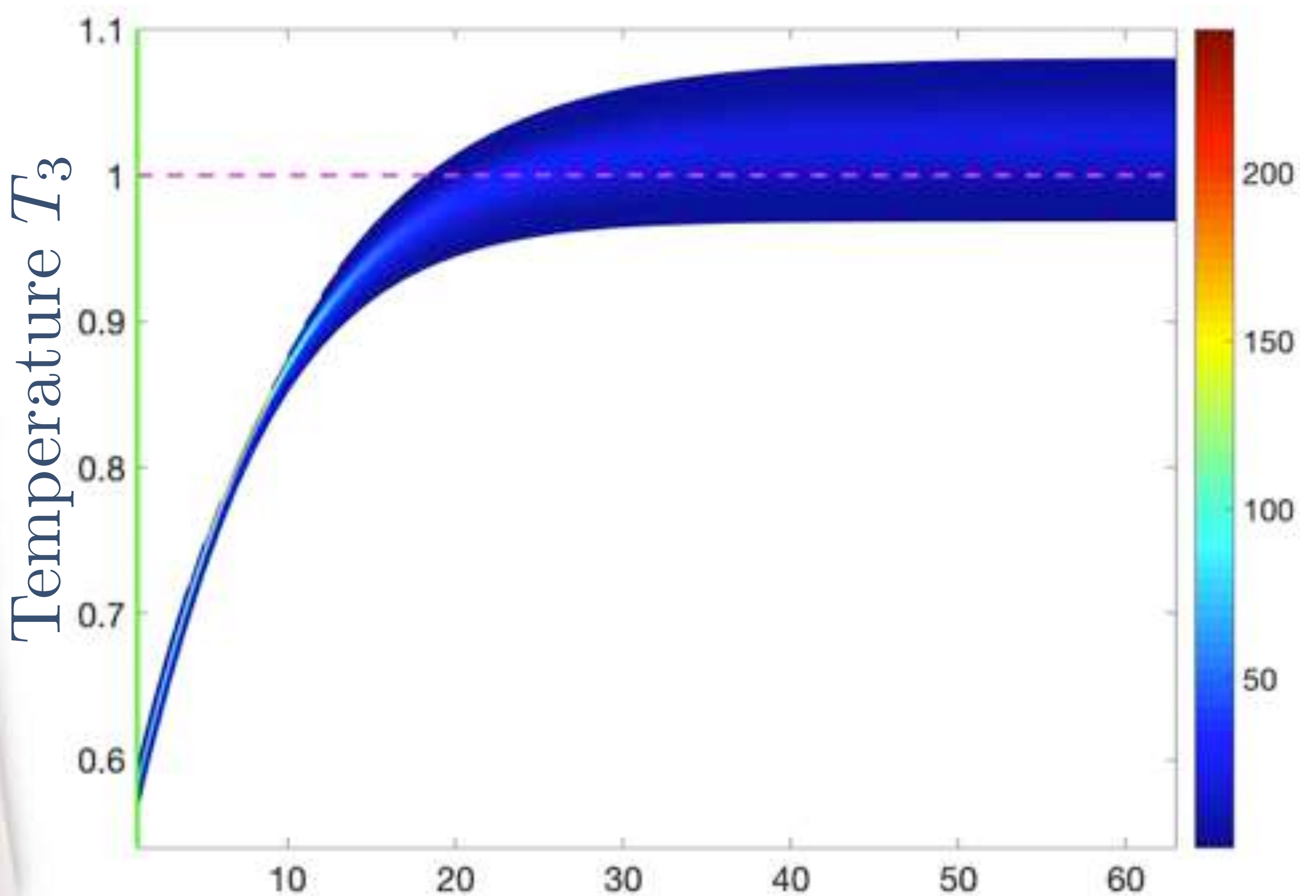


With control:







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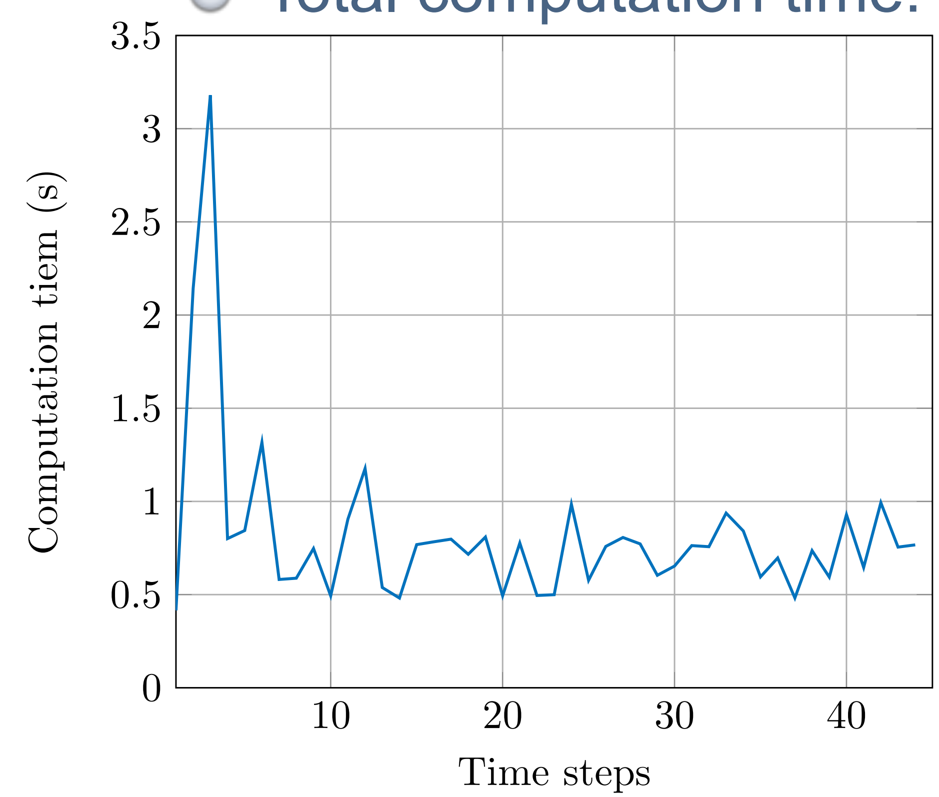
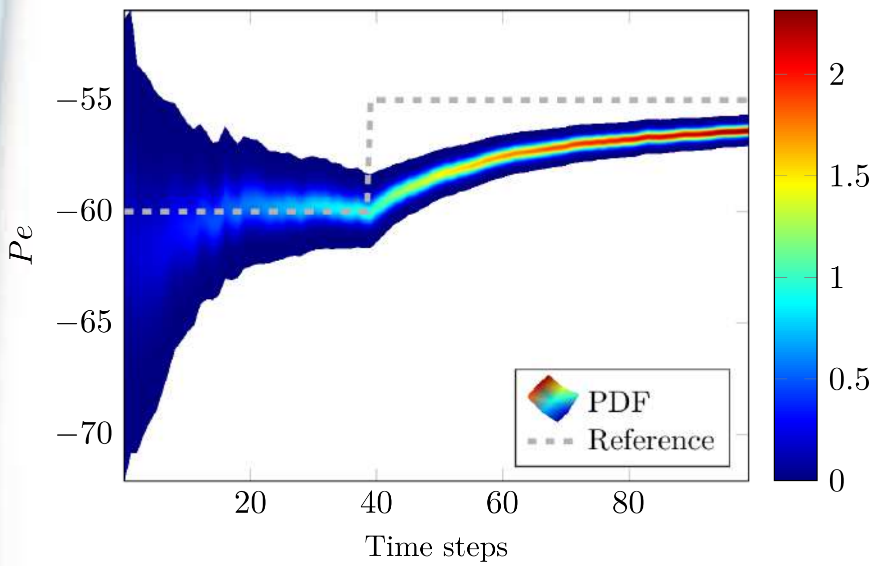
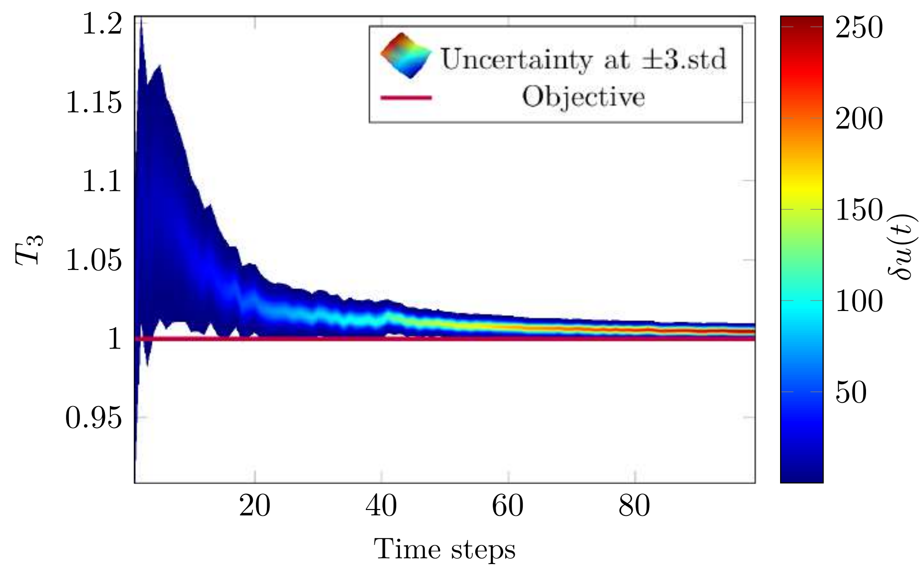
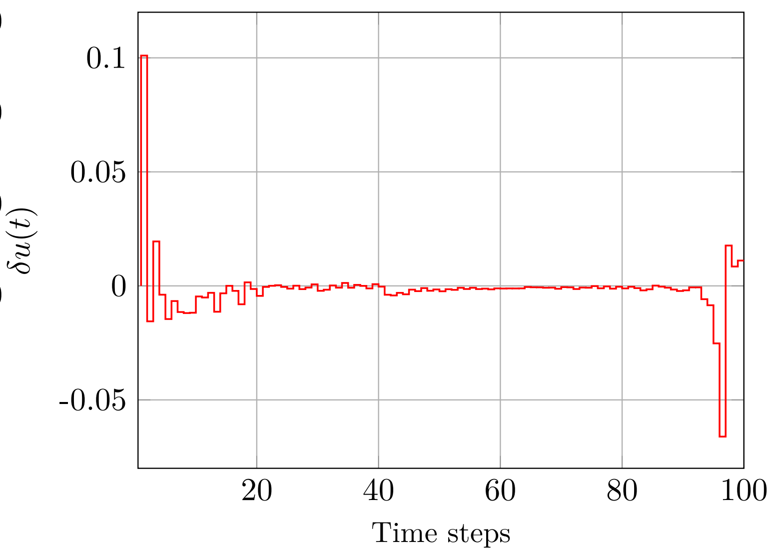


