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# TWINS

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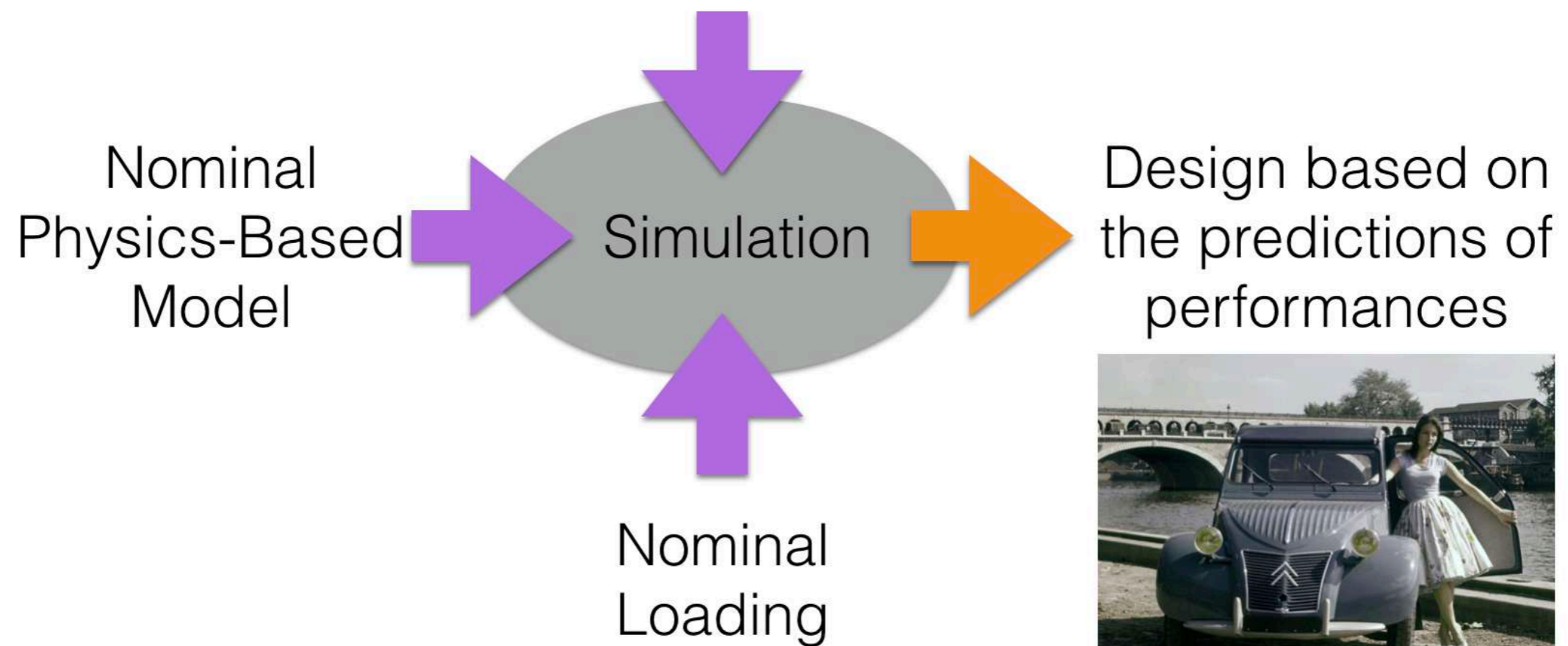


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# SBE@XX

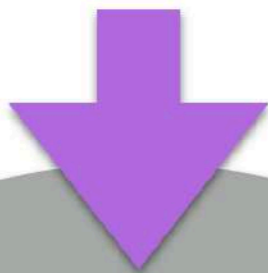
Few Data for Model Calibration



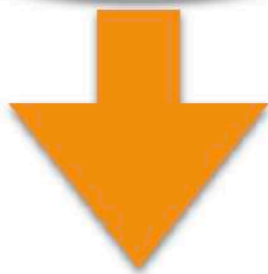
# SBE@XXI

Real load

$$\tau \leq t$$



Real system  
in service at  
time  $t$



Prediction  $\tau \geq t$

*World is changing. Today we do not sell aircraft engines, but hours of flight, we do not sell electric drills but good quality holes, ... We are nowadays more concerned by the performance management than by the products themselves ...*

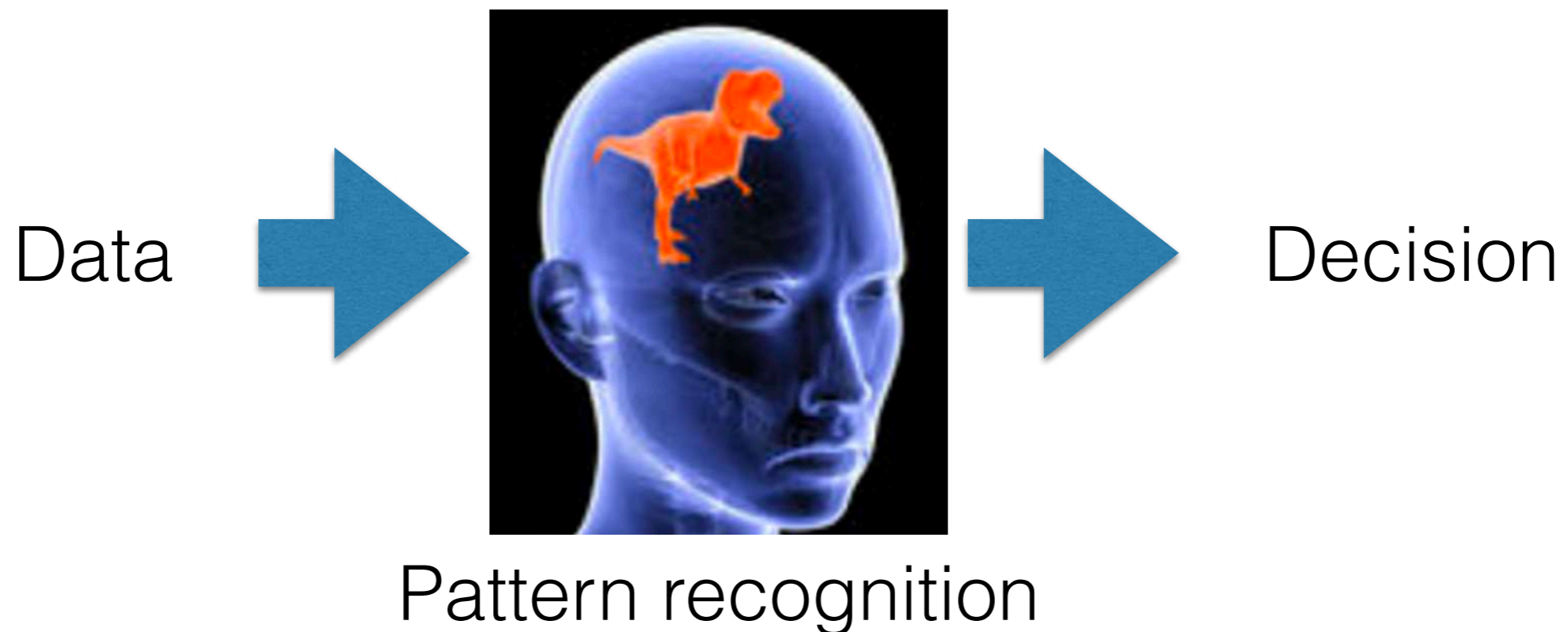


# A Twin is not

**A monitored and controlled**  
material, process, structure or system

*Automation was the 3rd industrial revolution*

Model-based or Data-Driven  
Transfer Function





# A Twin is not

## **Simulation - Based Engineering Science**

*Revolutionizing Engineering Science  
through Simulation*

*February 2006*

*Report of the National Science Foundation  
Blue Ribbon Panel on  
Simulation-Based Engineering Science*

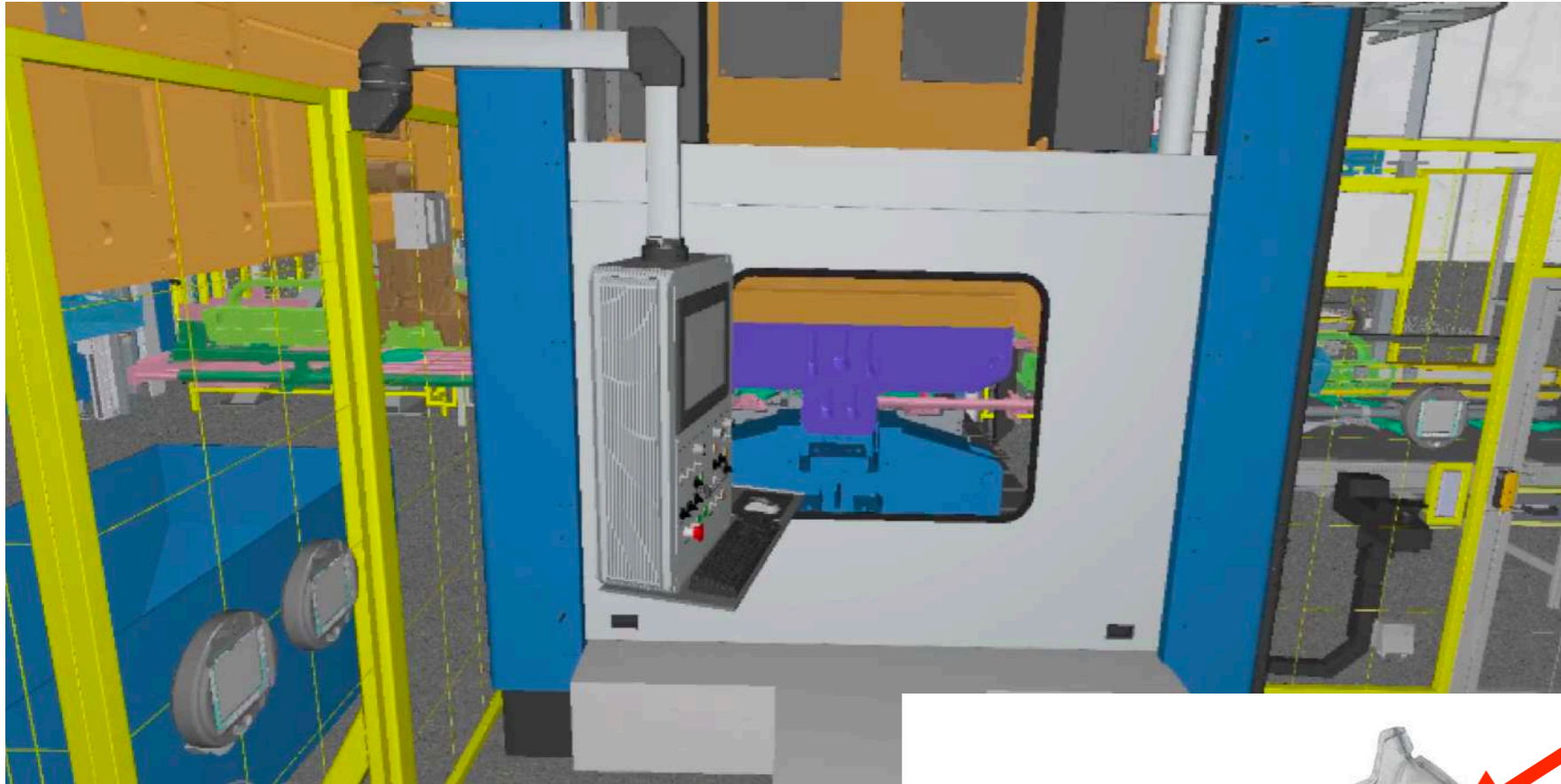


A simply **DDDAS**  
-Dynamic Data Driven  
Application Systems-

Models, even when calibrated (data-assimilation) rarely represent accurately the reality, in particular when addressing systems of systems combining complexity, size and uncertainty ...