Command Synthesis

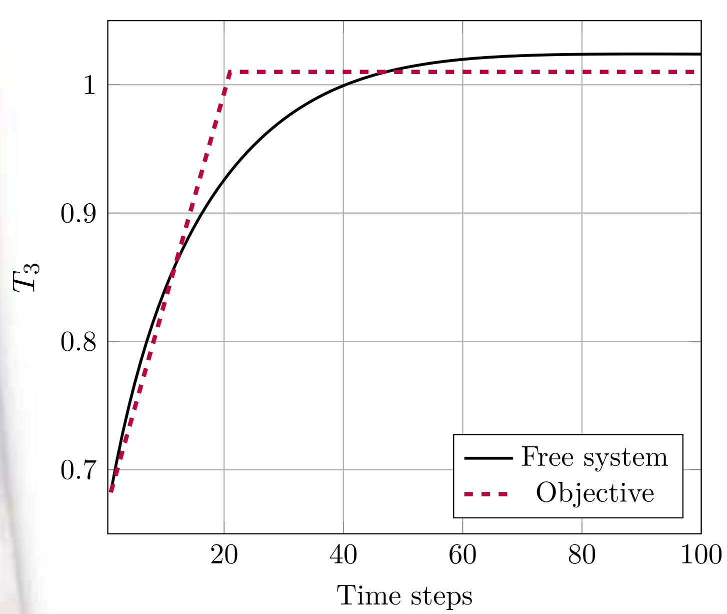
Control with evolving process parameters

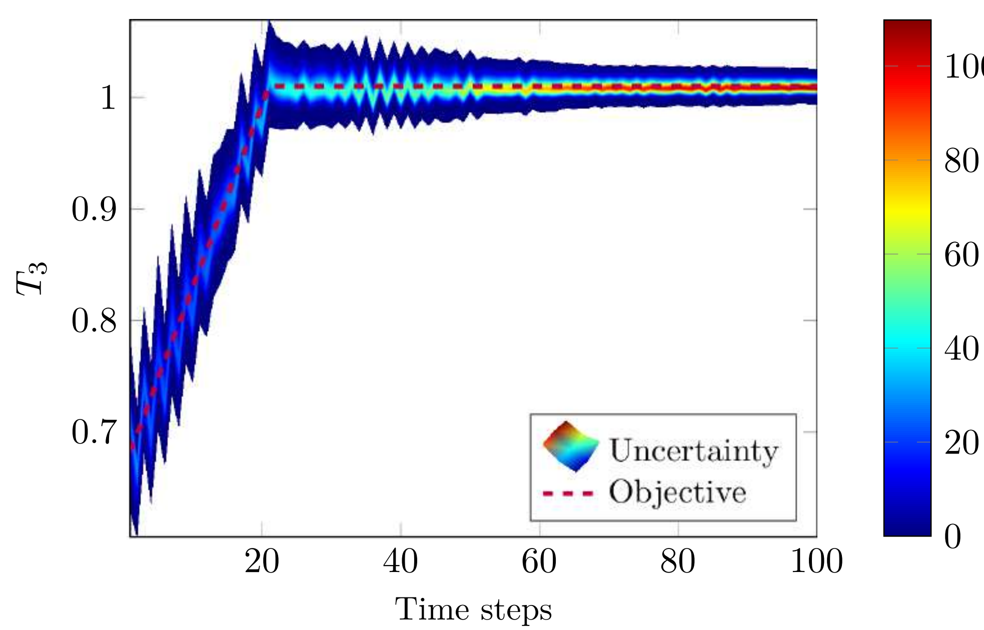


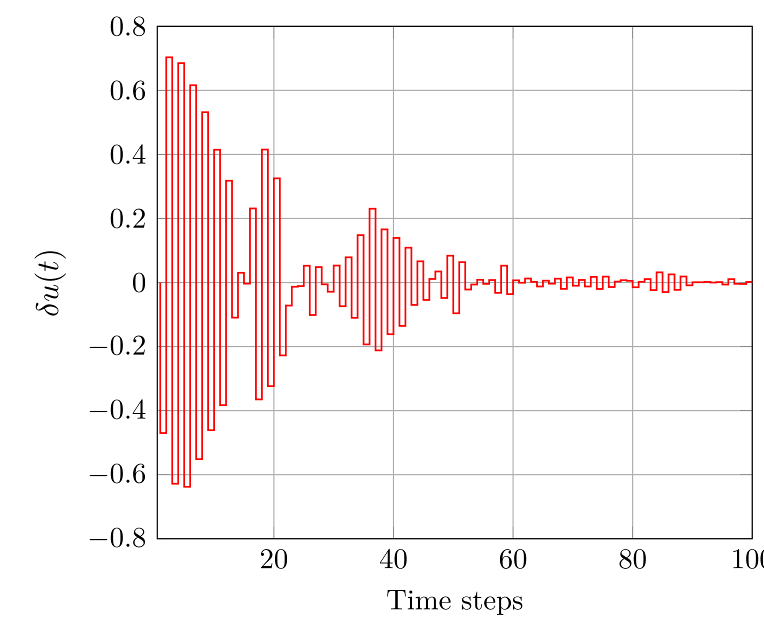




Control with prescribed time path



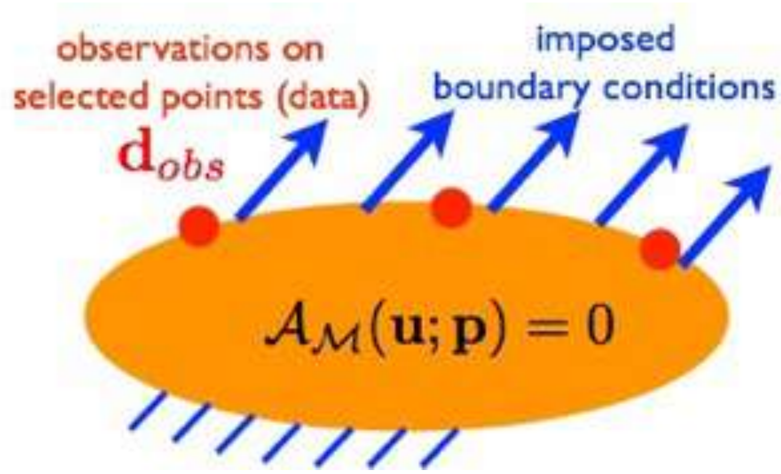




Outline

1. Reduced order modeling
2. Fully stochastic approach for sequential data assimilation
3. Online model bias correction & feedback control
4. Alternative coupled deterministic-stochastic approach
5. Ongoing projects & new challenges

Model Updating



find optimal $\mathbf{p}$: inverse problem (ill-posed)

- deterministic approaches → cost function to minimize + regularization

[Bui 93, Kern 02, Bonnet & Constantinescu 05]

[Tikhonov & Arsenin 77, Hansen 98]

$$\boxed{\mathbf{p}_{sol} = argmin_{\mathbf{p}\in\mathcal{P}}\mathcal{F}(\mathbf{p})}$$

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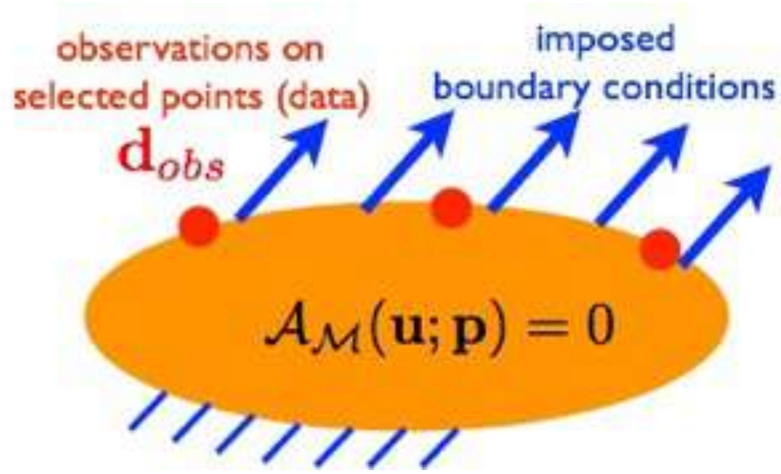

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→ $\mathbf{p}_{sol} = argmin_{\mathbf{p}\in\mathcal{P}}\mathcal{F}(\mathbf{p})$

LS functional: $\mathcal{F}(\mathbf{p}) = \frac{1}{2}(\mathbf{d}(\mathbf{u}(\mathbf{p})) - \mathbf{d}_{obs})^T\mathbb{G}_{obs}^{-1}(\mathbf{d}(\mathbf{u}(\mathbf{p})) - \mathbf{d}_{obs})$

Link with Bayesian: $\pi(\mathbf{p}|\mathbf{d}_{obs}) = \frac{1}{C}\pi(\mathbf{d}_{obs}|\mathbf{p}).\pi(\mathbf{p}) = Ce^{-S(\mathbf{p})}$

$C_0.e^{-\frac{1}{2}(\mathbf{p}-\mathbf{p}_0)^T\Sigma_0^{-1}(\mathbf{p}-\mathbf{p}_0)}$

$C_1.e^{-\frac{1}{2}(\mathbf{d}_{obs}-\mathbf{d}(\mathbf{u}(\mathbf{p})))^T[\Sigma_{obs}+\Sigma_{mod}]^{-1}(\mathbf{d}_{obs}-\mathbf{d}(\mathbf{u}(\mathbf{p})))}$

with $S(\mathbf{p}) = \frac{1}{2}(\mathbf{d}_{obs} - \mathbf{d}(\mathbf{u}(\mathbf{p})))^T[\Sigma_{obs} + \Sigma_{mod}]^{-1}(\mathbf{d}_{obs} - \mathbf{d}(\mathbf{u}(\mathbf{p}))) + \frac{1}{2}(\mathbf{p} - \mathbf{p}_0)^T\Sigma_0^{-1}(\mathbf{p} - \mathbf{p}_0)$

mCRE Framework

- **Modified CRE (reliability of info)** [Ladevèze *et al.* 94, Chouaki *et al.* 96]

$$\boxed{\mathcal{E}^2_{mCRE}(\hat{\mathbf{u}},\hat{\sigma};\mathbf{p}) = \underbrace{\mathcal{E}^2_{CRE}(\hat{\mathbf{u}},\hat{\sigma};\mathbf{p})}_{\text{modeling error term}} + \frac{\alpha}{2}\underbrace{(\mathbf{d}(\hat{\mathbf{u}})-\mathbf{d}_{obs})^T\mathbb{G}^{-1}_{obs}(\mathbf{d}(\hat{\mathbf{u}})-\mathbf{d}_{obs})}_{\text{distance to measurements}}}$$

The problem is split in : - reliable part (BC, equilibrium,…) $\longrightarrow$ admissible solution

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→ enforces **reliable theor./exp. info**: admissibility (regularization from physics)
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→ hybrid formulation
→ robust with noisy/corrupted data [Allix 05, Feissel & Allix 07]
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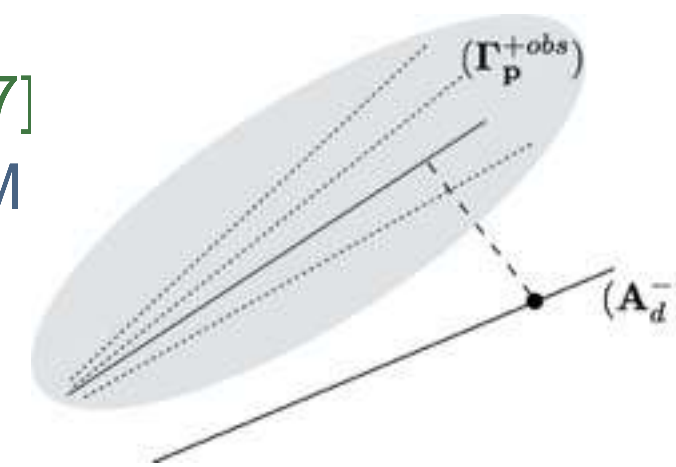