# A Twin should be

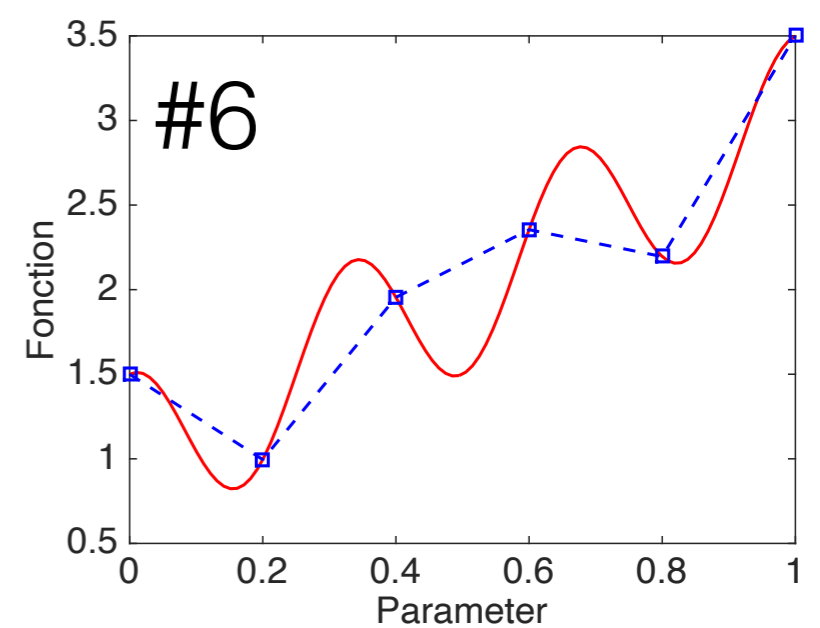
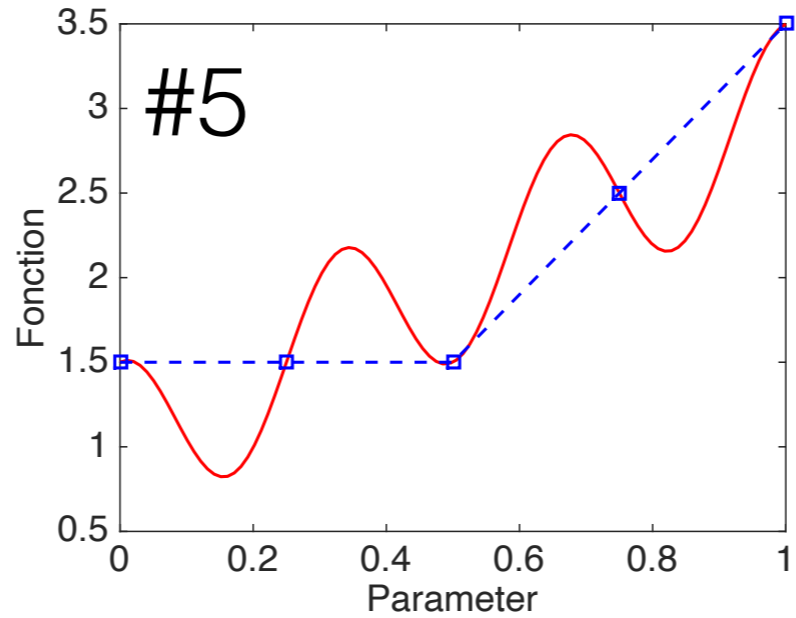
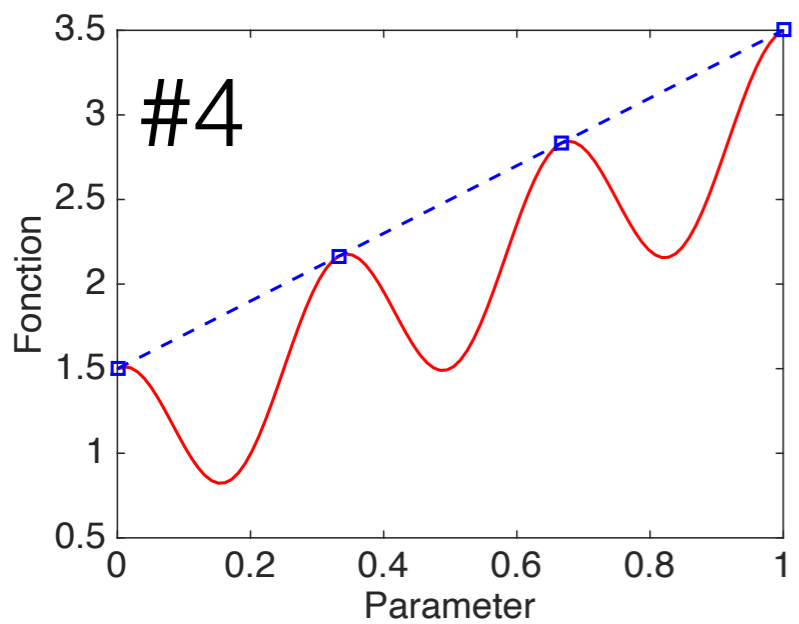
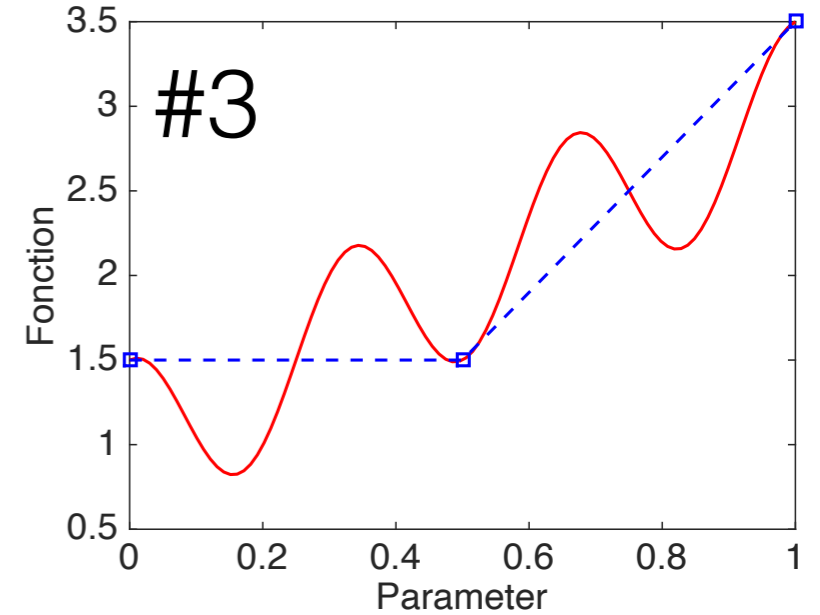
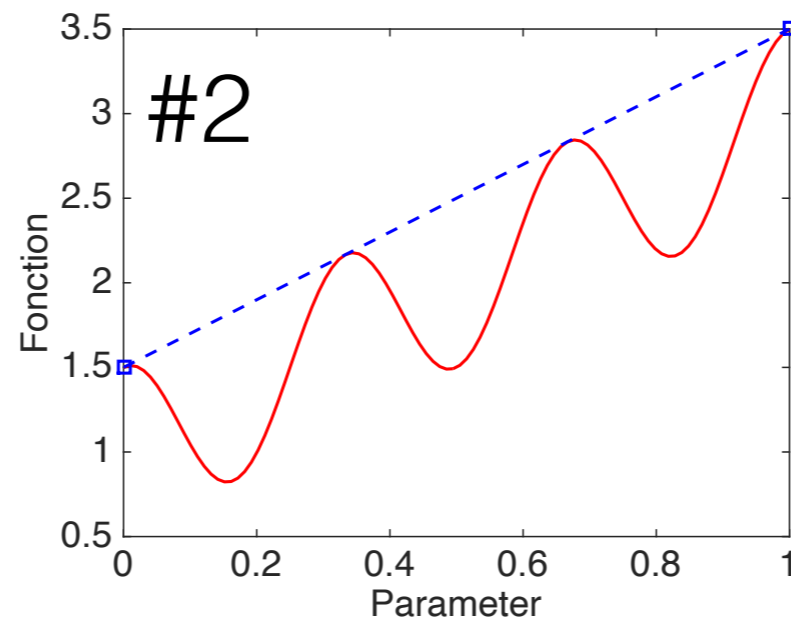
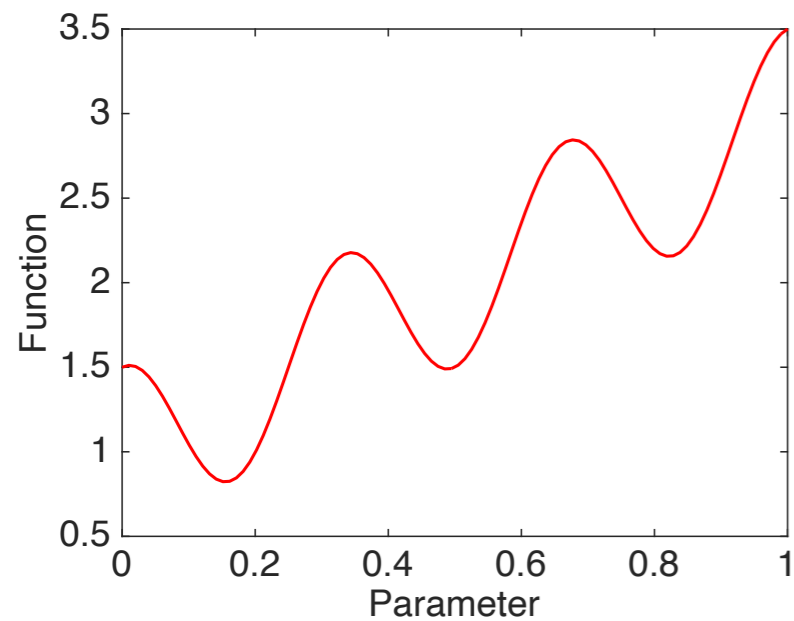


Thickness  
measurement

Inferred residual  
stress

**Frugal, Intrinsic, Holistic, Fast, Accurate,  
Explainable, Certifiable & Usable**

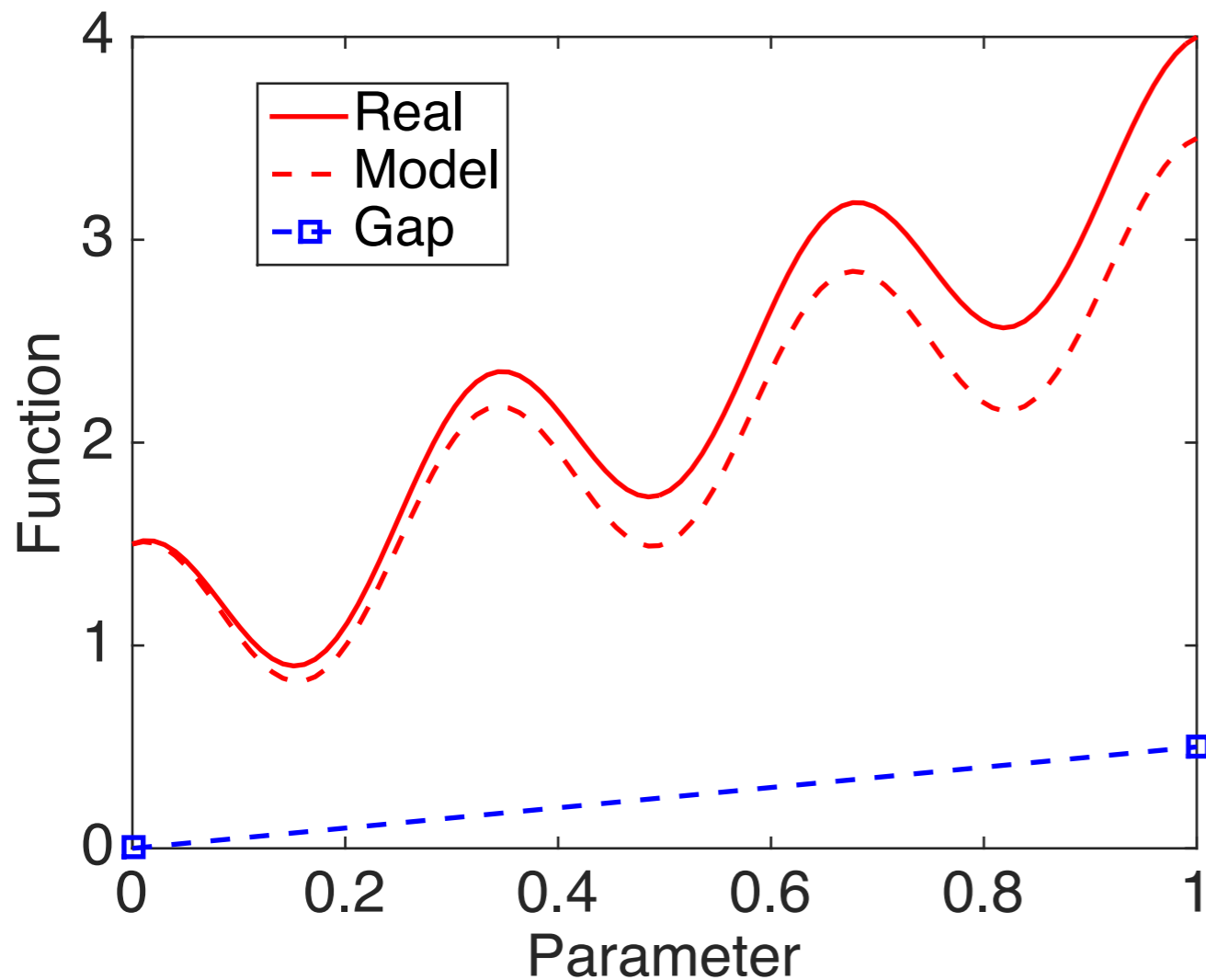
# Data-Based Representation



Enormous amounts of data are compulsory for representing strongly nonlinear functions

# The Hybrid Paradigm

Behavior = Model + Deviation (Ignorance)

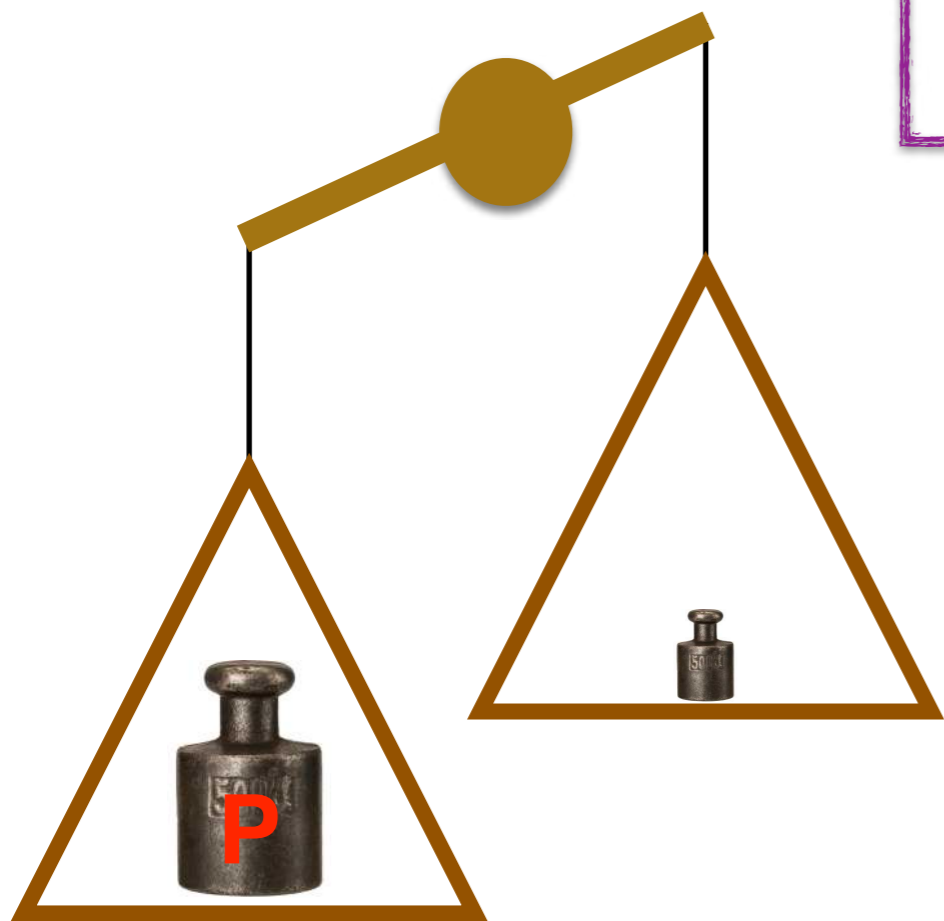


In this example the deviation being lineal it is accurately approximated with only two data

In the general case, being the deviation much less nonlinear, its approximation requires much less data

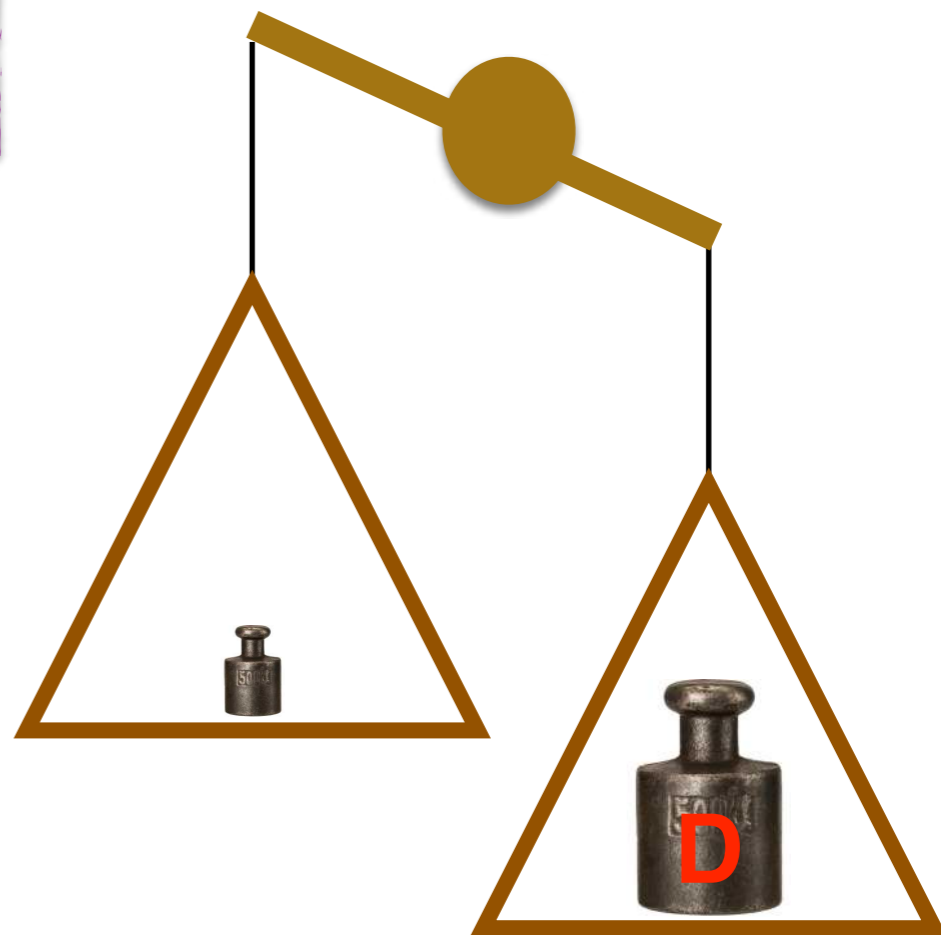


# Virtual Twin

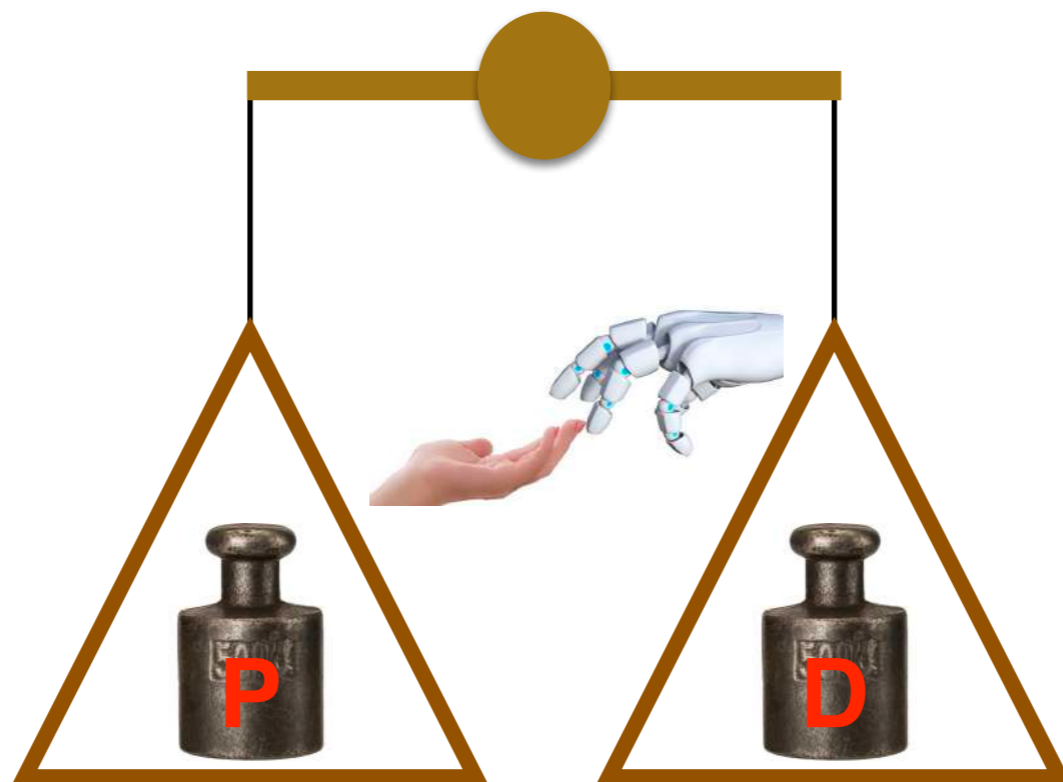


**P**: Physics  
**D**: Data

# Digital Twin



# Hybrid Twin



# PROS

The Physics-Based Model  
reduces the data needs

The Physics-Based Model  
enables certification

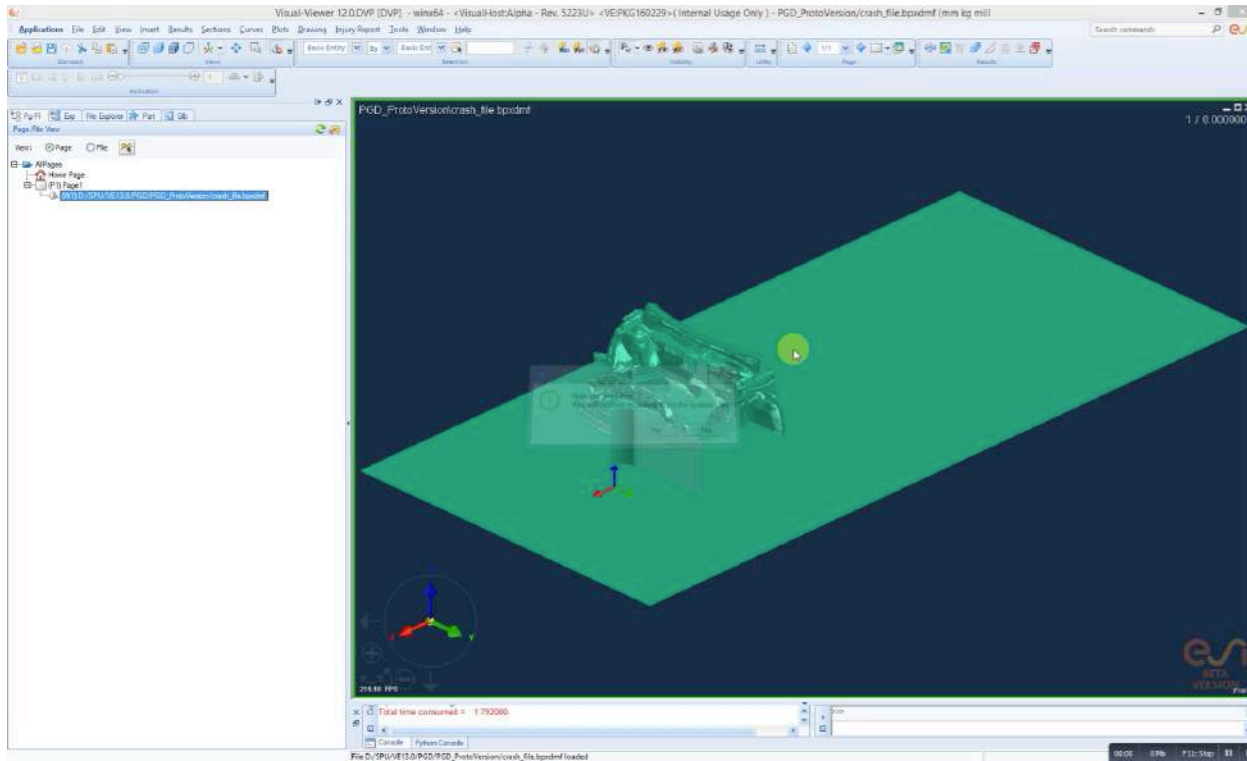
# Issues = Opportunities

The Physics-Based Model must be solved in real-time

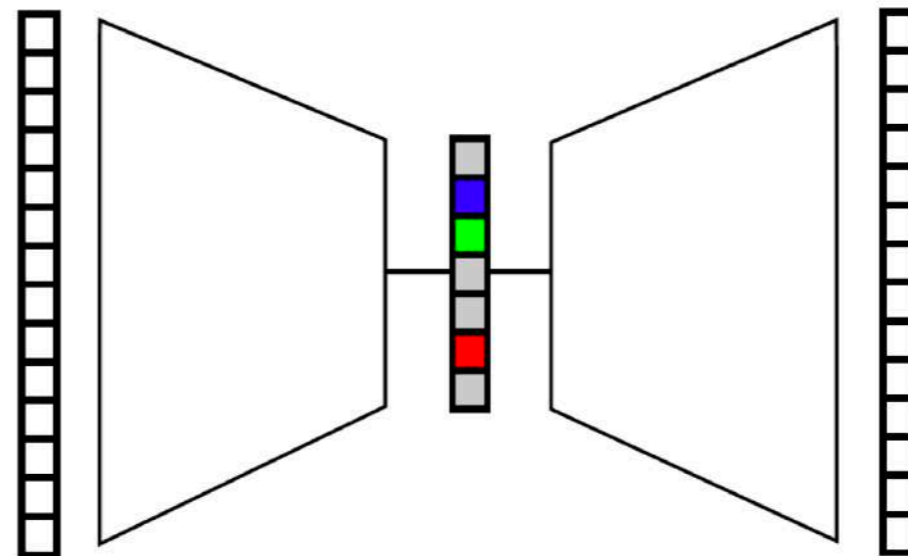
The Data-Driven Model must be learned in-the-fly while using few amount of data



# What is needed



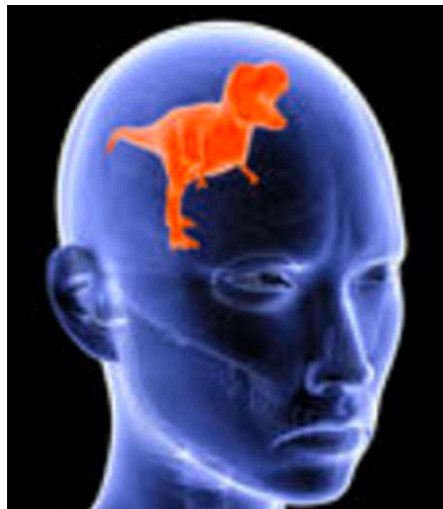
**Real-Time Physics +  
Physics aware AI**



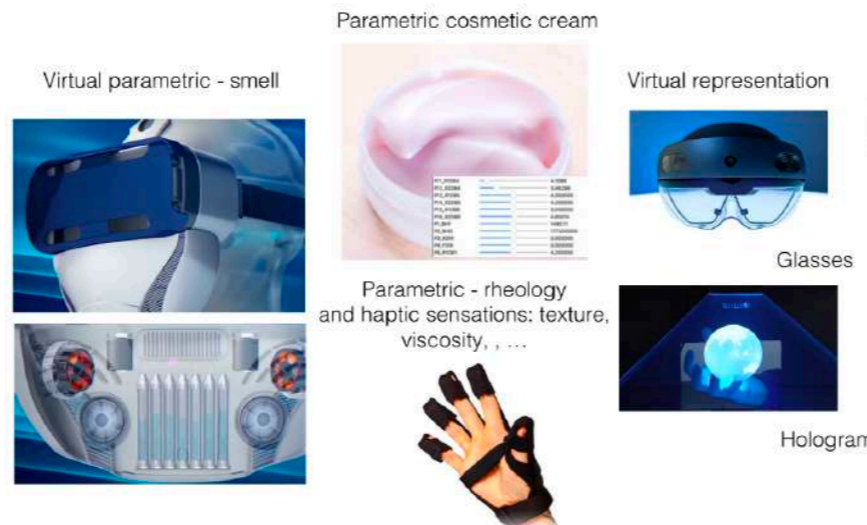
to model the  
gap between  
model &  
measurements