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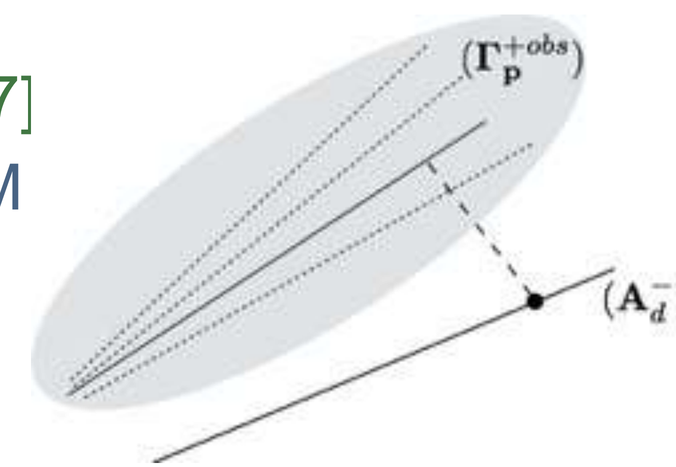

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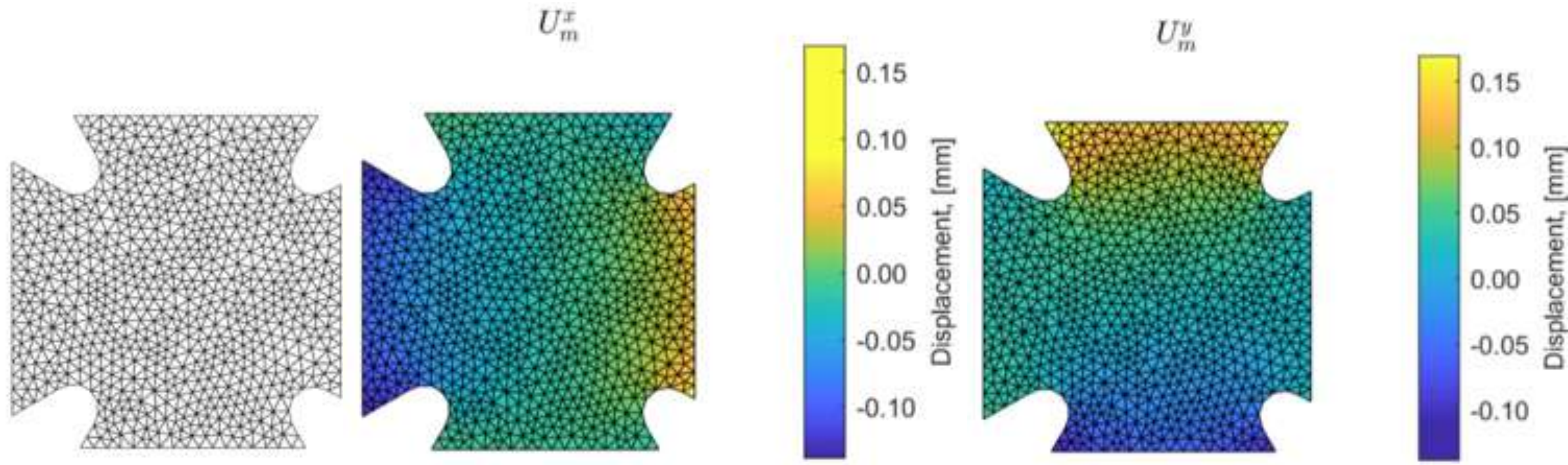
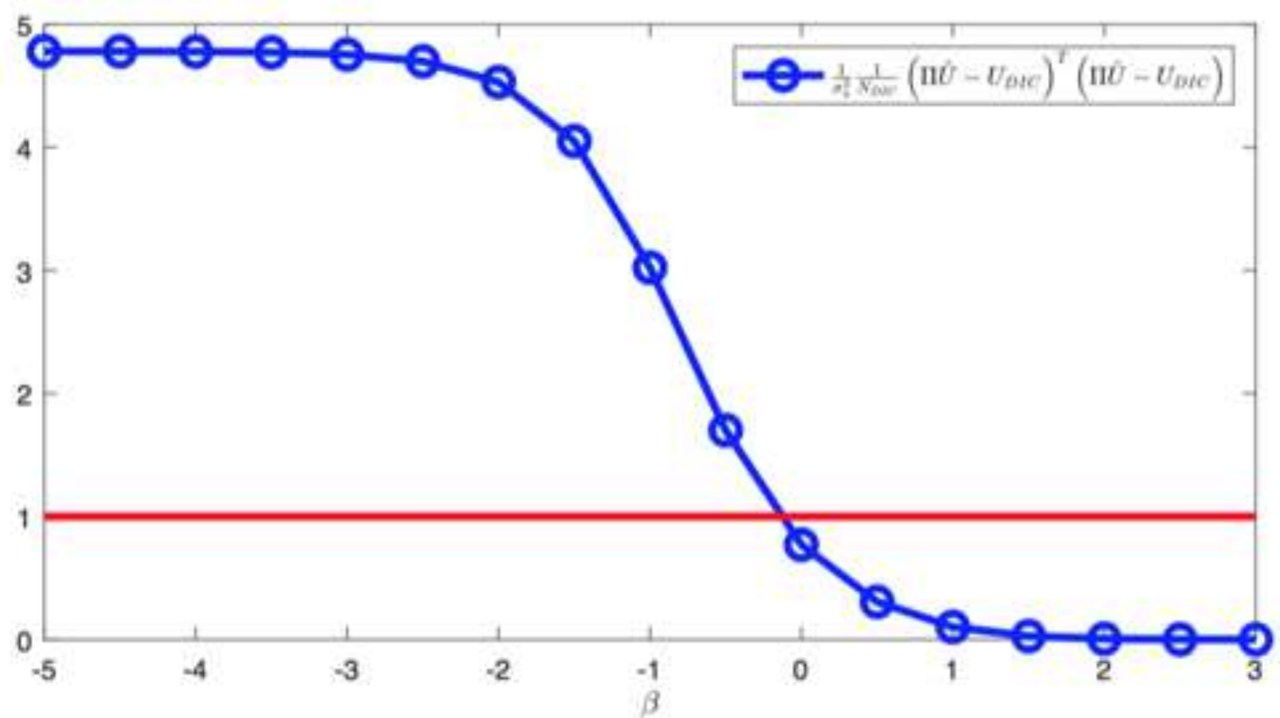
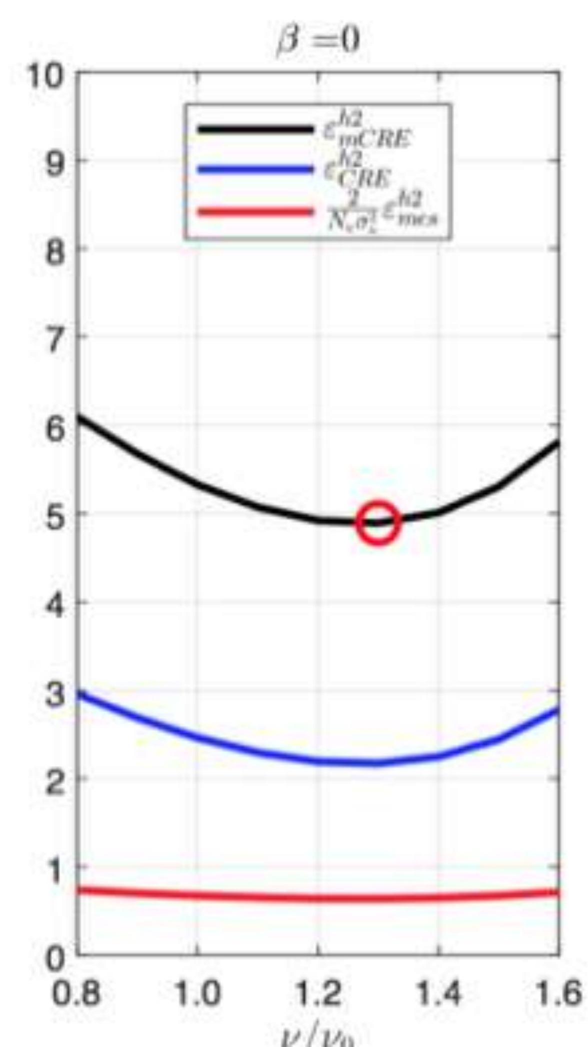
Example 1: Full-field Measurements

[PhD N. Nguyen 21]







Example 2: Viscoplasticity

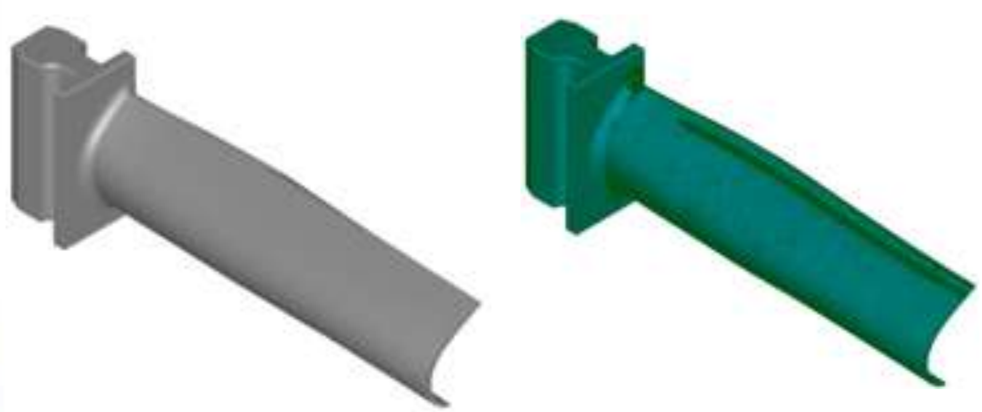
[Marchand *et al.* 15]

- extension to NL constitutive models using dual convex thermodynamics potentials

→ CRE measure from Legendre-Fenchel residuals (sym. Bergman divergence) [Ladevèze & Moës 99]

$$\boxed{\mathcal{E}^2_{CRE|t} = \int_\Omega \eta_\psi(\hat{\mathbf{e}}_e, \hat{\mathbf{s}}) + \int_0^t \int_\Omega \eta_\varphi(\dot{\hat{\mathbf{e}}}_p, \hat{\mathbf{s}})}$$

Rotating turbine blade (viscoplasticity)