# and human centric

**Hybrid modeling**  
physics-based  
and data-driven



**Control, pattern  
recognition and  
decision making**



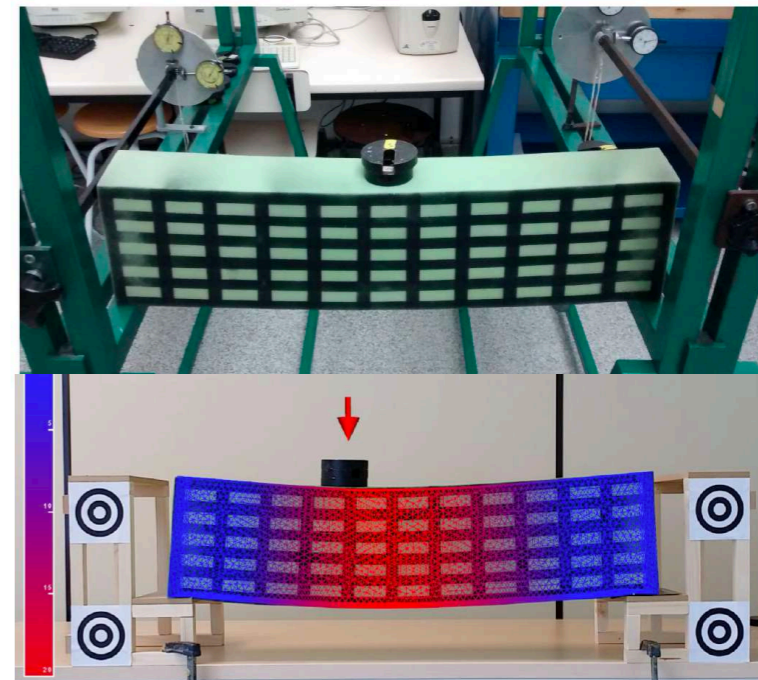
*Multisensorial  
&  
Emotionnal*



**Virtual, augmented & hybrid reality**

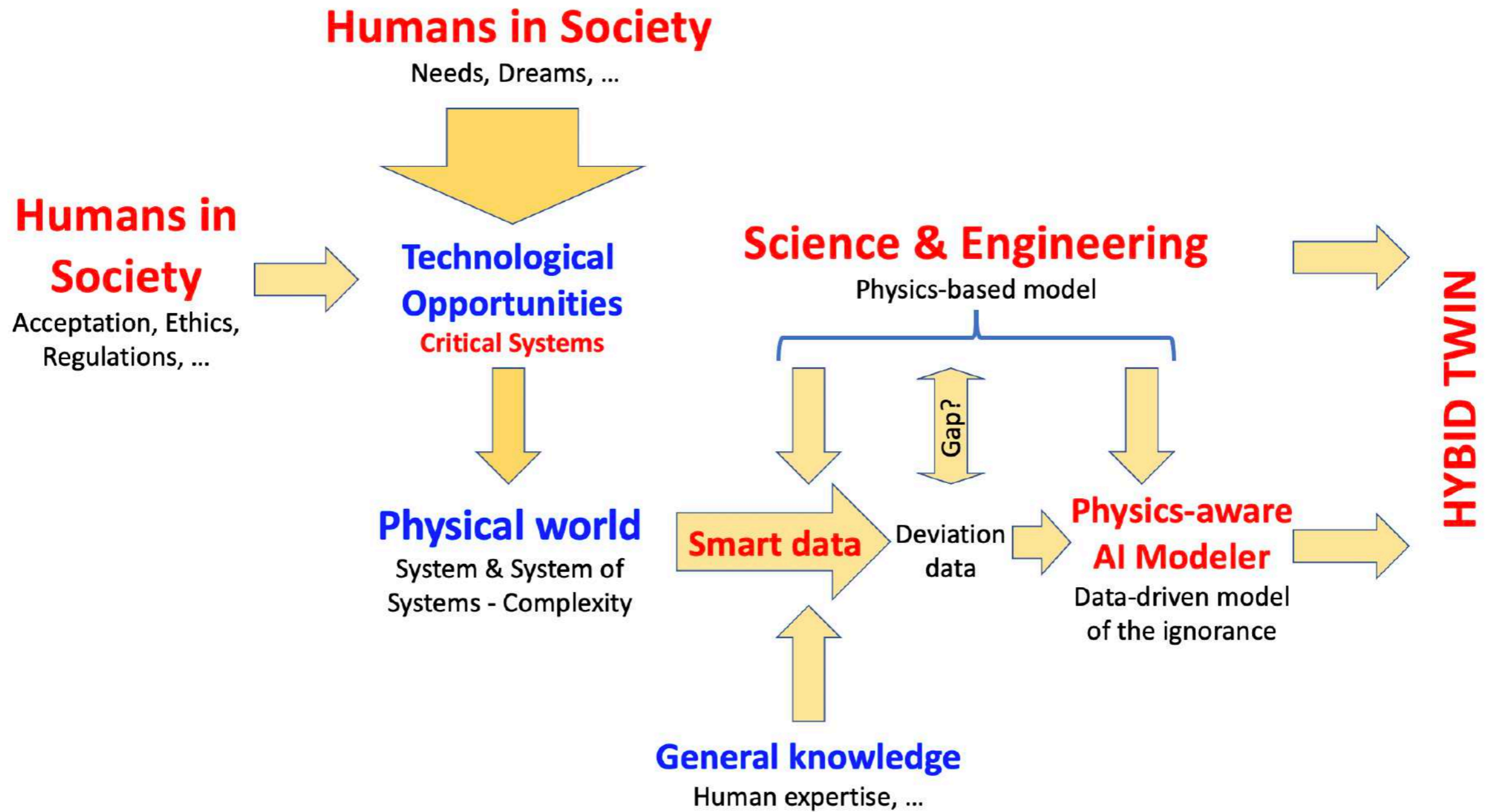


*DT that learns themselves*



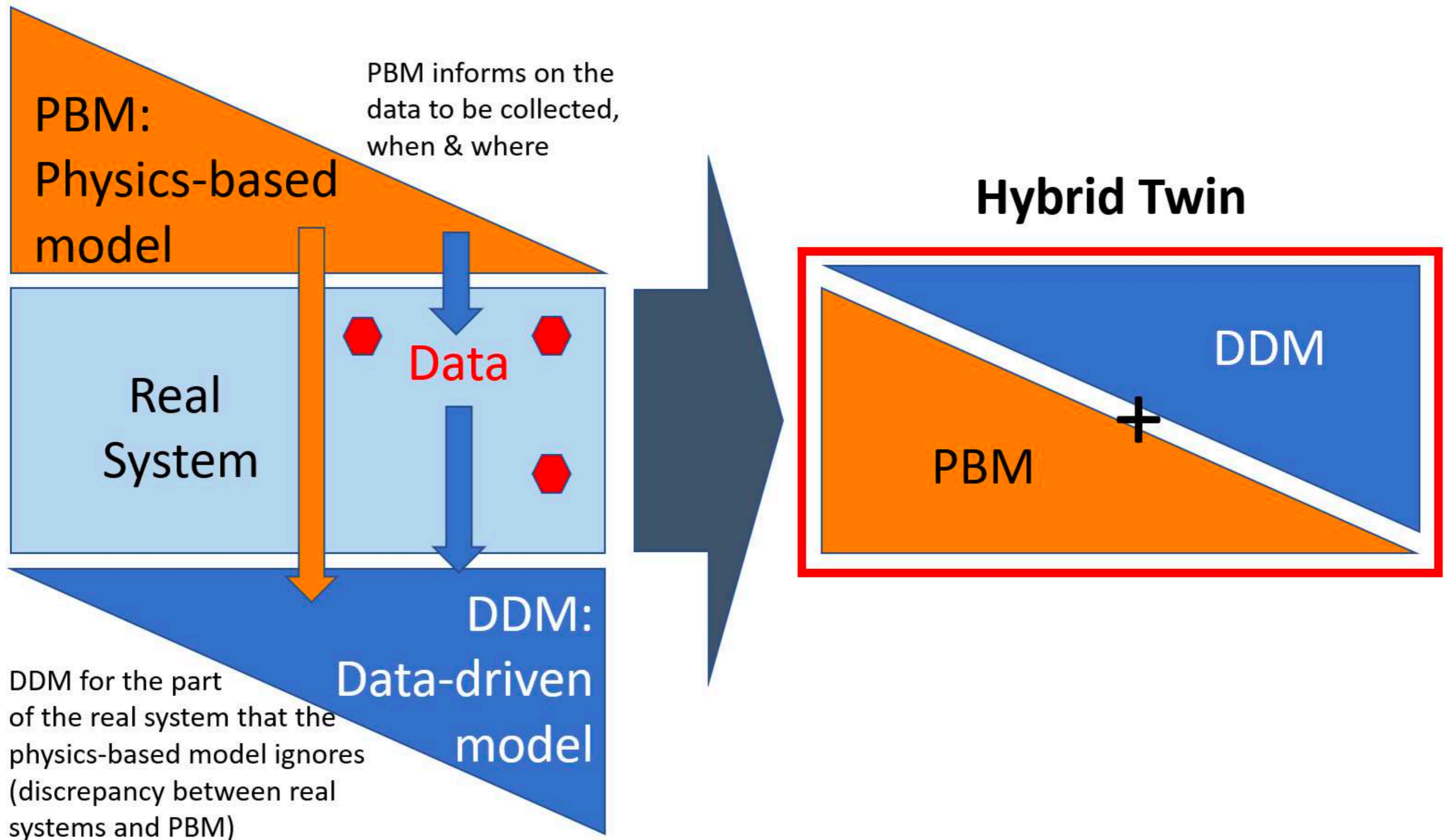


# The HT Platform





# The HT Platform



# Real-time simulation via MOR

**FEM**

$$u(x, t) \approx \sum_{i=1}^N U_i(t) N_i(x)$$

$$\mathbf{KU} = \mathbf{F}$$

**POD**

learning  $\longrightarrow \phi_i(x), i = 1, \dots, R$

$$u(x, t) \approx \sum_{i=1}^R u_i(t) \phi_i(x)$$

$$\mathbf{ku} = \mathbf{f}$$

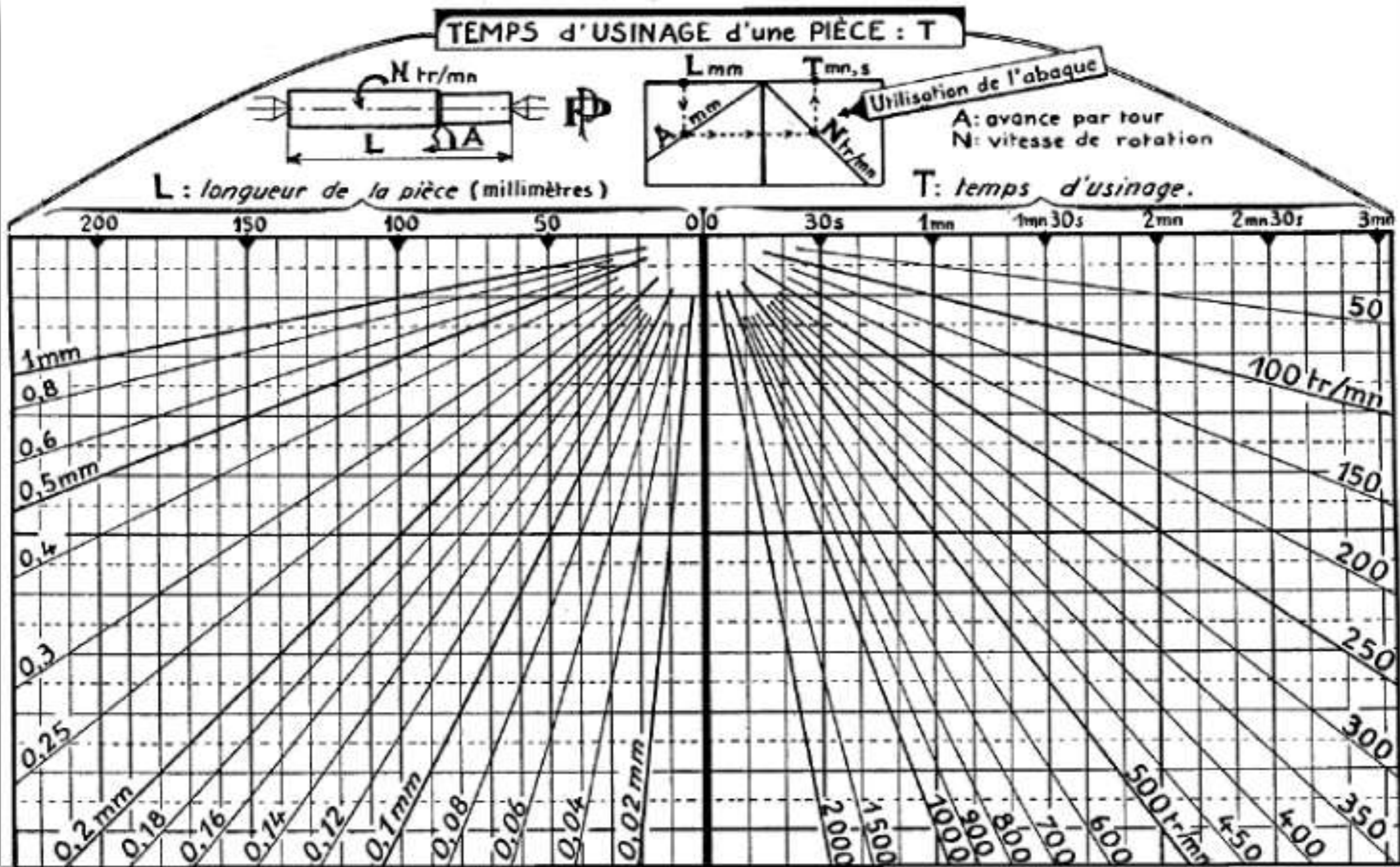
**PGD**

$$u(x, t) \approx \sum_{i=1}^M T_i(t) X_i(x)$$

$$\text{Vademecum } u(x, t, p) \approx \sum_{i=1}^M T_i(t) X_i(x) P_i(p)$$



# Machining vademecum





# Parametric solutions in action

$$u(x, t, p) \approx \sum_{i=1}^M T_i(t) X_i(x) P_i(p)$$

**Almost**

- Real-time simulation
- Real-time optimization
- Real-time inverse analysis
- Real-time uncertainty propagation
- Real-time control

# Non-Intrusive PGD-based meta-modeling

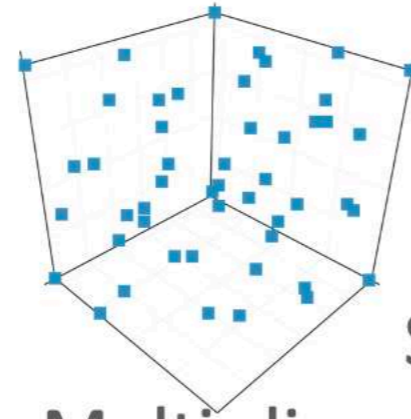
Commercial  
software

SSL, sPGD, ...

**Parametric solutions**

$$u(x, t, p) \approx \sum_{i=1}^M T_i(t) X_i(x) P_i(p)$$

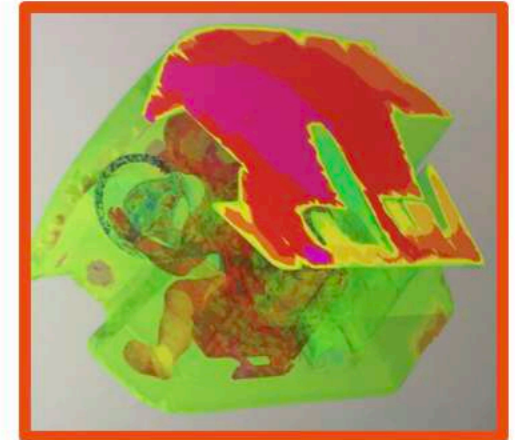
Vademecum



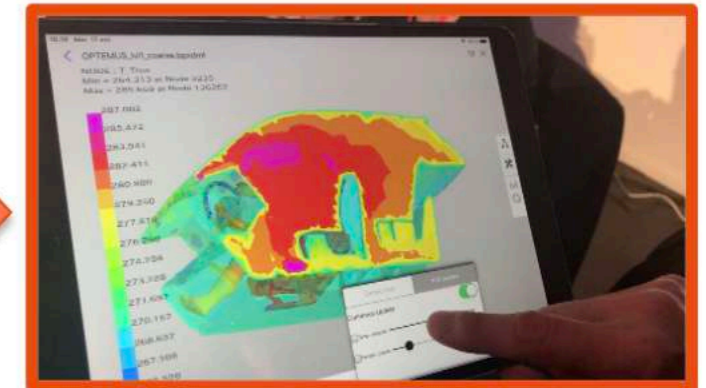
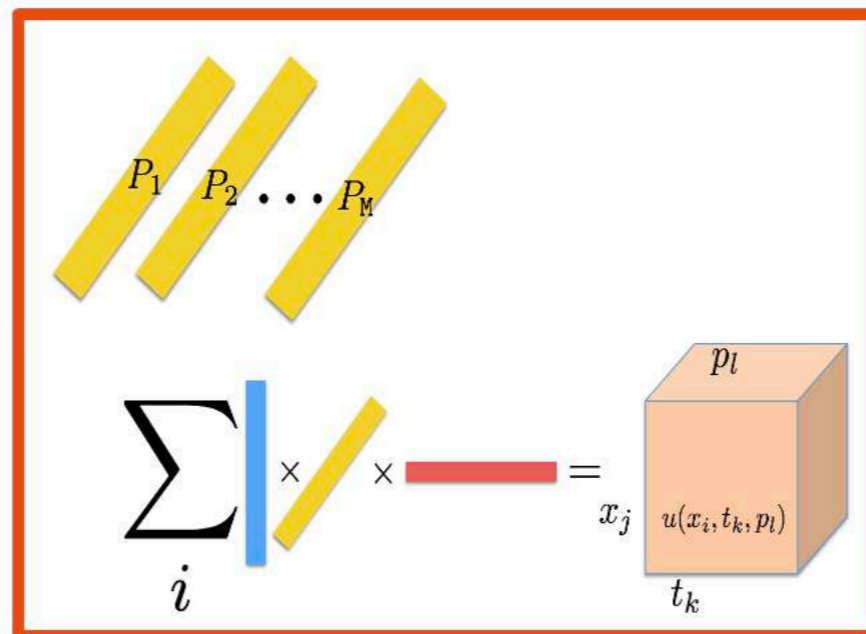
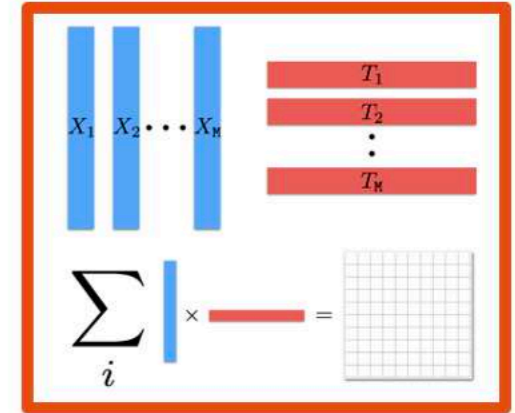
Sparse  
Multi-dimensional  
DOE



Runs of the  
HF solver



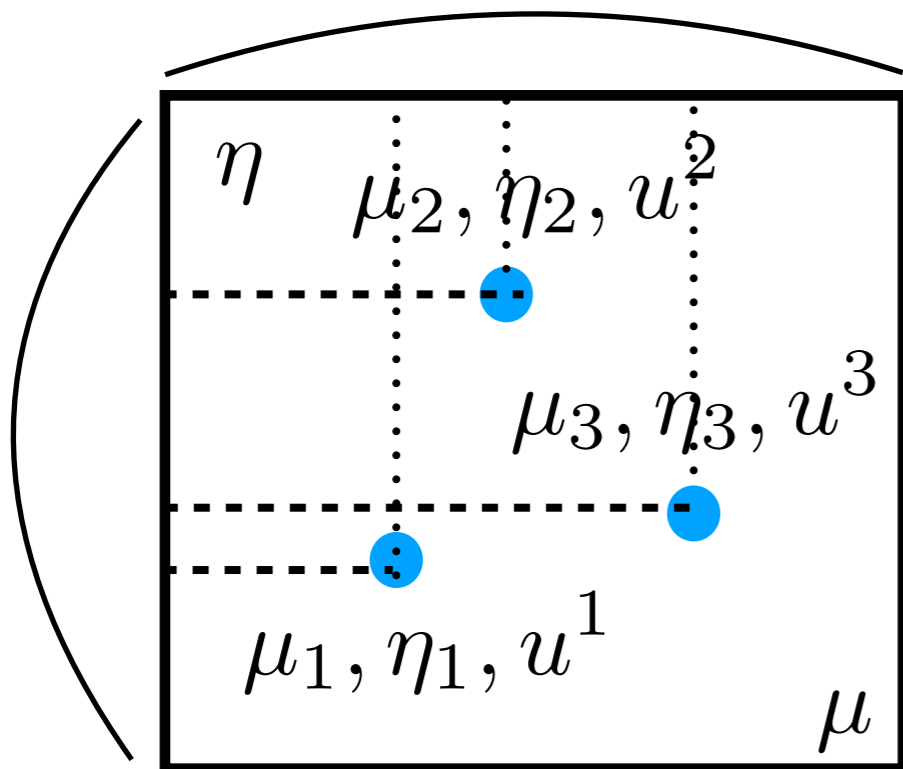
Solution compression (RB):  
POD, PGD, CUR, EIM, Cross, ...



SSL,  
sPGS,  
msPGD,  
s2PGD,  
ANOVA-PGD, ...



# sPGD



**Linear regression**

$$u(\mu, \eta) = a + b\mu + c\eta$$

**sPGD**

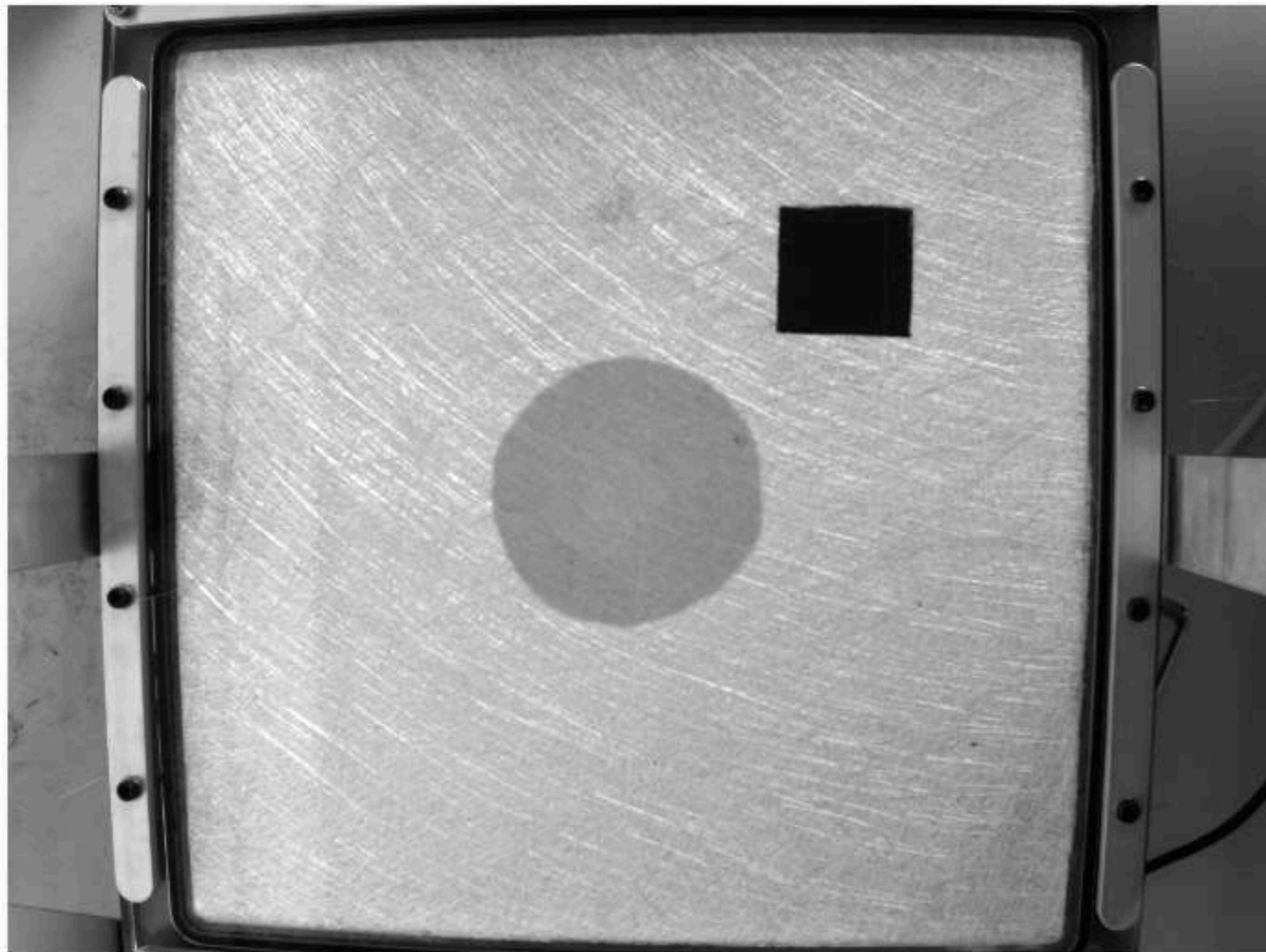
$$u(\mu, \eta) = \sum_i M_i(\mu) E_i(\eta)$$

$$[1, \mu, \eta, \mu^2, \mu\eta, \eta^2, \mu^2\eta, \mu^2\eta^2, \mu\eta^2]$$

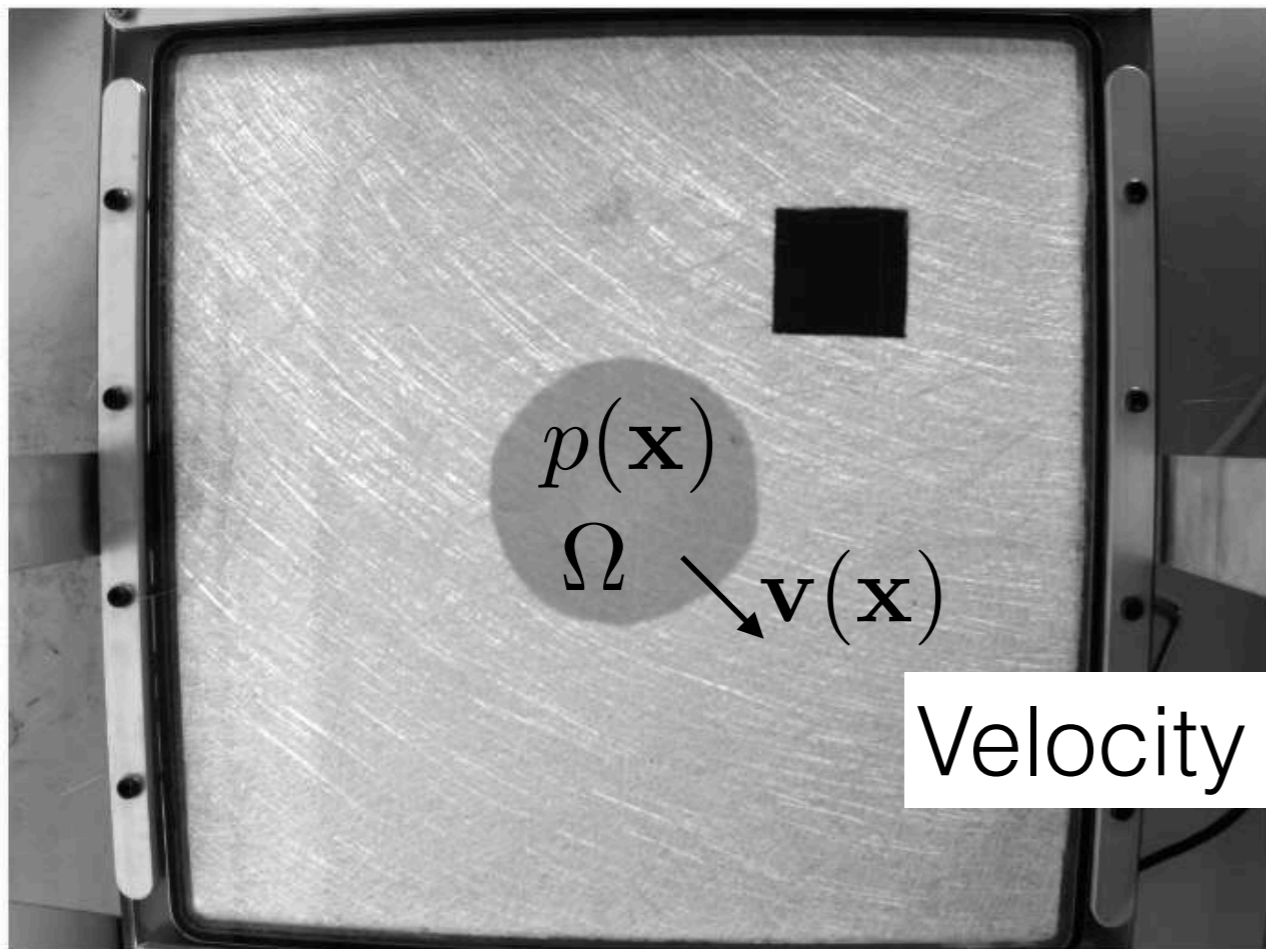
specially adapted for addressing nonlinear regression  
in multi-parametric settings in the low data limit

# Example

The system & the goal



# The existing knowledge



Fluid domain  $\Omega$

Point in the fluid domain  $\mathbf{x}$

Pressure in the fluid  $p(\mathbf{x})$

Velocity of each point in the fluid  $\mathbf{v}(\mathbf{x})$

Fluid behavior  $\mathbf{v}(\mathbf{x}) = -\mathbf{K} \cdot \nabla p(\mathbf{x})$

Fluid conservation  $\nabla \cdot \mathbf{v}(\mathbf{x}) = 0$

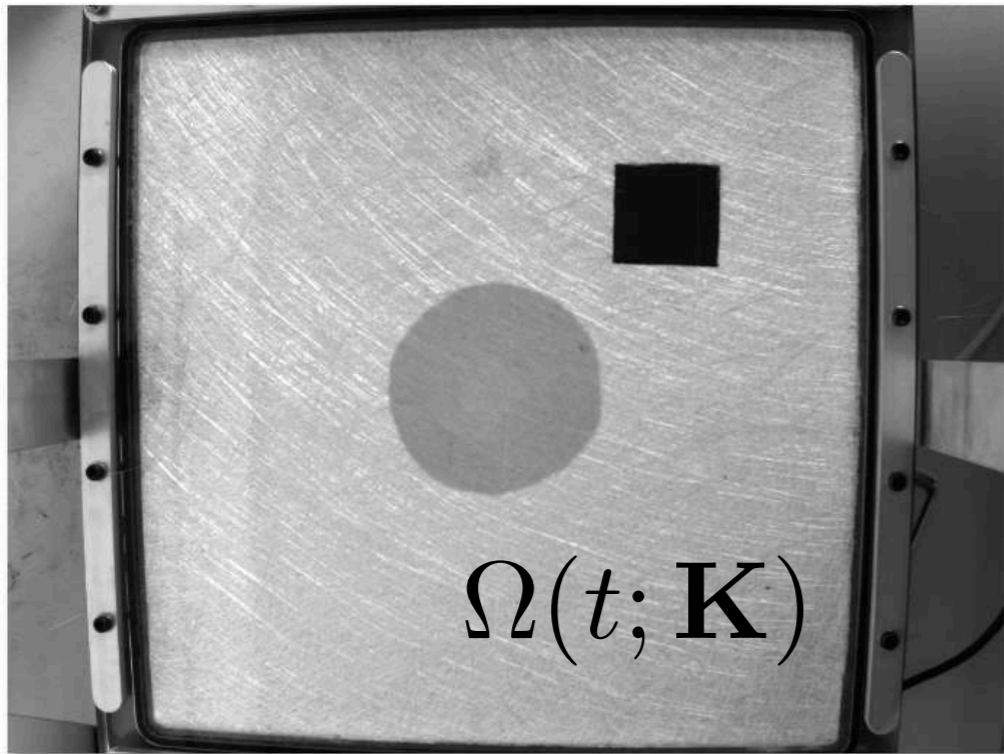
$$\nabla \cdot \{\mathbf{K} \cdot \nabla p(\mathbf{x})\} = 0$$



# Physics in real time

$$\nabla \cdot \{\mathbf{K} \cdot \nabla p(\mathbf{x})\} = 0$$

Commercial Software &  
Model Order Reduction

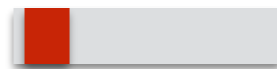
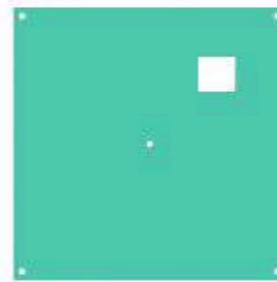


$$\Omega(t; \mathbf{K})$$

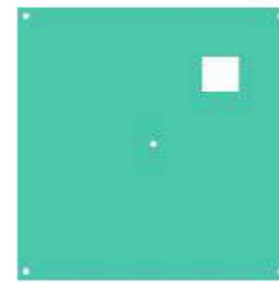
esi  
get it right®

ARTS  
ET MÉTIERS  
ParisTech

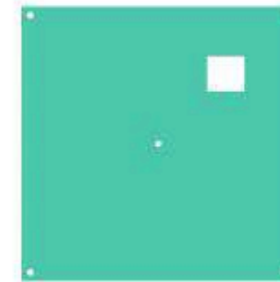
$$k = 7.7e-12 \text{ m}^2$$



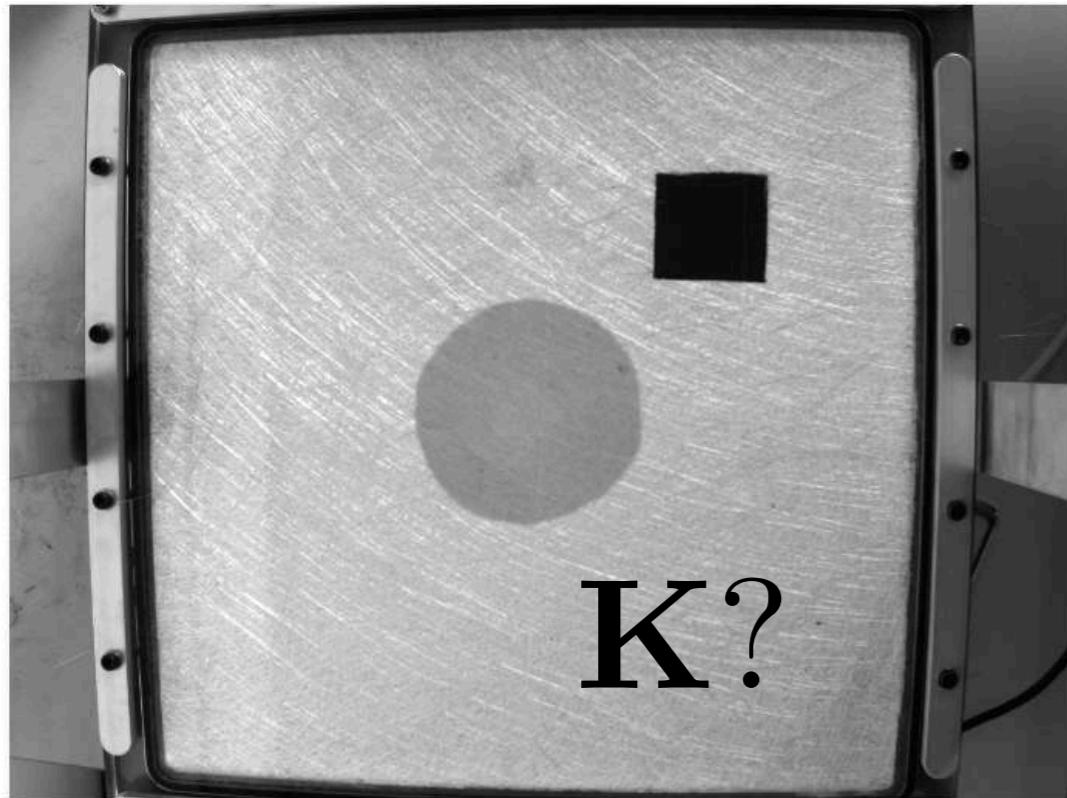
$$k = 7.7e-11 \text{ m}^2$$



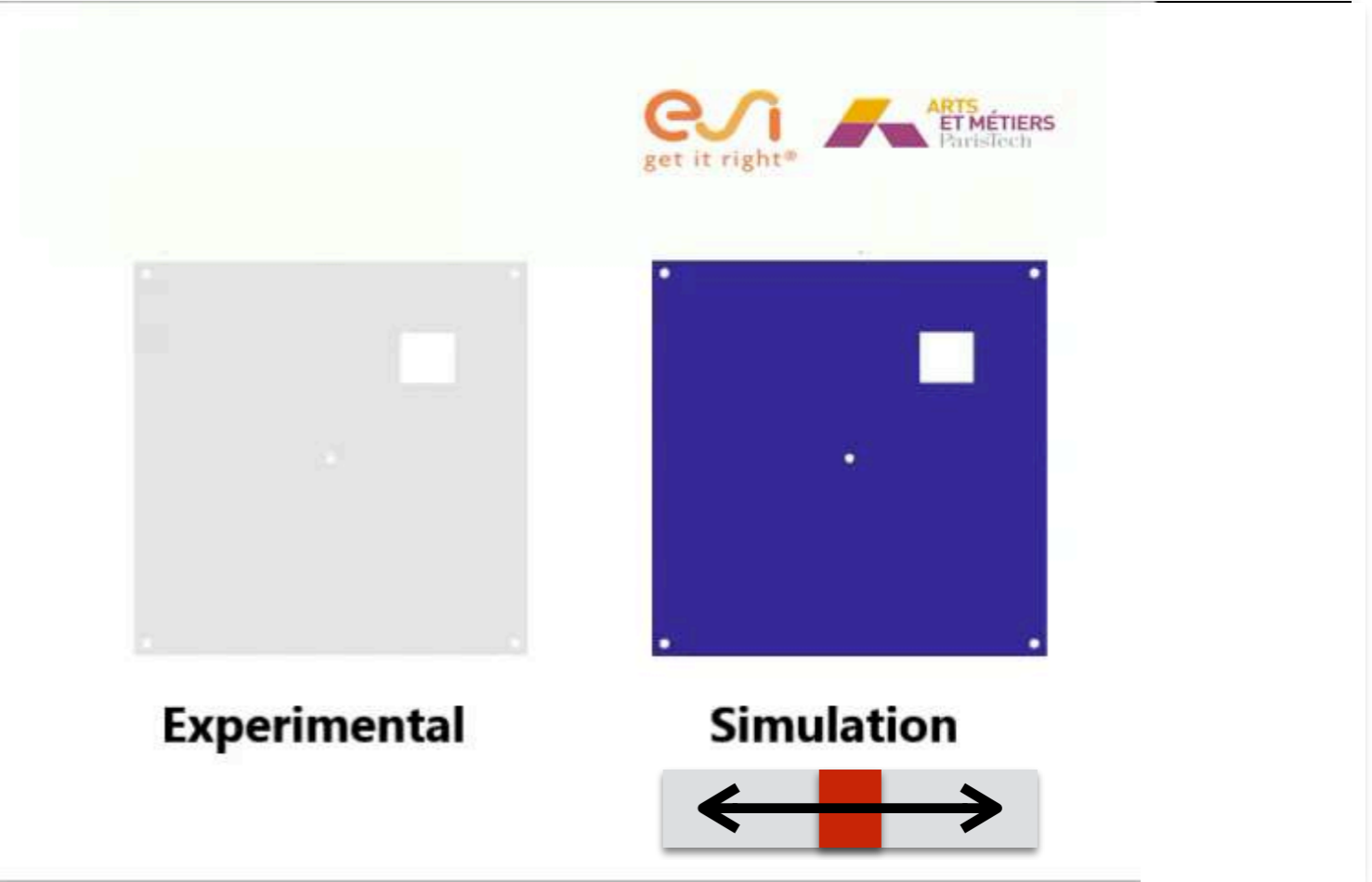
$$k = 7.7e-10 \text{ m}^2$$



# Real time model calibration



$$K = \operatorname{argmin}_{K^*} |\Omega^{\text{num}}(K^*) - \Omega^{\text{exp}}|$$