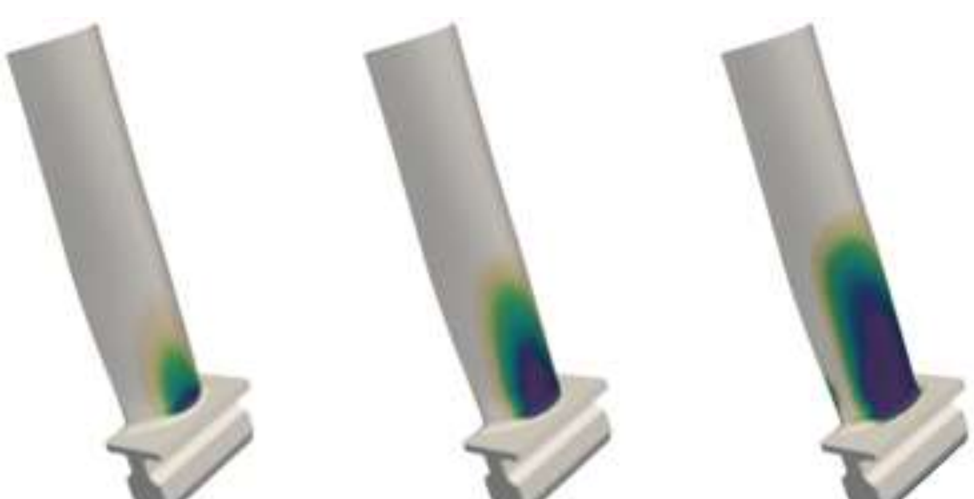


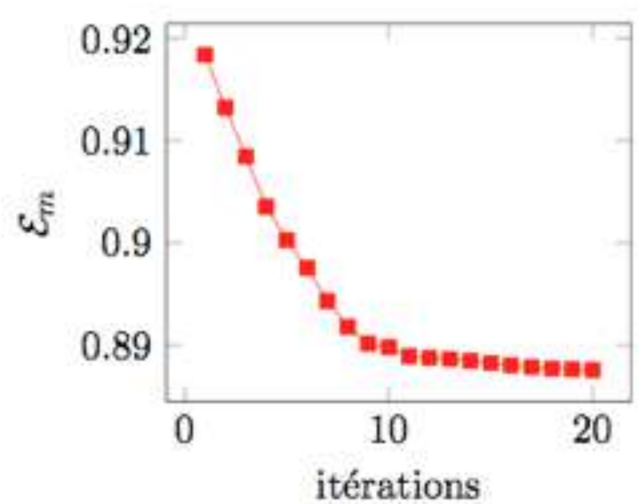
Prandlt-Reuss model
with isotropic hardening

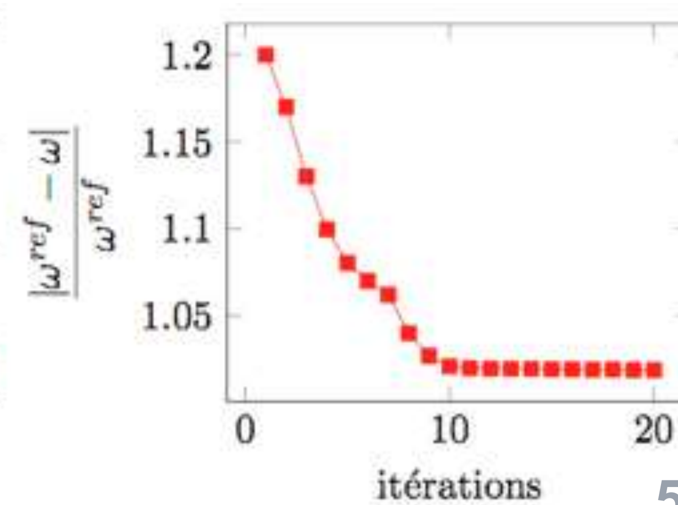
$$\begin{aligned} \psi(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}^{vp}, p) &= \frac{1}{2}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{vp}) : \boldsymbol{\mathcal{C}} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{vp}) + g(p) \\ \varphi^\star(\boldsymbol{\sigma}, R) &= \frac{k_v}{n_v+1} \left\langle \|\boldsymbol{\sigma}^D\| - (R + R_0) \right\rangle_+^{n_v+1} \end{aligned}$$

8 sensors on extrado +
8 sensors on intrado

$$\begin{aligned} \psi^\star(\boldsymbol{\sigma}, R) &= \frac{1}{2}\boldsymbol{\sigma} : \boldsymbol{\mathcal{C}}^{-1} : \boldsymbol{\sigma} + g^\star(R) \\ \varphi(\dot{\boldsymbol{\varepsilon}}^{vp}, -\dot{p}) &= R_0 \|\dot{\boldsymbol{\varepsilon}}^{vp}\| + \frac{1}{(n_v+1)k_v^{n_v}} \|\dot{\boldsymbol{\varepsilon}}^{vp}\|^{n_v+1} \end{aligned}$$







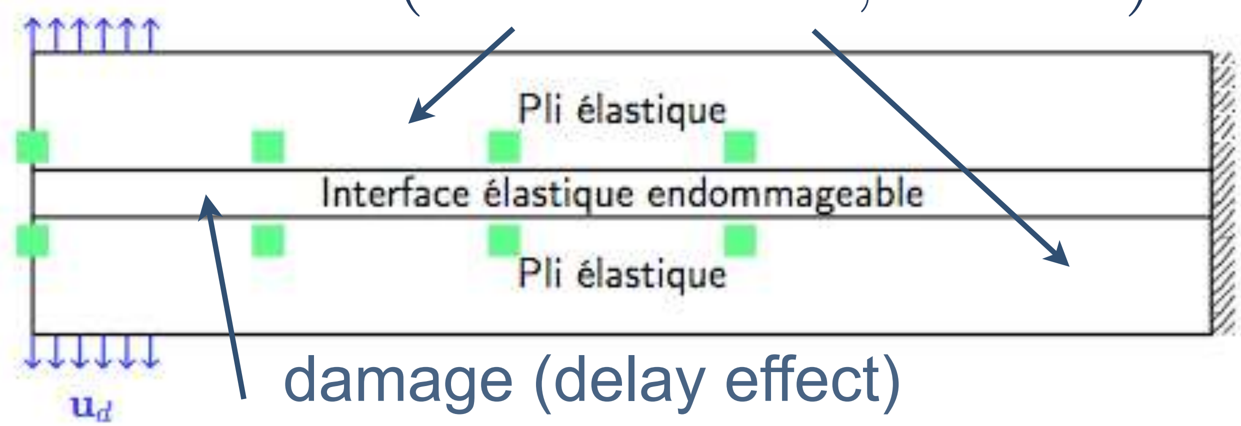
Example 3: Damage

[Marchand *et al.* 15]

DCB test on a composite

$$\dot{d} = \frac{k}{a}\left(1 - exp\left[-a\left\langle \frac{Y - Y_0}{Y_c - Y_0} - d^2 \right\rangle_+\right]\right)$$

$$(E = 123GPa, \nu = 0.3)$$



damage (delay effect)

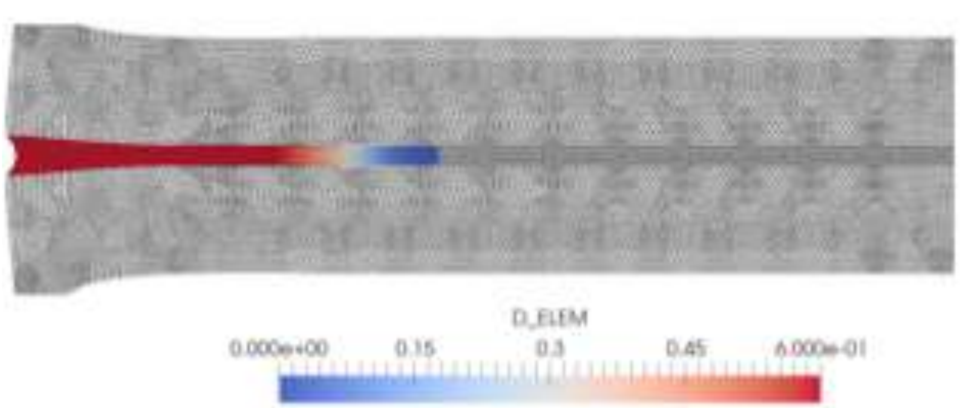
$$(E = 6GPa, \nu = 0.3, Y_c = 60, Y_0 = ???, a = 1, k = 1)$$

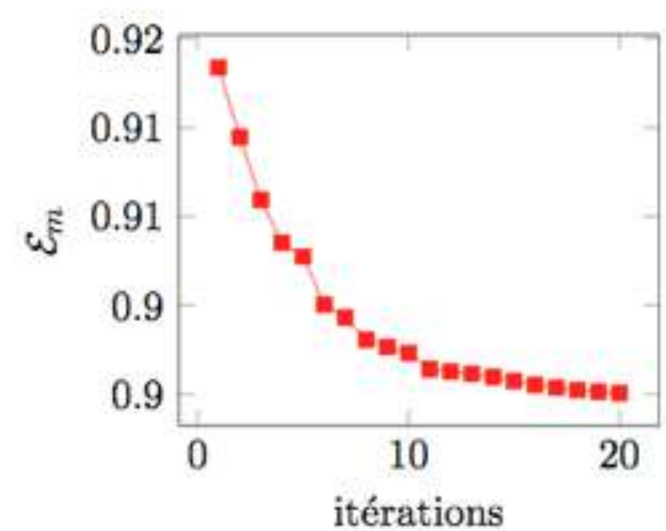
$$\psi(\boldsymbol{\sigma}, d, \alpha) = \frac{1}{2}\frac{\boldsymbol{\sigma} : \mathcal{C}^{-1} : \boldsymbol{\sigma}}{1-d} + \frac{(Y_c - Y_0)}{3}\alpha^3 + \Psi_{d\leq 1}$$

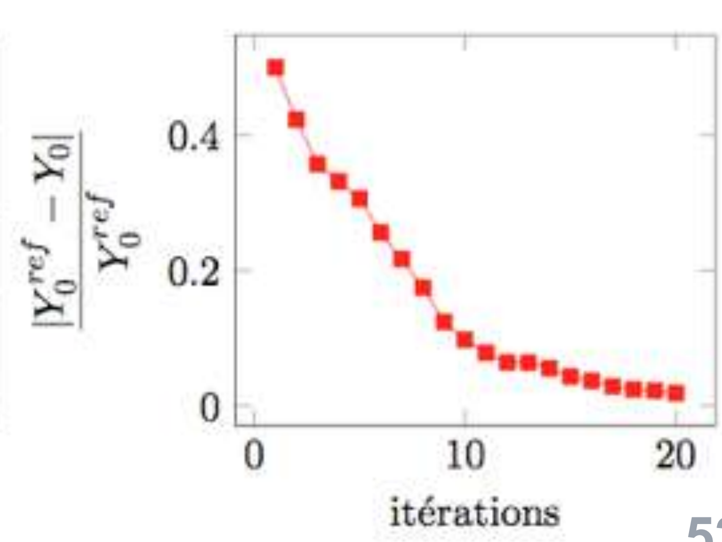
$$\varphi^\star(Y, \beta) = \frac{k}{a}\left[\langle Y - Y_0 - \beta\rangle_+ + \frac{Y_c - Y_0}{a}\left(\exp\left(-\frac{a\langle Y - Y_0 - \beta\rangle_+}{Y_c - Y_0}\right) - 1\right)\right]$$

$$\psi^\star(\boldsymbol{\varepsilon}^e, Y, \beta) = Y + \frac{2}{3}\frac{|\beta|^{3/2}}{\sqrt{Y_c - Y_0}} + \Psi_{Y \leq \frac{1}{2}\boldsymbol{\varepsilon}^e : \mathcal{C} : \boldsymbol{\varepsilon}^e}$$

$$\varphi(\dot{d}, -\dot{\alpha}) = \dot{d}\frac{(a-1)Y_0 + Y_c}{a} + \left(\frac{k}{a} - \dot{d}\right)\ln\left(1 - \frac{a}{k}\dot{d}\right)$$