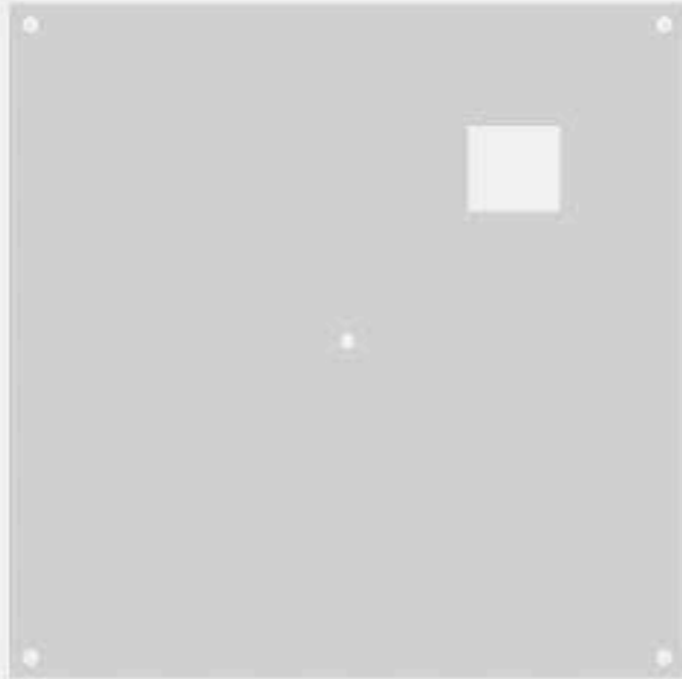


**K?**

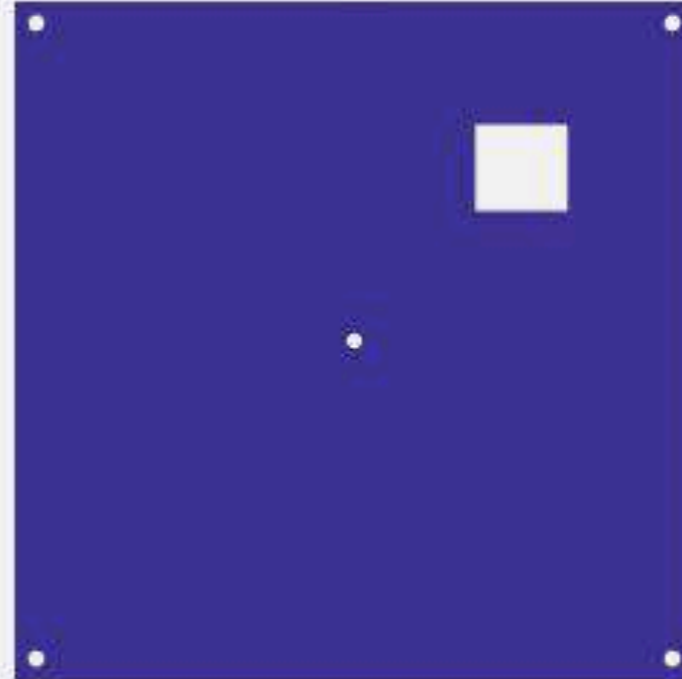
# Model prediction versus measures

time = 0 seconds

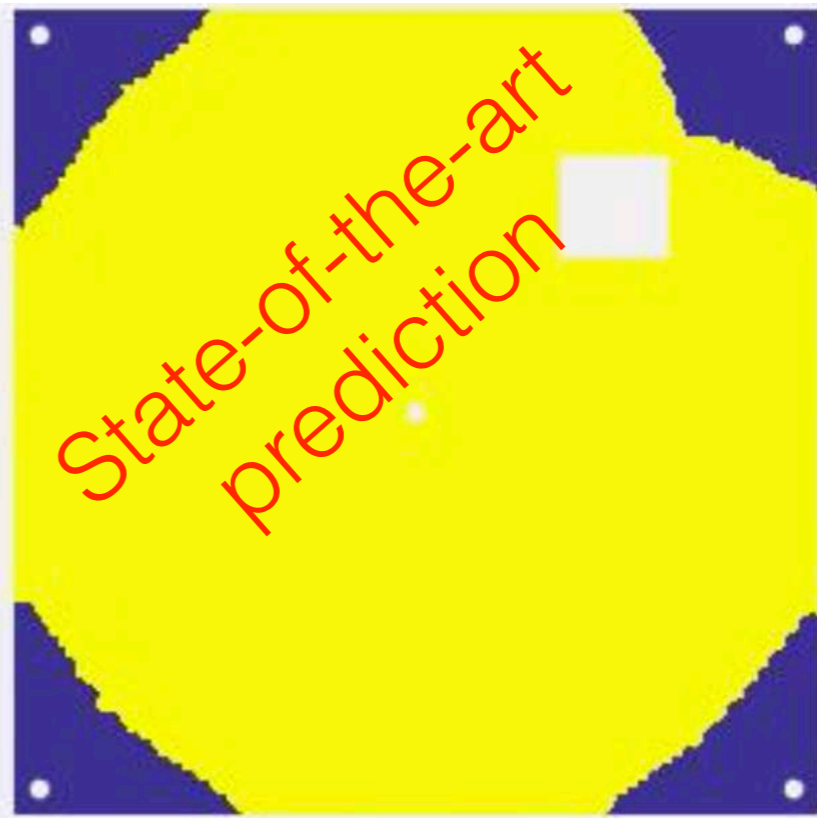
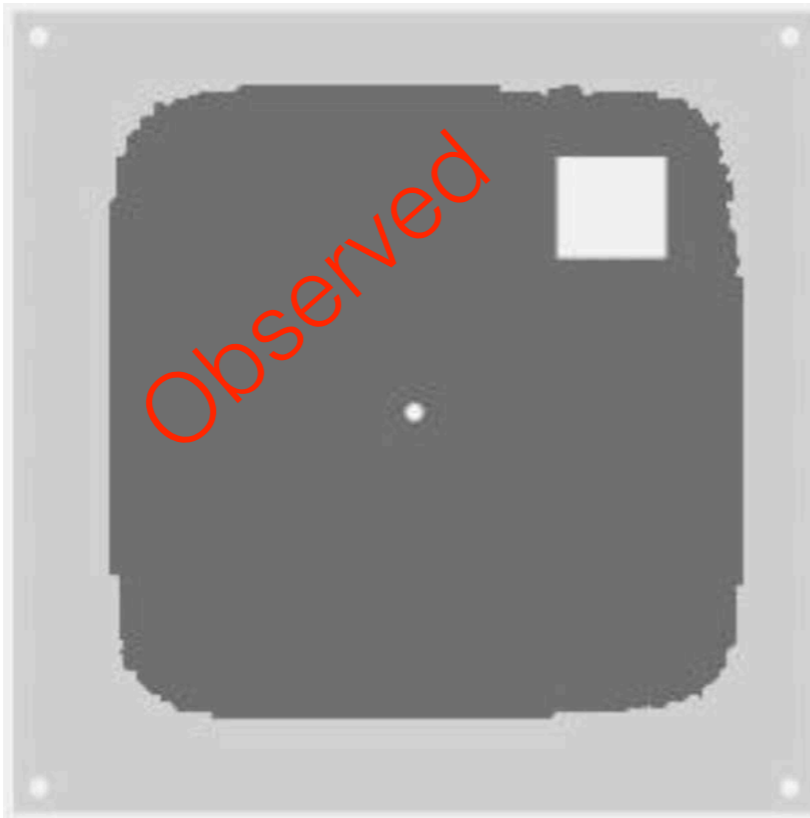
**Experimental emulation**



**Numerical prediction**







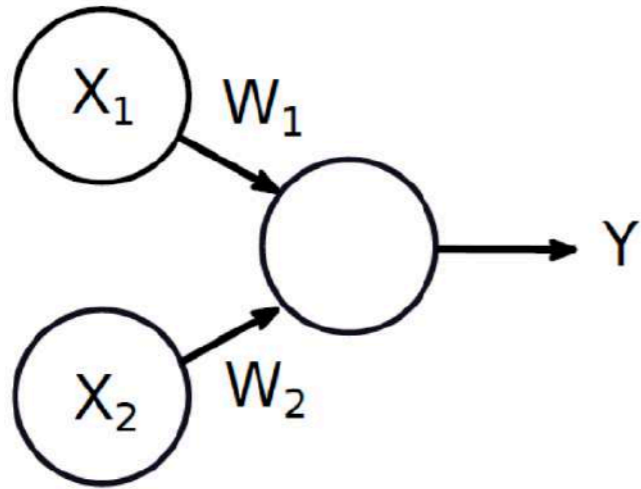
# Data-driven model of the deviation

... in the scarce data limit

Some of our proposals:

- **Multi-Local Sparse PGD**-Based NL Regression
- ***Code2Vect*** for heterogeneous / scarce data
- **Reduced Incremental Dynamic Mode Decomposition**
- **Thermodynamically Consistent ML**
- **Physically Informed Neural Networks:**  
combining tensor-flow and tensor formats

# Data-Driven modeling: Neural Network



$$Y = W_1 X_1 + W_2 X_2$$

with N inputs 
$$Y = \sum_{i=1}^N W_i X_i$$

$$\varepsilon(\mathbf{W}) = \sum_{k=1}^P \left( Y^k - \mathbf{W}^T \mathbf{X}^k \right)^2$$

$$\left. \begin{array}{l} \mathbf{Y}^T = (Y^1 \ Y^2 \ \dots \ Y^P) \\ \mathbf{X} = (\mathbf{X}^1 \ \mathbf{X}^2 \ \dots \ \mathbf{X}^P) \end{array} \right\} \varepsilon(\mathbf{W}) = \frac{1}{2} (\mathbf{Y}^T - \mathbf{W}^T \mathbf{X})^2$$

Nonlinear: 
$$\varepsilon(\mathbf{W}) = \frac{1}{2} (\mathbf{Y}^T - \sigma(\mathbf{W}^T \mathbf{X}))^2$$



# Ensuring thermodynamic consistency

$$\dot{\mathbf{z}}_t = \mathbf{L}(\mathbf{z}_t) \nabla E(\mathbf{z}_t) + \mathbf{M} \nabla S(\mathbf{z}_t), \quad \mathbf{z}(0) = \mathbf{z}_0$$



with

Poisson matrix:  
reversibility

Friction matrix:  
irreversibility

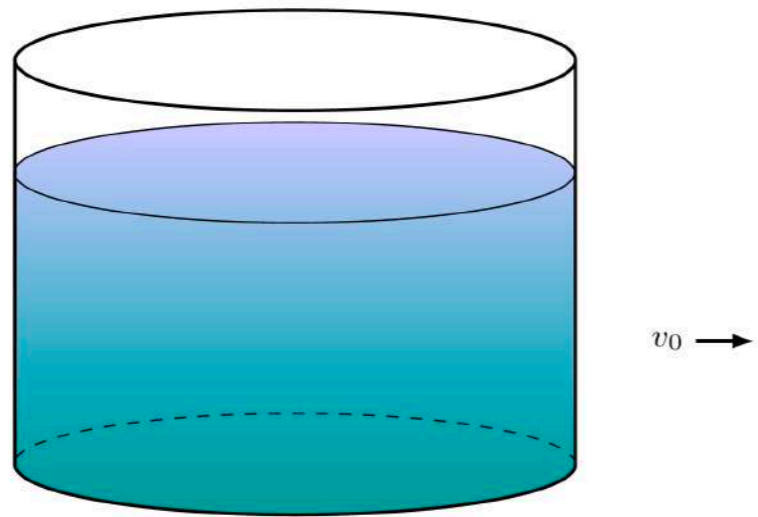
$$\begin{aligned} \mathbf{L}(\mathbf{z}) \cdot \nabla S(\mathbf{z}) &= \mathbf{0}, \\ \mathbf{M}(\mathbf{z}) \cdot \nabla E(\mathbf{z}) &= \mathbf{0}. \end{aligned}$$

$$\frac{\mathbf{z}_{n+1} - \mathbf{z}_n}{\Delta t} = \mathbf{L}(\mathbf{z}_{n+1}, \mathbf{z}_n) \mathbf{DE}(\mathbf{z}_{n+1}, \mathbf{z}_n) + \mathbf{M}(\mathbf{z}_{n+1}, \mathbf{z}_n) \mathbf{DS}(\mathbf{z}_{n+1}, \mathbf{z}_n)$$

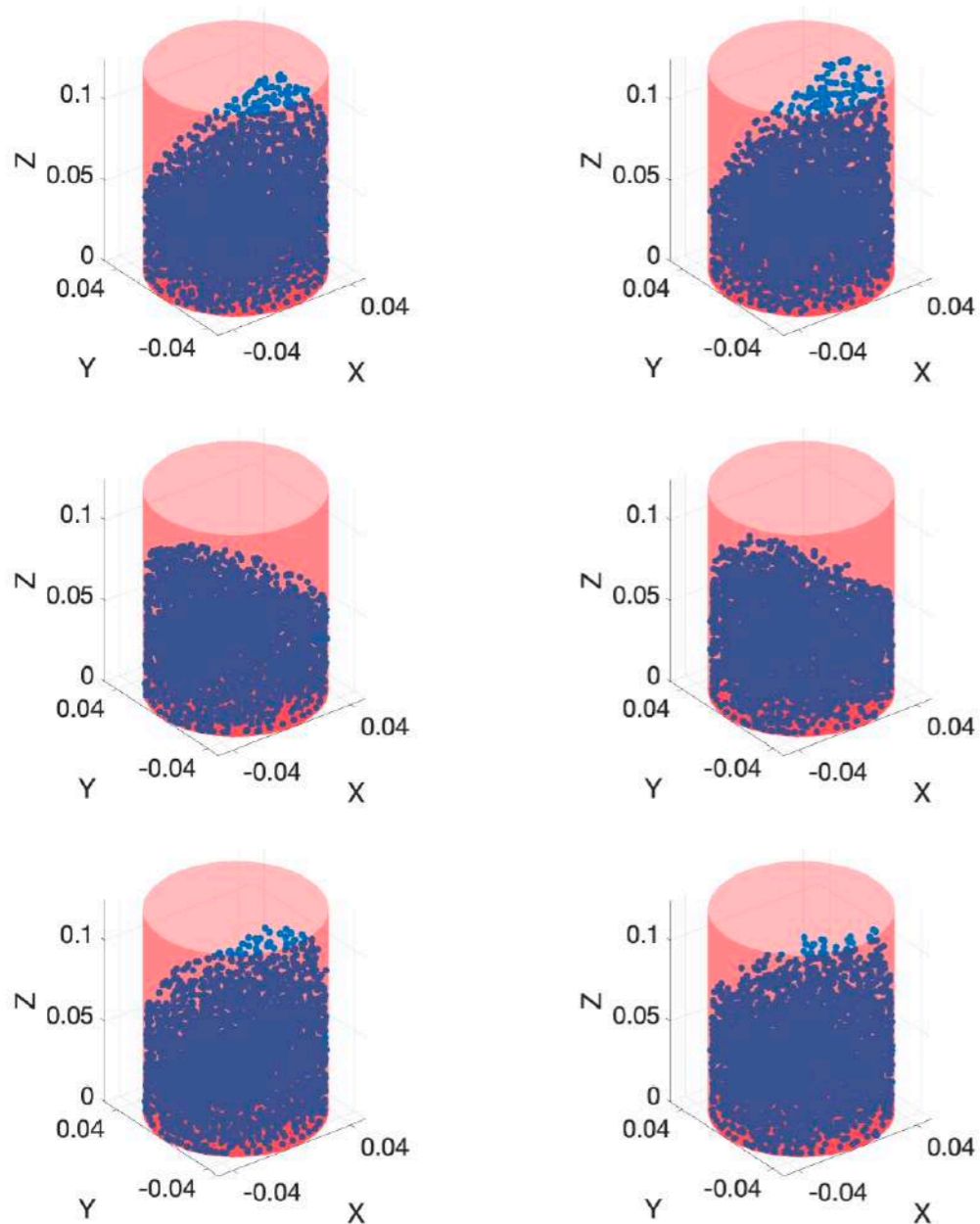
$$\boldsymbol{\mu} = \{\mathbf{L}, \mathbf{M}, \mathbf{DE}, \mathbf{DS}\} = \arg \min_{\boldsymbol{\mu}^*} \|\mathbf{z}(\boldsymbol{\mu}) - \mathbf{z}^{\text{meas}}\|$$

with

$$\begin{aligned} \mathbf{DE} &= \mathbf{A}\mathbf{z} \\ \mathbf{DS} &= \mathbf{B}\mathbf{z} \end{aligned}$$



$$\dot{z}_t = \mathbf{L}(z_t)\nabla E(z_t) + \mathbf{M}\nabla S(z_t), \quad z(0) = z_0$$