Kalman Filtering

Dynamical system: $\begin{cases} \mathbf{u}^{(k+1)} & = \mathcal{M}^{(k)}\mathbf{u}^{(k)} + \mathbf{e_u}^{(k)} \\ \mathbf{s}^{(k)} & = \mathcal{H}^{(k)}\mathbf{u}^{(k)} + \mathbf{e_s}^{(k)} \end{cases}$

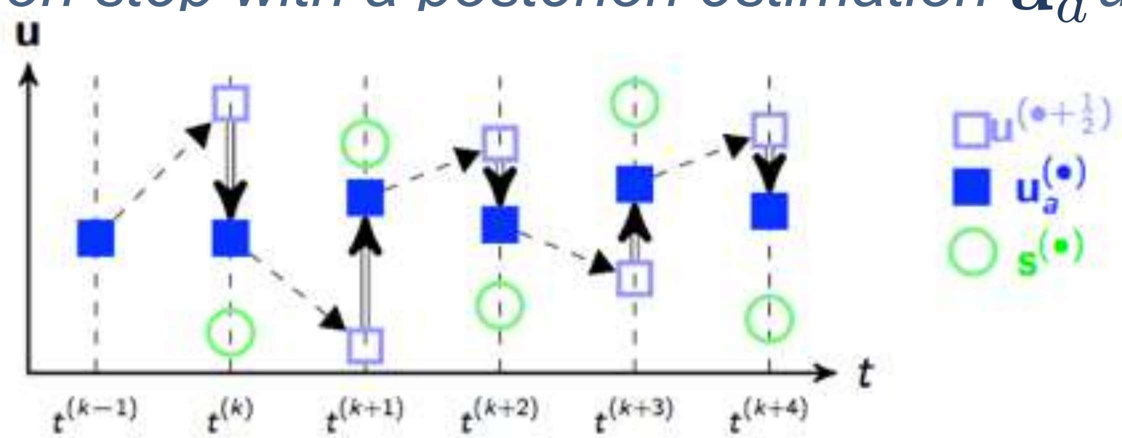
Bayes theorem: $\boxed{\pi\left(\mathbf{u}^{(k)}|\mathbf{s}^{(k)}\right) = \dfrac{\pi\left(\mathbf{s}^{(k)}|\mathbf{u}^{(k)}\right)\pi\left(\mathbf{u}^{(k)}|\mathbf{s}^{(0:k-1)}\right)}{\pi\left(\mathbf{s}^{(k)}|\mathbf{s}^{(0:k-1)}\right)}}$

→ under the assumptions:
- state $\mathbf{u}^{(k)}$ is a Markov process
- observations $\mathbf{s}^{(k)}$ are statistically independent of state history

Kalman filter: Bayesian filter combined with Maximum A Posteriori for Gaussian PDFs [Kalman 60, Corigliano *et al.* 01, Mariani & Ghisi 07, Chapelle *et al.* 09]

2 steps: 1) *prediction step with a priori estimation* $\mathbf{u}^{(k+\frac{1}{2})}$ *of the state*

2) *correction step with a posteriori estimation* $\mathbf{u}_a$ *using new observations*



→ *considers mean & covariance (linearized version of Bayesian inference)*

Kalman Filtering

The Kalman filter is a convenient tool to solve inverse problems
[Mariani & Corigliano 04, Moireau & Chapelle 11]

↳ introduce the parameters vector $\boldsymbol{\xi} \in \mathbb{R}^{n_p}$ in the dynamical system
no *a priori* knowledge ⟶ stationarity assumption

$$\frac{\partial \xi}{\partial t} \simeq 0 \quad \Rightarrow \quad \xi^{(k+1)} = \xi^{(k)} + \boxed{\mathbf{e}_{\xi}^{(k)}}$$

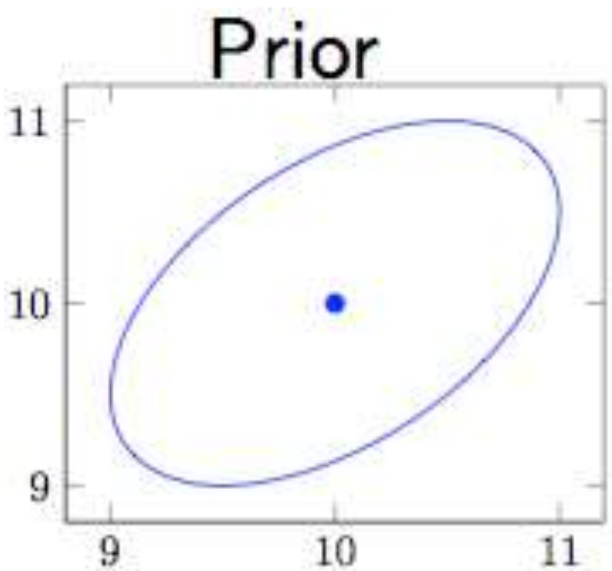
2 formulations

Joint Kalman Filter	Dual Kalman Filter
$\begin{cases} \bar{\mathbf{u}}^{(k+1)} = \bar{\mathcal{M}}^{(k)} \bar{\mathbf{u}}^{(k)} + \bar{\mathbf{e}}_M^{(k)} \\ \mathbf{s}^{(k)} = \bar{\mathcal{H}}^{(k)} \bar{\mathbf{u}}^{(k)} + \mathbf{e}_s^{(k)} \end{cases}$ $\begin{bmatrix} \mathbf{u}^{(k)} \\ \xi^{(k)} \end{bmatrix}$	$\begin{cases} \xi^{(k+1)} = \xi^{(k)} + \mathbf{e}_{\xi}^{(k)} \\ \mathbf{s}^{(k)} = \mathcal{H}^{(k)} \underbrace{\mathbf{u}^{(k)}(\xi^{(k)})} + \mathbf{e}_s^{(k)} \end{cases}$ computed with another Kalman filter

⟶ nonlinear system

Kalman Filtering

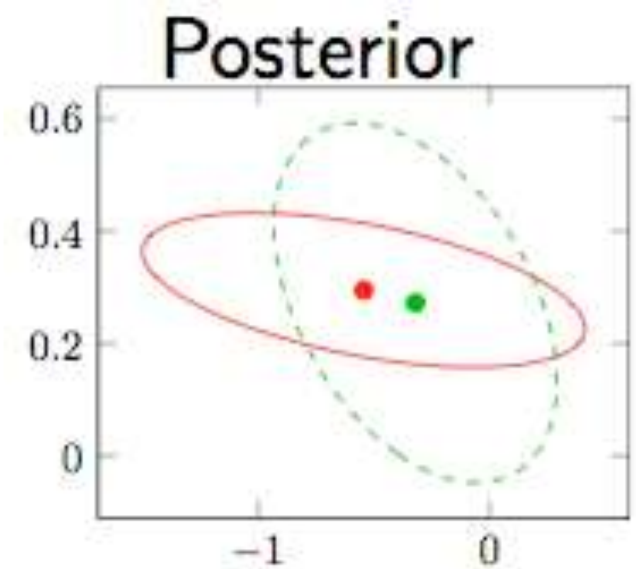
Gaussian $\mathcal{N}(\bar{\mathbf{x}}, \mathbf{C}_{\mathbf{x}})$ → nonlinear operator $\mathcal{A}$ → Gaussian $\mathcal{N}(\bar{\mathbf{y}}, \mathbf{C}_{\mathbf{y}})$

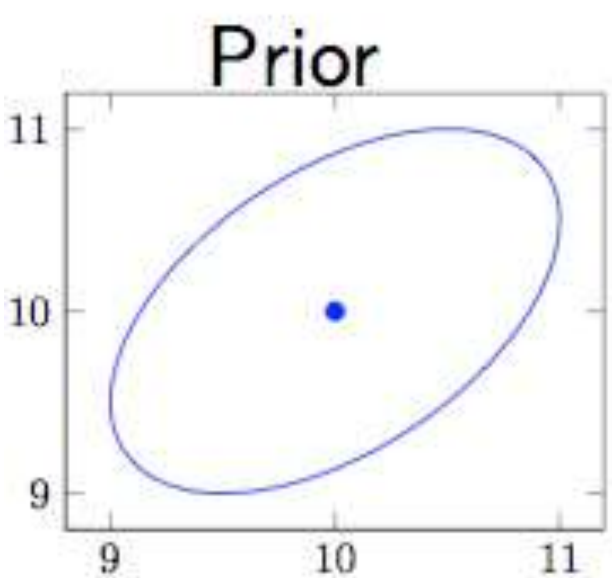


Extended Kalman Filter (EKF)

(first order linearization)

$$\begin{aligned}\mathbf{A} &= \nabla_{\mathbf{x}}\mathcal{A}\\ \bar{\mathbf{y}} &= \mathcal{A}(\bar{\mathbf{x}})\\ \mathbf{C}_{\mathbf{y}} &= \mathbf{A}\mathbf{C}_{\mathbf{x}}\mathbf{A}^T\end{aligned}$$



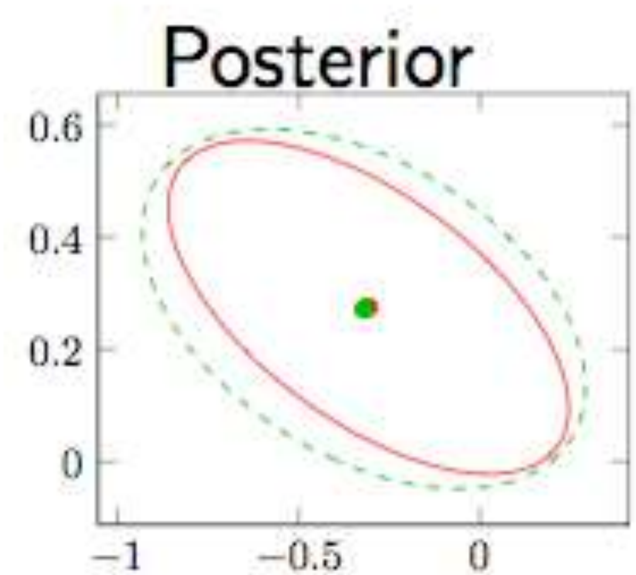


Unscented transform (UKF)

(propagation on σ- points)

$$\{x_i\}_{i=1,\ldots,2N+1}$$
$$\{y_i\} = \mathcal{A}(\{x_i\})$$

[Julier & Uhlmann 97]



mKF Strategy

[Marchand *et al.* 16]

Kalman filtering is well-adapted to dynamical systems and DDDAS paradigm

BUT: → very costly if the number of dofs/parameters is large

→ accuracy strongly depends on data noise [Li *et al.* 16]