# Physically sound, self-learning digital twins for sloshing fluids

B. Moya, I. Alfaro, D. González, F. Chinesta, E. Cueto

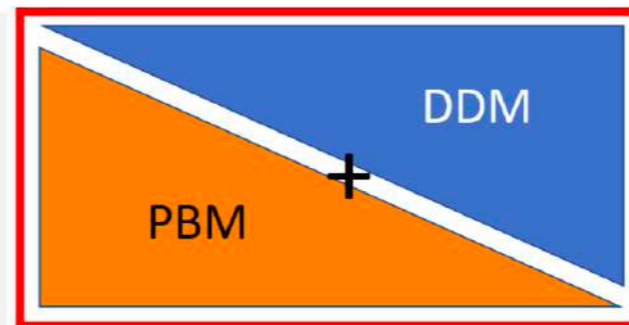


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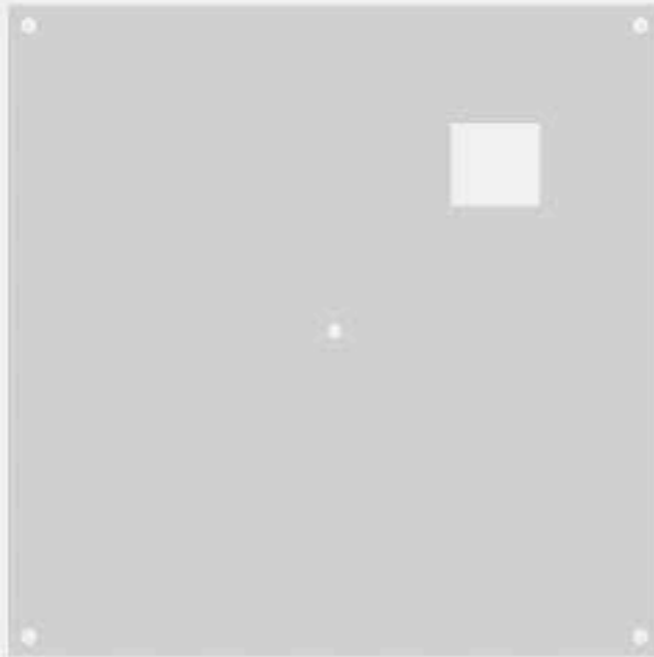
**unizar.es**

# HT in action

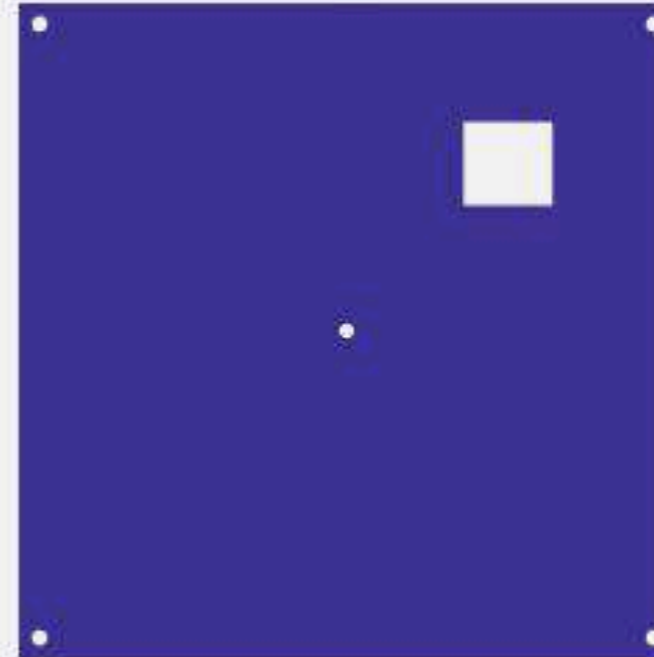
Hybrid Twin



Experimental emulation



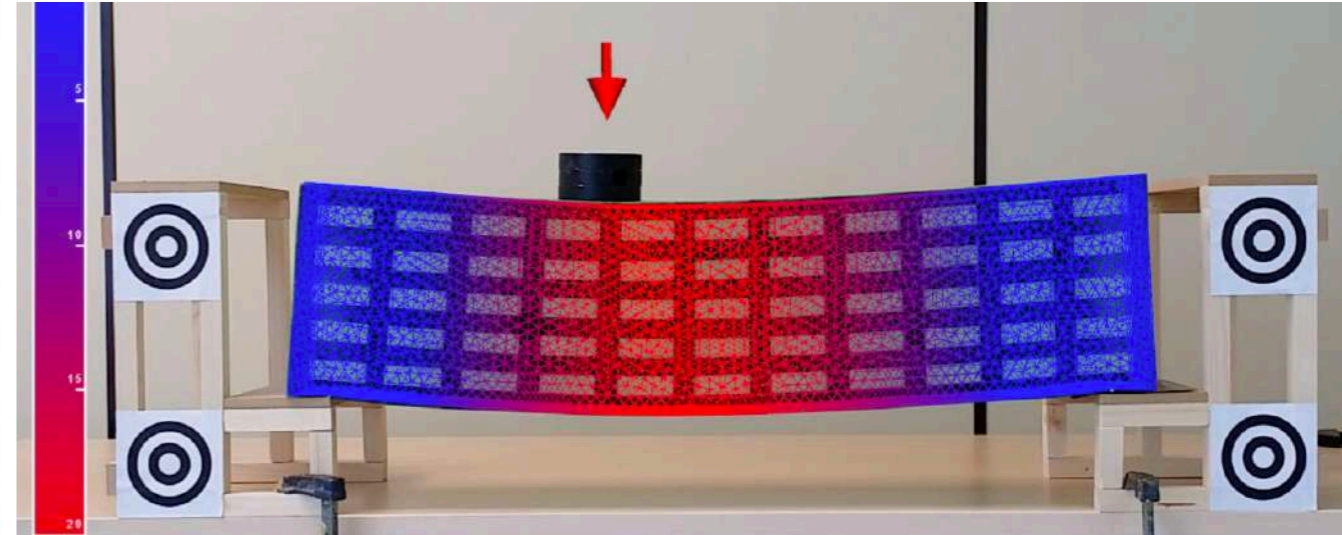
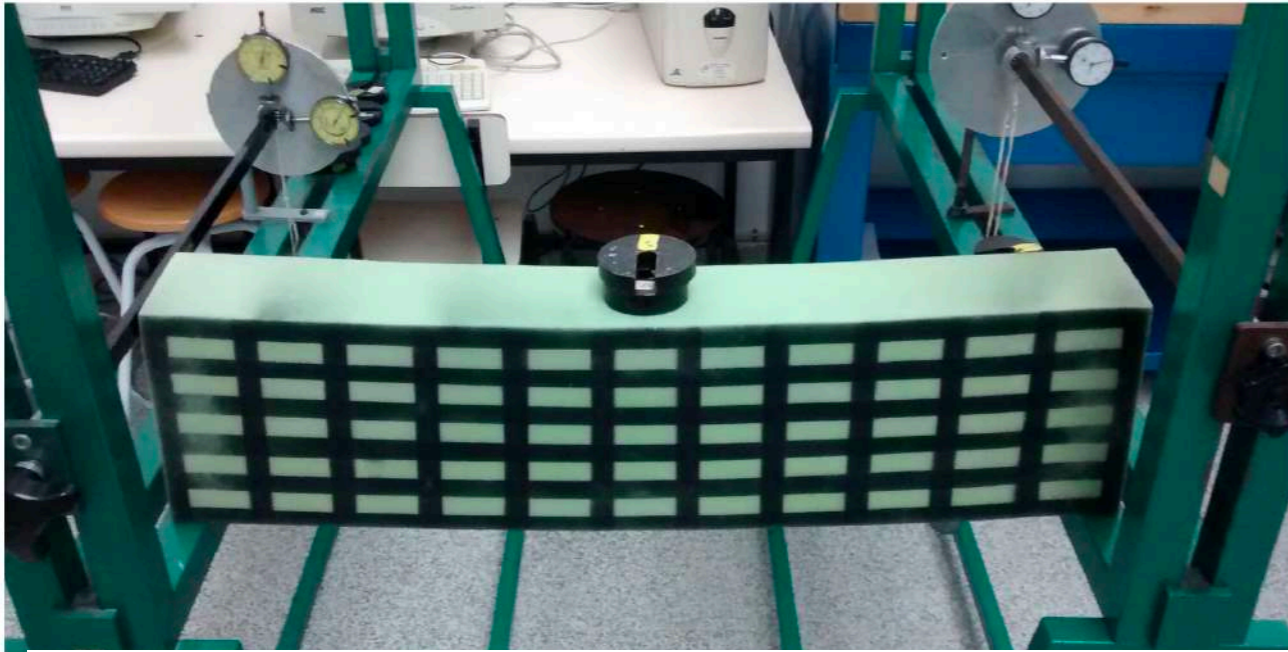
Numerical prediction





# Digital twins that learn and correct themselves†

Beatriz Moya<sup>1</sup> | Alberto Badías<sup>1</sup> | Icíar Alfaro<sup>1</sup> | Francisco Chinesta<sup>2</sup> | Elías Cueto<sup>1</sup>

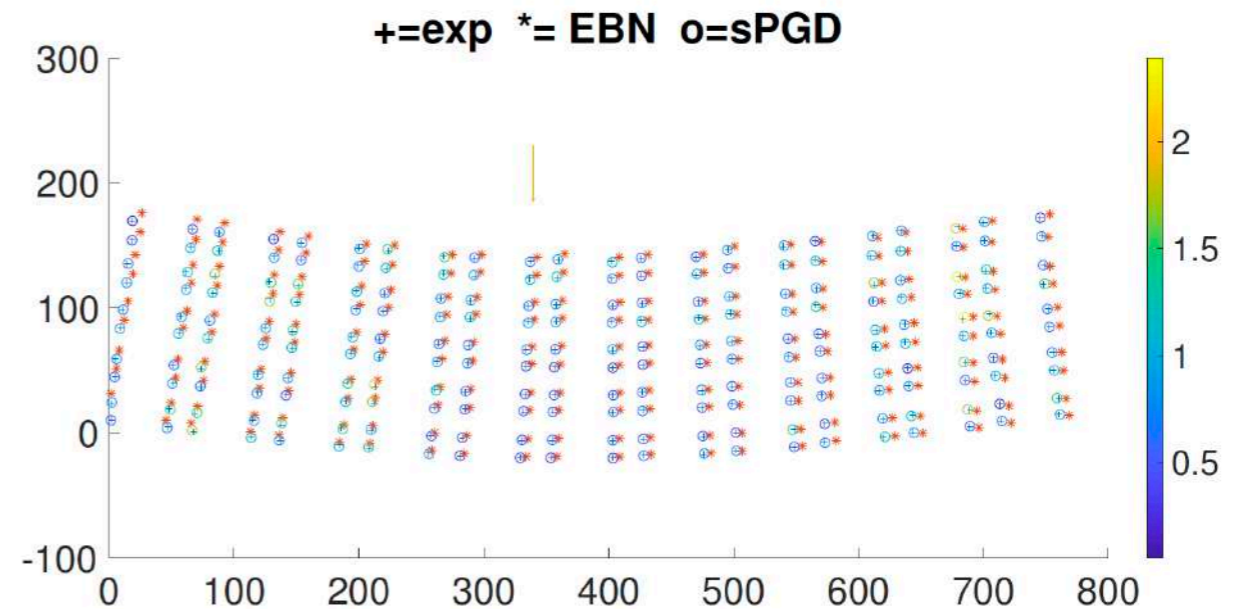


$$\begin{cases} \dot{\mathbf{X}} = \mathbf{A}(\mathbf{X}, t, \boldsymbol{\mu}) + \mathbf{B}(\mathbf{X}, t) + \mathbf{C}(\mathbf{X}) + \mathbf{R} \\ \mathbf{Y} = \mathbf{D}(\mathbf{X}) + \mathbf{R}' \\ \mathbf{Z} = \mathbf{G}(\mathbf{X}) + \mathbf{R}'' \end{cases}$$

Parametric deterministic contribution PGD  
 Data-Driven correction  
 Control  
 Noise

Filters: Kalman, Bayesian, ...

Measures involving noise



# Visco-Hyper-Elasticity as a Data-Driven correction

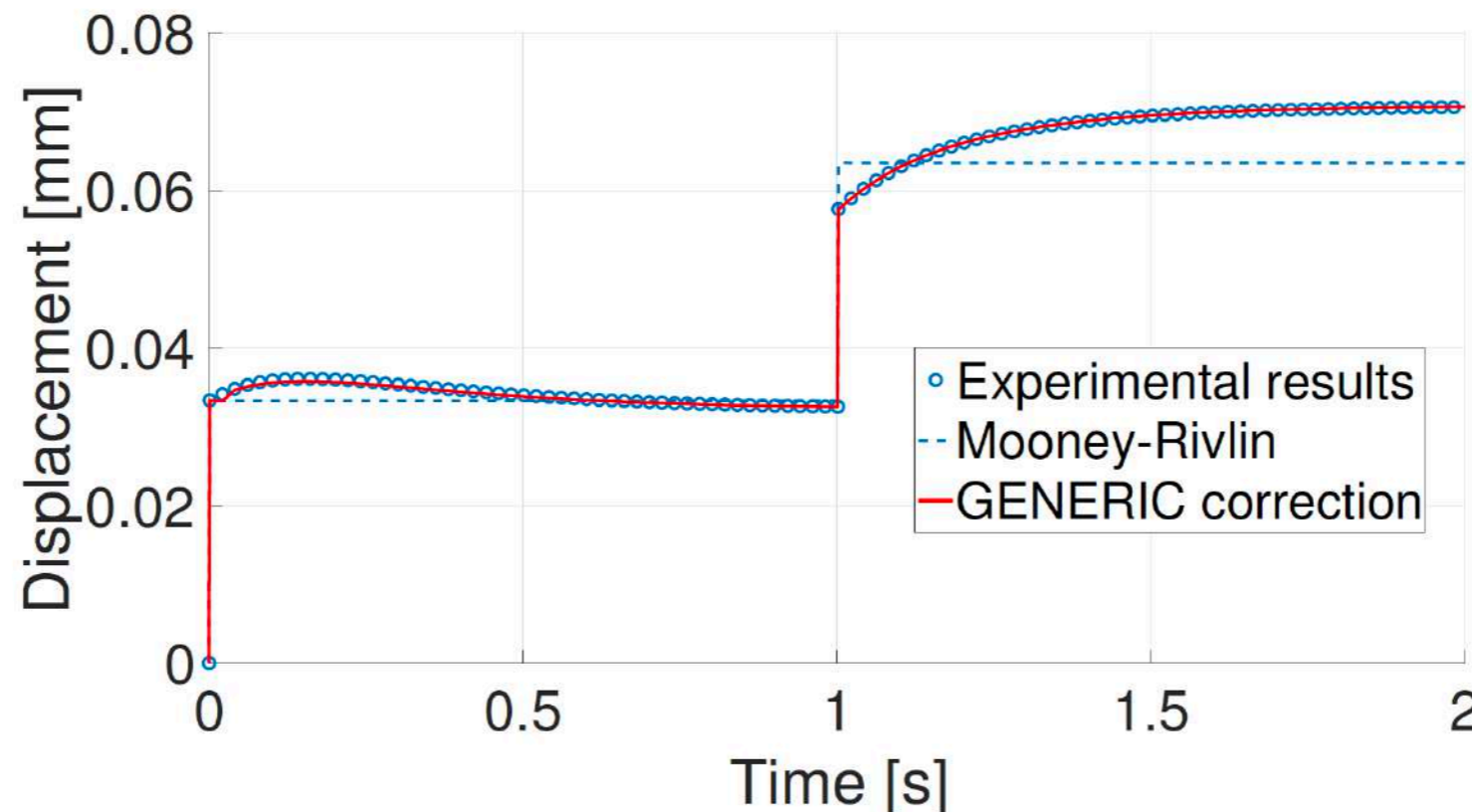
thermodynamically consistent of a purely-hyper-elasticity

$$\dot{z}^{\text{exp}} = \mathbf{L}^{\text{model}} (\nabla E^{\text{model}} + \nabla E^{\text{corr}}) + (\mathbf{M}^{\text{model}} + \mathbf{M}^{\text{corr}}) (\nabla S^{\text{model}} + \nabla S^{\text{corr}})$$

$$\dot{z}^{\text{exp}} = \mathbf{L}^{\text{model}} (\nabla E^{\text{model}} + \nabla E^{\text{corr}}) + \mathbf{M}^{\text{corr}} \nabla S^{\text{corr}}$$

$$\frac{z_{n+1}^{\text{exp}} - z_n^{\text{exp}}}{\Delta t} = \mathbf{L} \text{DE}(z_{n+1}^{\text{exp}}) + \mathbf{M}(z_{n+1}^{\text{exp}}) \text{DS}(z_{n+1}^{\text{exp}})$$

$$\mu^* = \{\mathbf{M}, \text{DE}, \text{DS}\} = \arg \min_{\mu} \|z(\mu) - z^{\text{meas}}\|$$





# Augmented reality

## Physics-aware interaction between virtual and physical objects in Mixed Reality

A. Badías, D. González, I. Alfaro, F. Chinesta, E. Cueto



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# Physics-based Interactive Holograms

A. Badías, D. González, I. Alfaro, F. Chinesta, E. Cueto

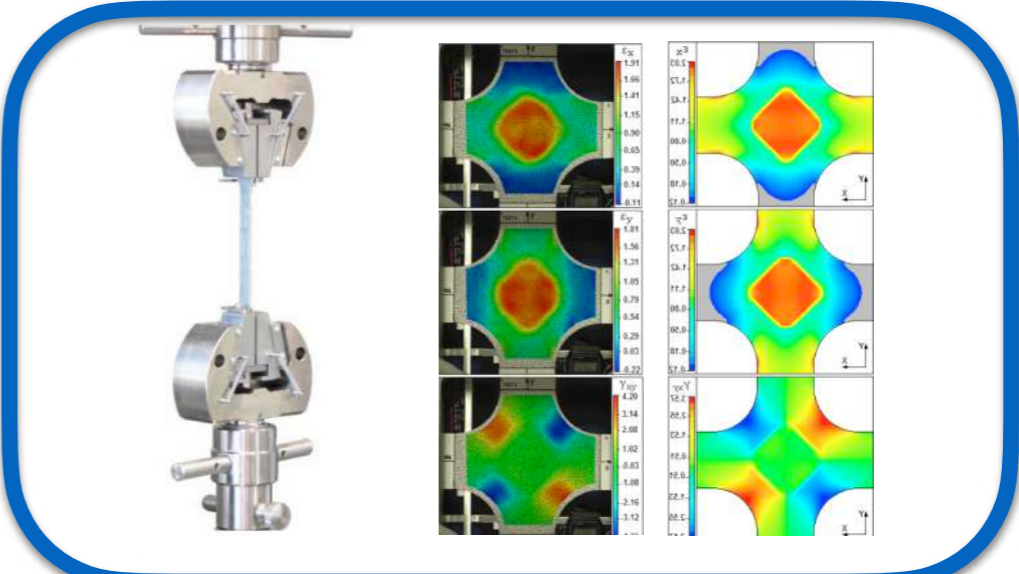


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# Plasticity correction



Reality  
e.g.