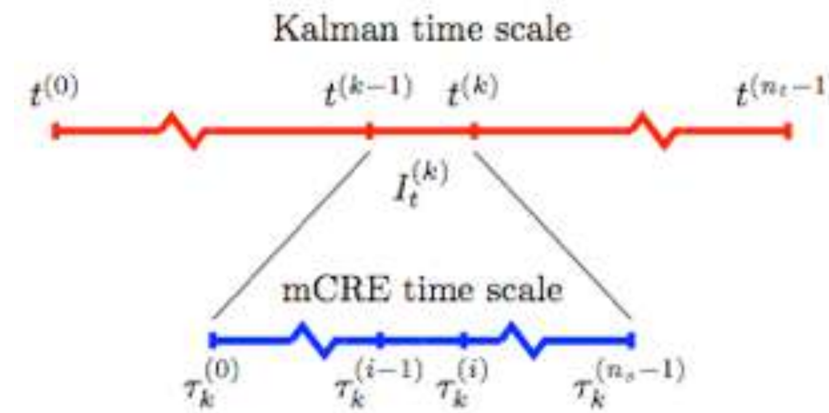
$$\begin{cases} \xi^{(k+1)} = \xi^{(k)} + \mathbf{e}_\xi^{(k)} \\ \mathbf{s}^{(k)} = \mathcal{H}^{(k)} \boxed{\mathbf{u}^{(k)}(\xi^{(k)})} + \mathbf{e}_s^{(k)} \end{cases}$$

$$\begin{cases} \xi^{(k+1)} = \xi^{(k)} + \mathbf{e}_\xi^{(k)} \\ \mathbf{s}^{(k)} = \boxed{\mathcal{H}_m^{(k)}(\xi^{(k)}; \mathbf{s}^{(k-1:k)})} + \mathbf{e}_s^{(k)} \end{cases}$$

defined from the mCRE functional

- Kalman filtering for parameters alone
- **change of metric in the observation space** (optimal admissible fields from mCRE)

mKF Strategy



computation of the state over $[t_{k-1}, t_k]$

solved with a Lagrangian functional

FE discretization

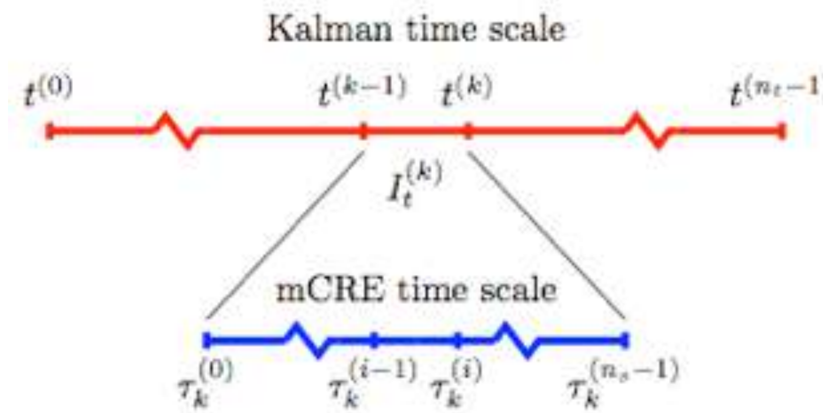
$$\begin{bmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & -\mathbf{C} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{u}} \\ \dot{\boldsymbol{\lambda}} \end{bmatrix} + \begin{bmatrix} \mathbf{K} & -\mathbf{K} \\ \delta\mathbf{\Pi}^T\mathbf{\Pi} & \mathbf{K} \end{bmatrix} \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{ext} \\ \delta\mathbf{\Pi}^T\mathbf{s} \end{bmatrix} \quad \forall t \qquad \begin{aligned} \boldsymbol{u}(\tau_k^{(0)}) &= \mathbf{u}^{(k-1)} \\ \boldsymbol{\lambda}(\tau_k^{(n_s-1)}) &= \mathbf{0} \end{aligned}$$

coupled parametrized forward-backward problem at each time step

→ used of PGD (decompositions computed simultaneously)

→ extra-parameters:
- space, time
- parameters ξ to identify
- observation data (at both ends)
- initial conditions (projection over a basis $u_0^{(k)} = \sum_{i=1}^{n_{init}} \alpha_i \psi_i(x)$)

mKF Strategy



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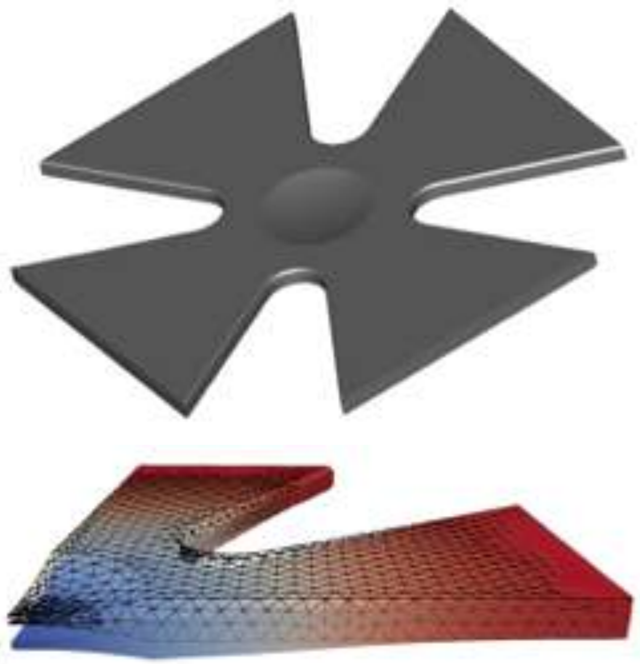
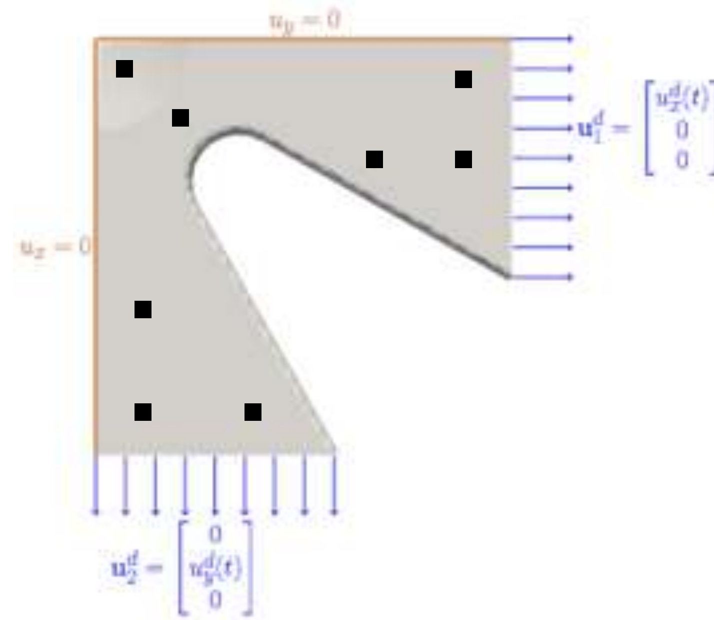
***offline* phase**	***online* phase (at each time step)**
▸ reduced basis computation for IC ▸ PGD admissible fields computation	▸ project IC on the reduced basis ▸ evaluate PGD solution for σ points ▸ project state into observation space ▸ Kalman update

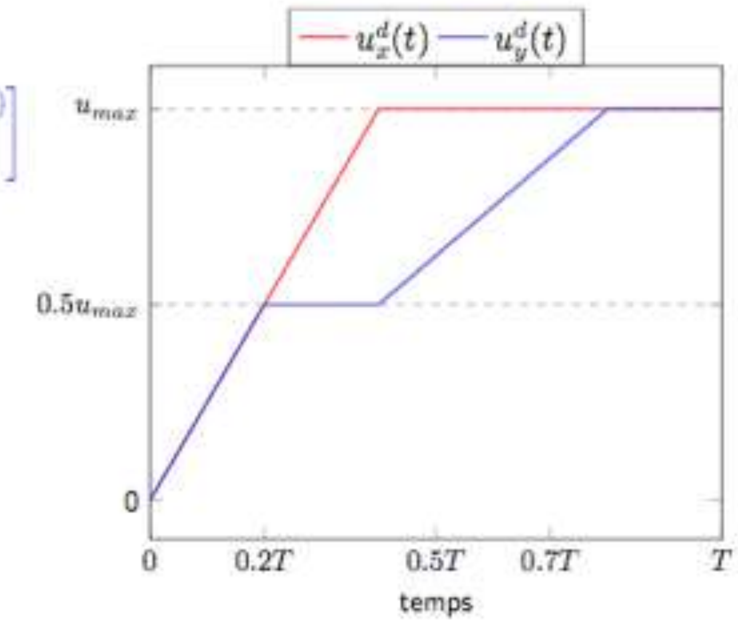
Illustration

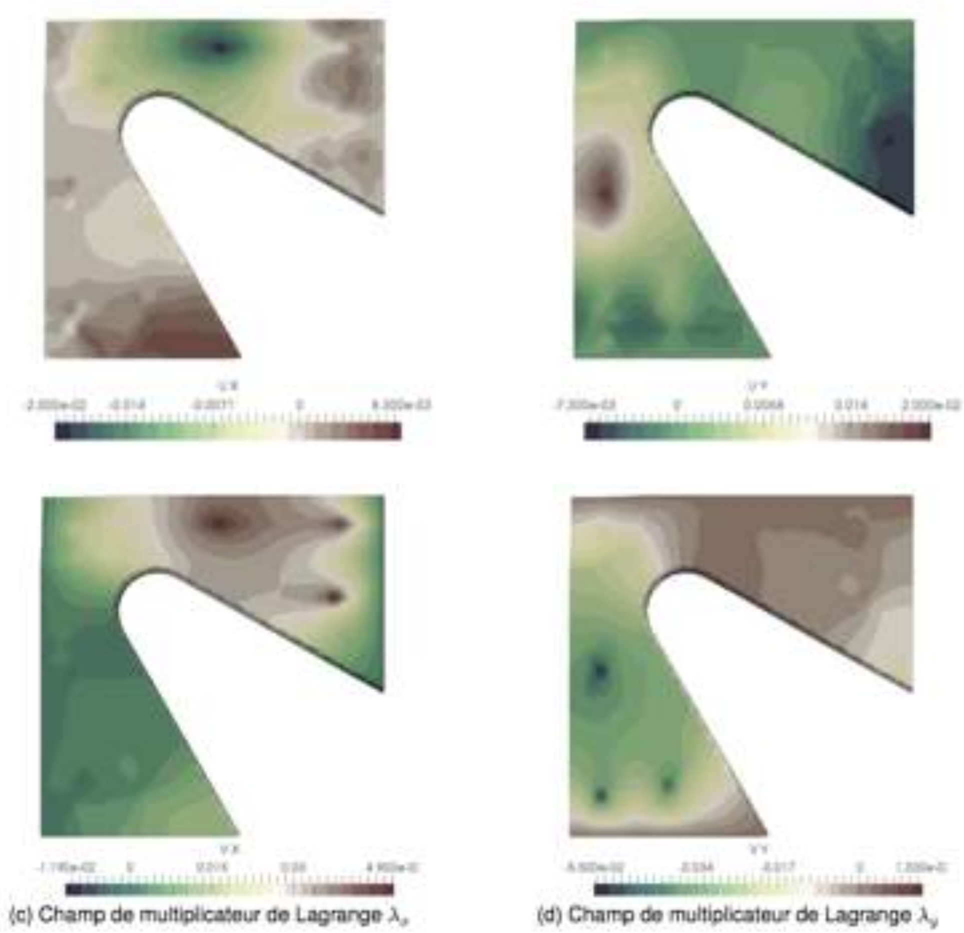
Viscoplasticity case (Prandtl-Reuss)