Barlat Yld2004-18p

=

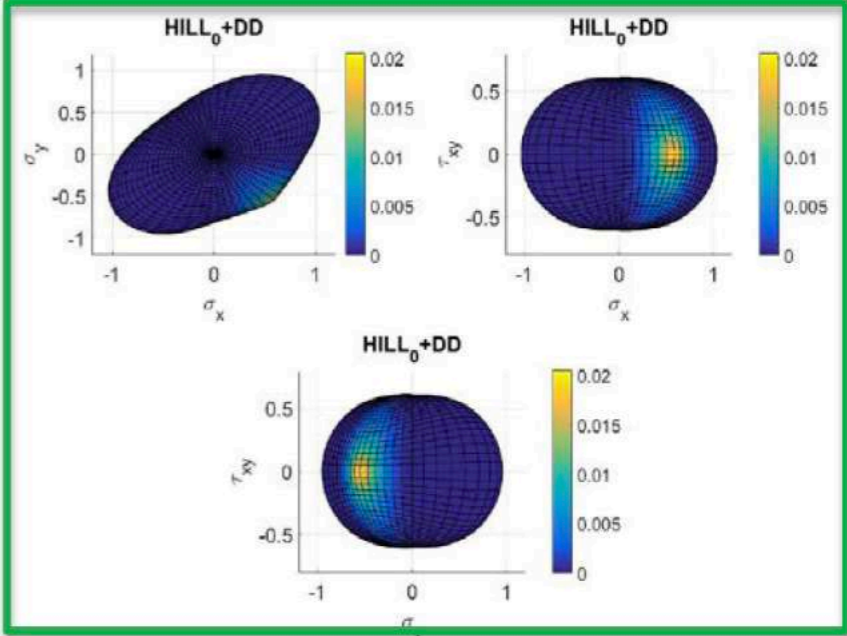
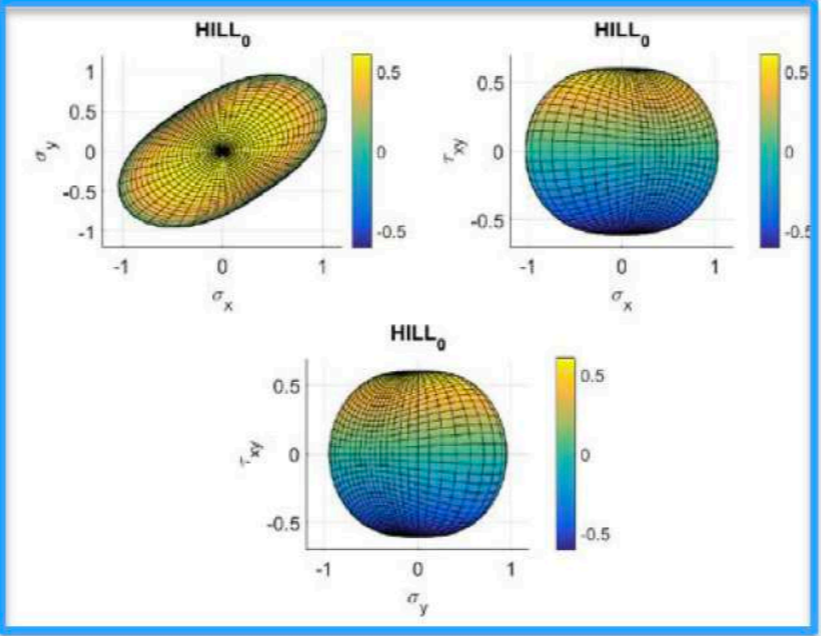
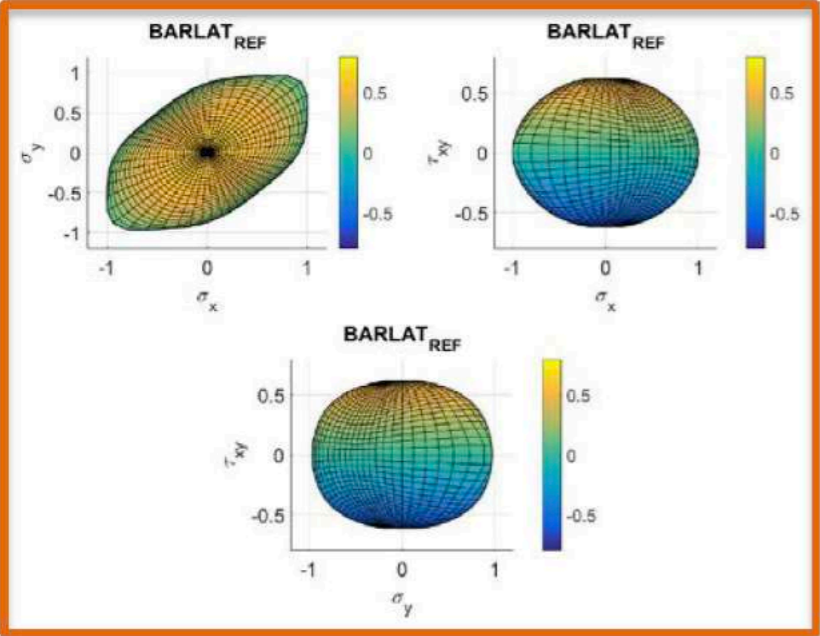
First order model  
e.g.

Quadratic Hill

+

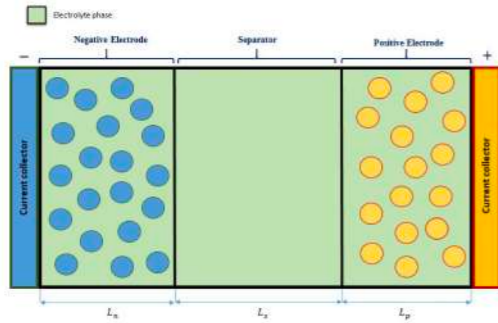
Deviation model

Perturbation Model



# Battery Hybrid Twin for real-time planning

## Micro



Newman's P2D model

$$\frac{\partial c_s}{\partial t} = \frac{D_s}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial c_s}{\partial r} \right); \quad \frac{\partial c_s}{\partial r} \Big|_{r=0} = 0; \quad D_s \frac{\partial c_s}{\partial r} \Big|_{r=R_s} = -\frac{j^{Li}}{a_s F}$$

$$\frac{\partial}{\partial x} \left( \kappa^{eff} \frac{\partial \phi_e}{\partial x} \right) + \frac{\partial}{\partial x} \left( \kappa_D^{eff} \frac{\partial}{\partial x} \ln(c_e) \right) + j^{Li} = 0; \quad \frac{\partial \phi_e}{\partial x} \Big|_{x=0} = \frac{\partial \phi_e}{\partial x} \Big|_{x=L} = 0$$

$$\frac{\partial}{\partial x} \left( \sigma^{eff} \frac{\partial \phi_s}{\partial x} \right) - j^{Li} = 0;$$

$$-\sigma^{eff} \frac{\partial \phi_s}{\partial x} \Big|_{x=0} = \sigma^{eff} \frac{\partial \phi_s}{\partial x} \Big|_{x=L} = \frac{I}{A}; \quad \frac{\partial \phi_s}{\partial x} \Big|_{x=0} = \frac{\partial \phi_s}{\partial x} \Big|_{x=L} = 0$$

$$\frac{\partial (c_e)}{\partial t} = \frac{\partial}{\partial x} \left( D_e^{eff} \frac{\partial c_e}{\partial x} \right) + \frac{1-t^0}{F} j^{Li}; \quad \frac{\partial c_e}{\partial x} \Big|_{x=0} = \frac{\partial c_e}{\partial x} \Big|_{x=L} = 0$$

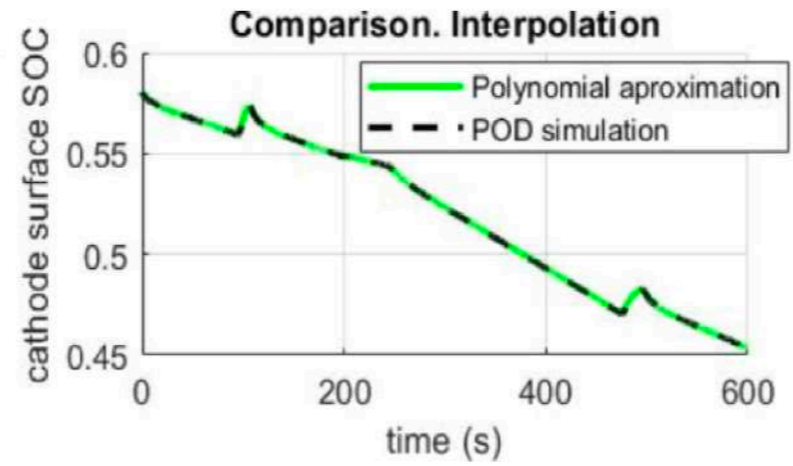
POD  
&  
PGD

## Meso

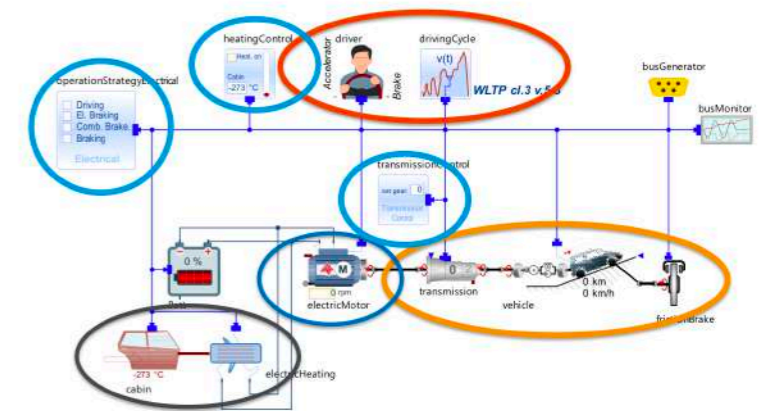
$$SOC(t) = f(SOC(t=0), Load, Env)$$

$$Voltage(t) = g(SOC(t=0), Load, Env)$$

## AI: sPGD & T-MD

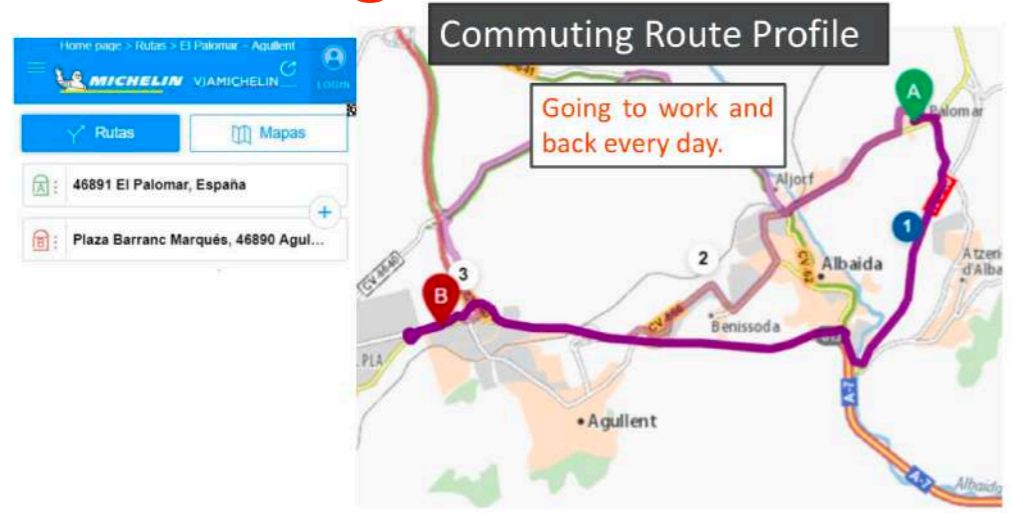


## Macro

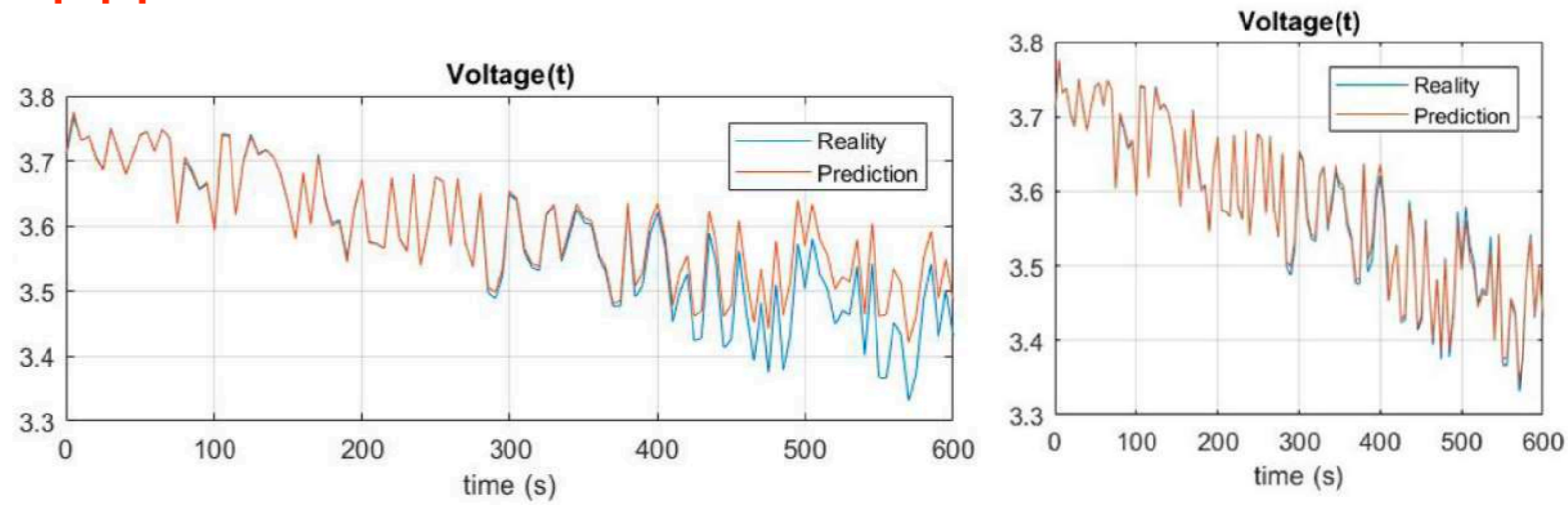


## System modelling

## Planning



## HT



## Data-Driven Correction