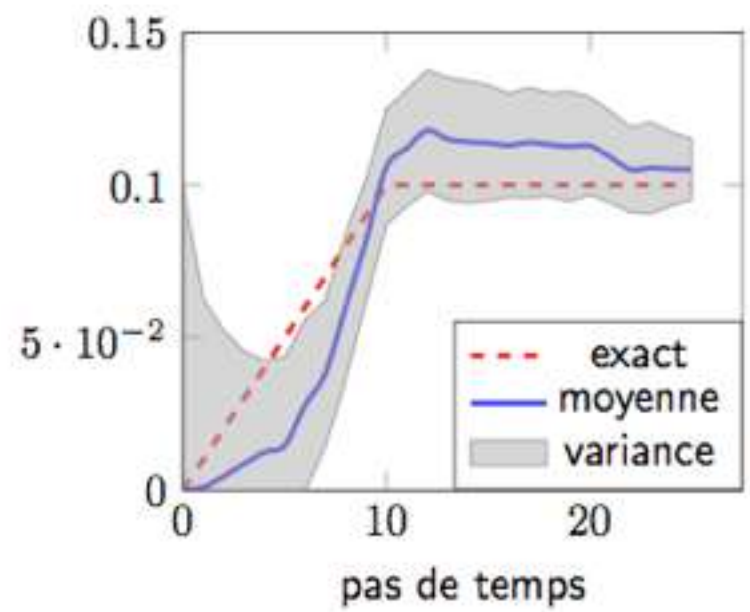
[Marchand *et al.*]

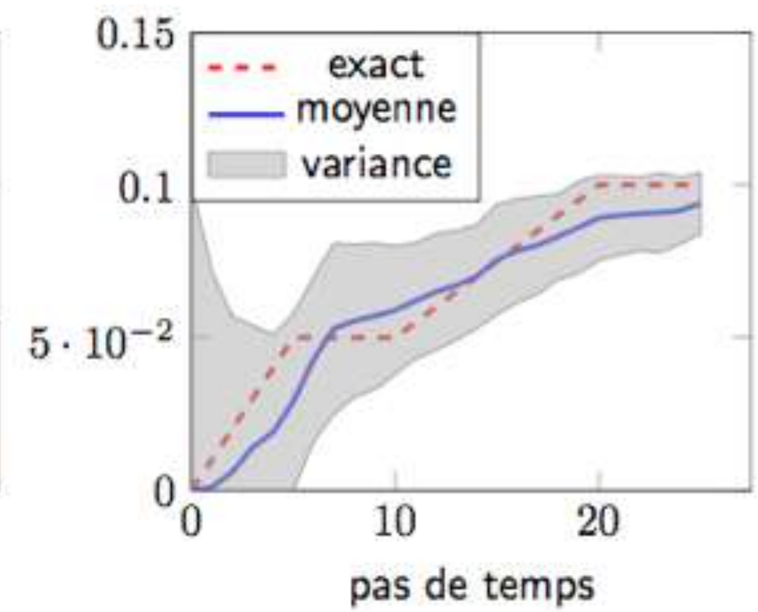












average CPU time $\approx$ 1s

Outline

1. Reduced order modeling
2. Fully stochastic approach for sequential data assimilation
3. Online model bias correction & feedback control
4. Alternative coupled deterministic-stochastic approach
5. Ongoing projects & new challenges

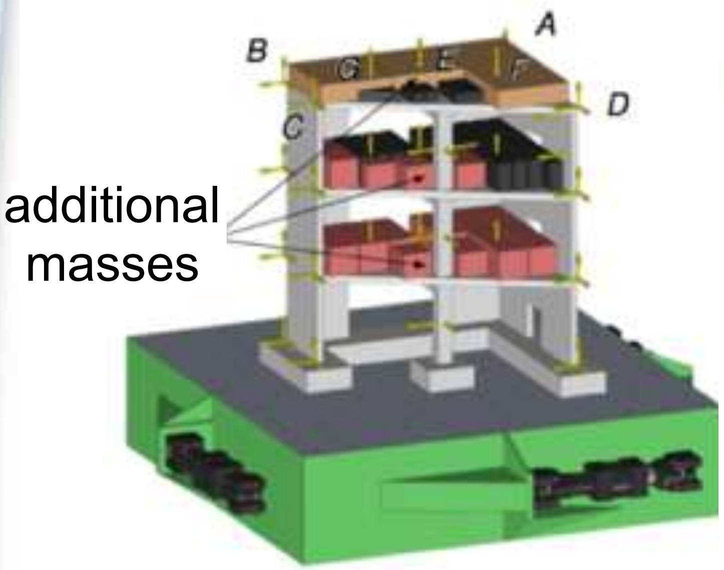
Conclusions & Prospects

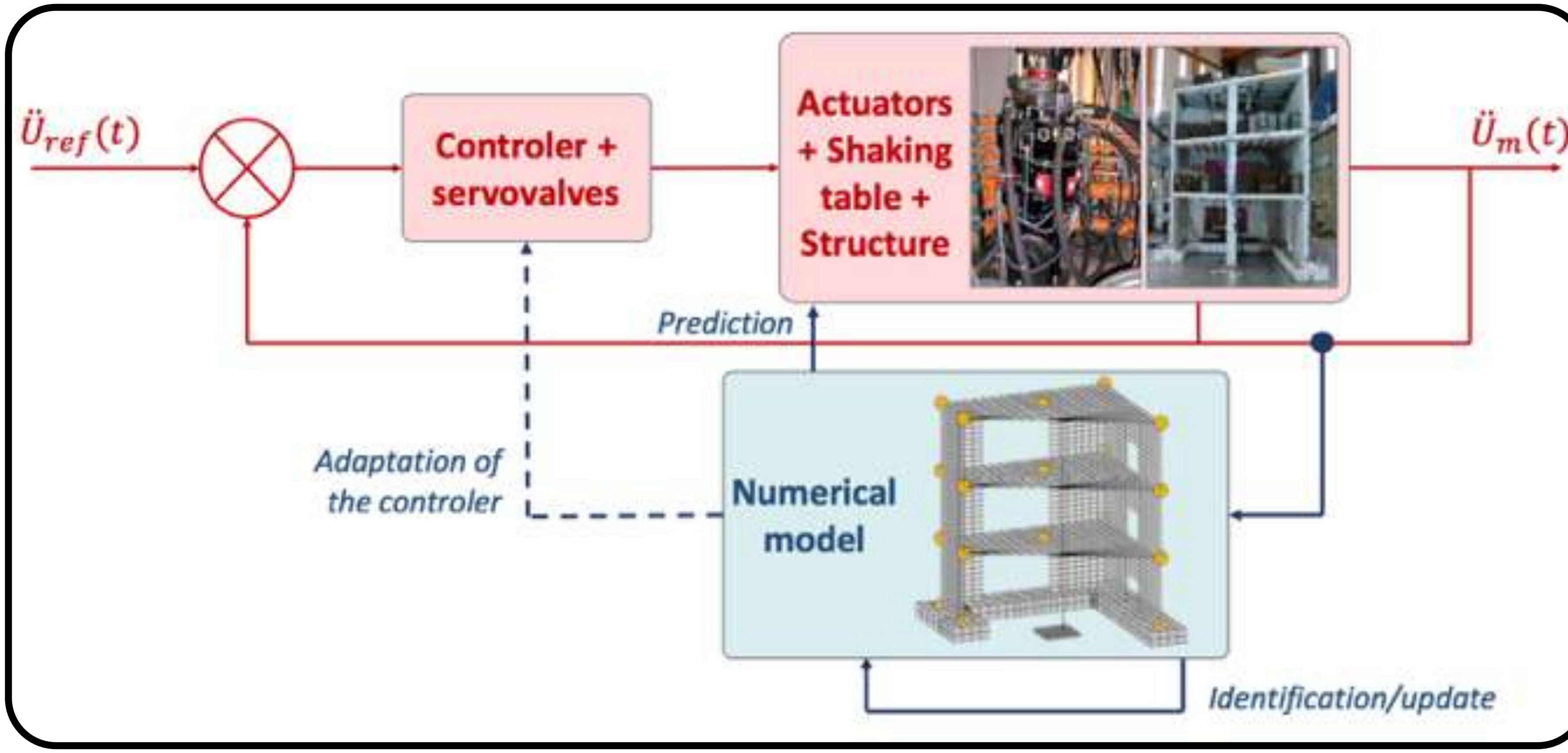
- **Conclusions:**
  - Two strategies for real-time data assimilation with hybrid formulation
  - PGD model reduction allows to get fast computations
  - Application on several linear/nonlinear structural mechanics problems

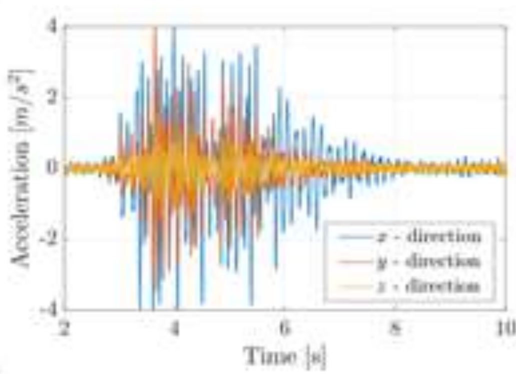
- **Prospects & challenges:**
  - Improve TM sampling (regression of map compositions, auto. order adaptivity)
  - More complex constitutive models with local multi-scale effects (e.g. damage in composites)
  - Investigate high-dimension problems (field identification)
  - Dynamic adaptive modeling (multi-fidelity) & data selection
  - Determine & interpret model enrichment with constrained-AI tools (learning)
  - Command synthesis on evolving systems with UQ (*Model Predictive Control*) [Le Coent *et al.* 17,18]

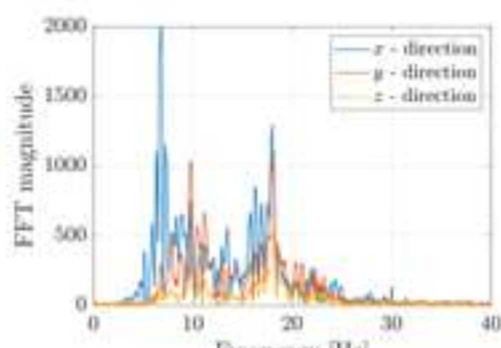
Control of Tests on Shake-Tables

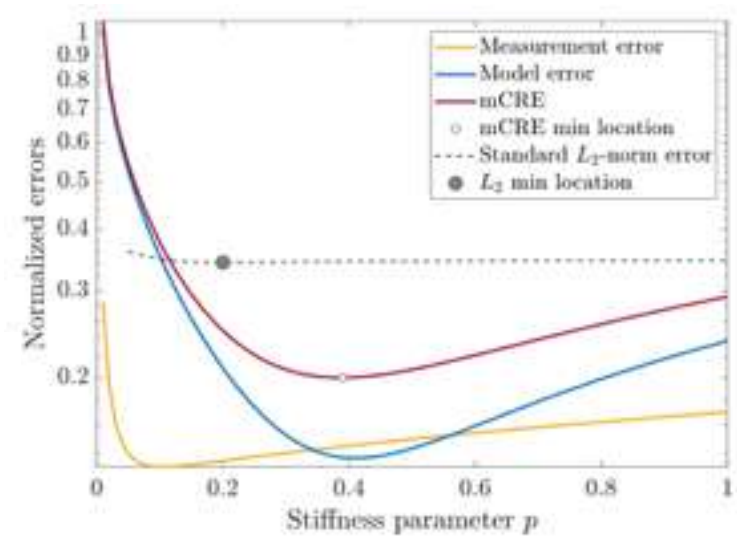
[PhD M. Diaz 2020-2023]

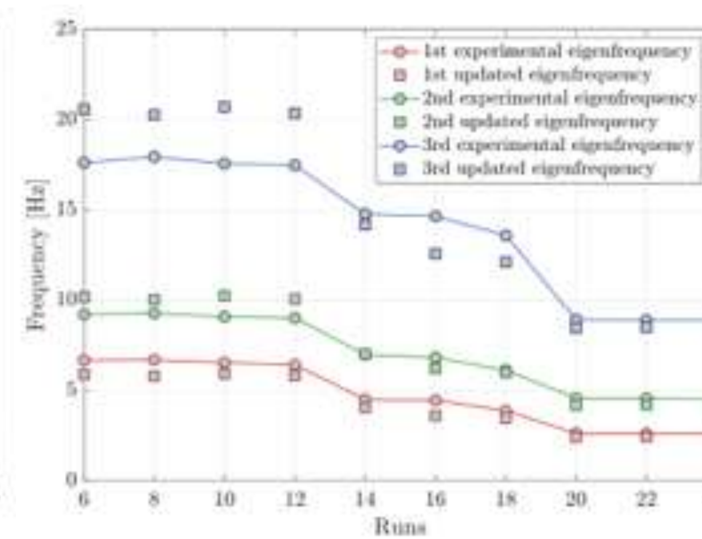


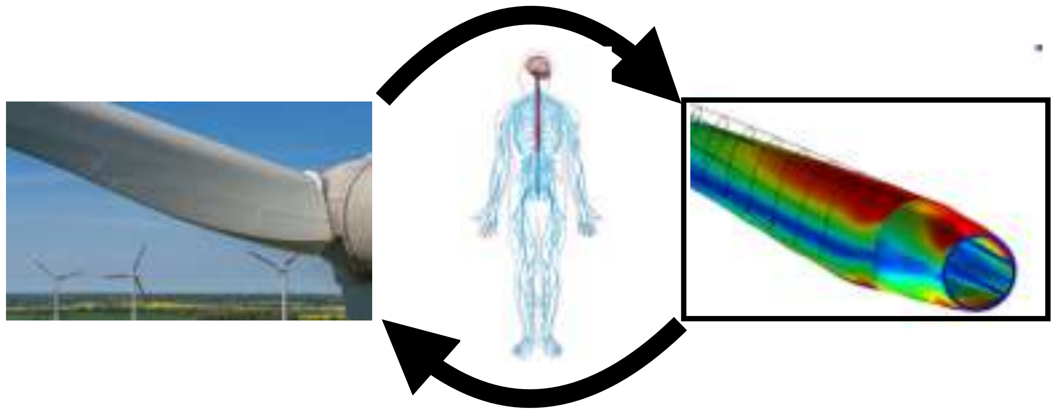






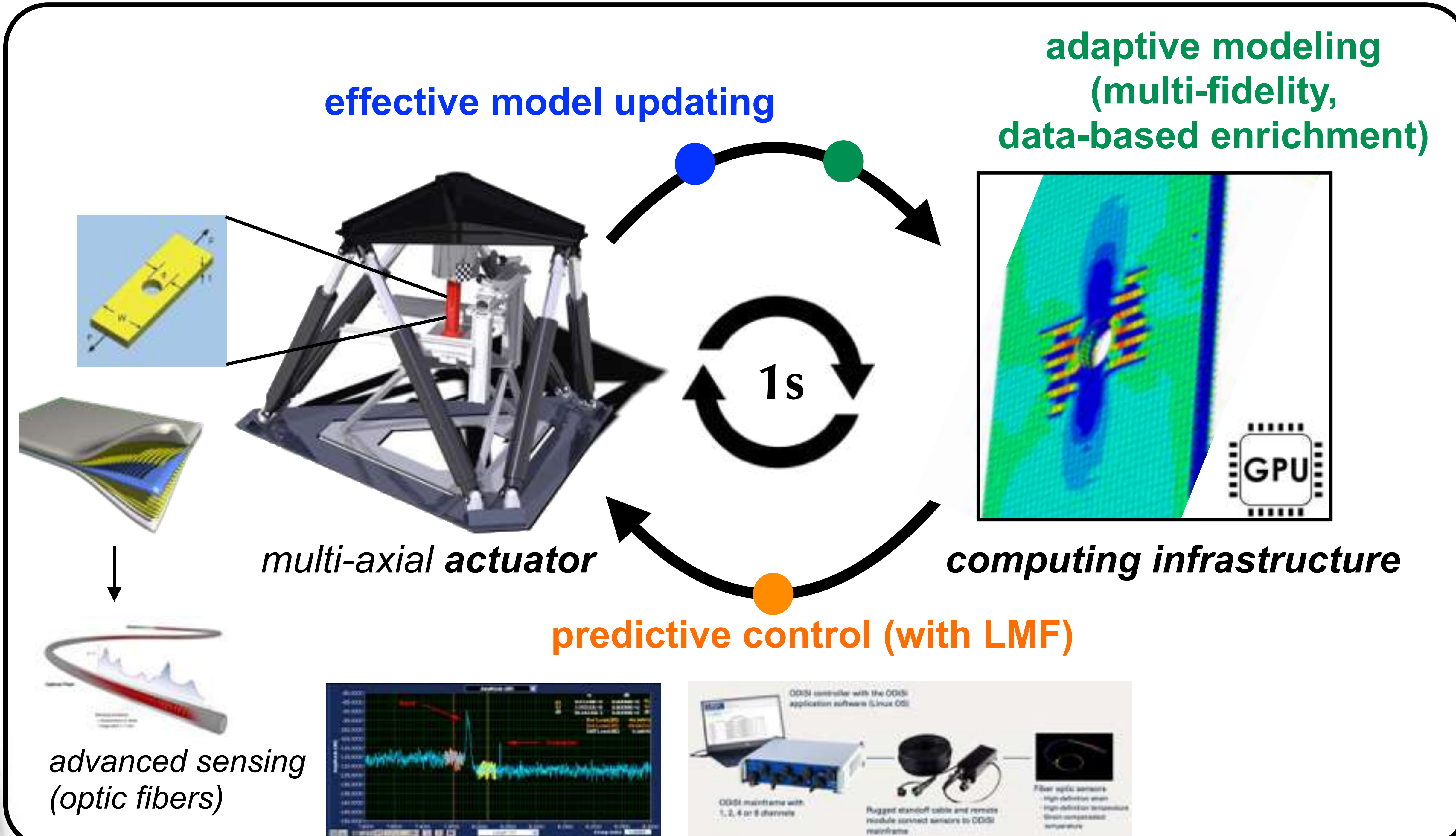




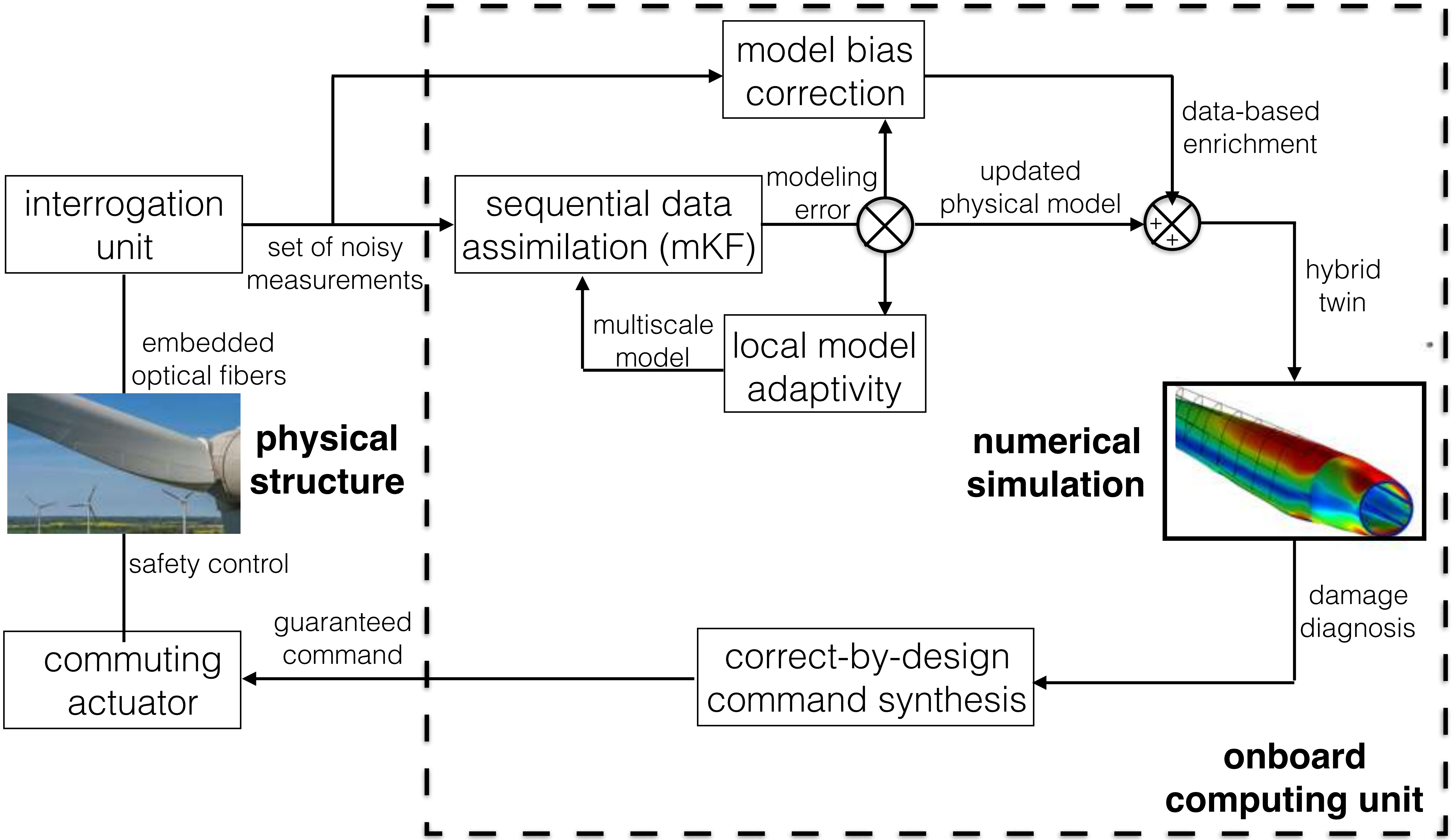


DREAM-ON Project

[ERC-CoG 2021-2026]



DREAM-ON Project



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Thank you!!