Uncertainty quantification for random neutronics

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Motivations

- Nuclear matter emits neutrons, detection is used to identify the type of matter.
- Extracted data are composed of a timelist file, which is a listing of detection times
- Given a timelist file, how can we estimate the fissile matter characteristics ?

Neutronic fluctuations

Neutron fluctuations are due to branching processes

- neutron reactions (fission, capture)
- source (spontaneaous fission)
- detection

Main random process is the fission branching process.
We analyse the timelist file as follows

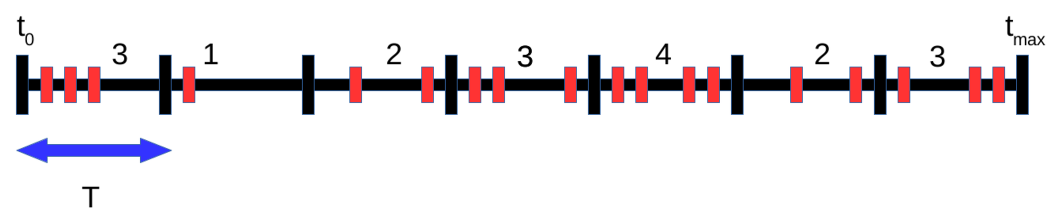


FIGURE – Sequential binning

It produces a count vector from which we compute empirical moments.

Point model and forward problem

- A stochastic point model was introduced by R.Feynman during the Manhattan project.
- More precisely : all the neutrons move at same speed in an infinite homogeneous, isotropic medium.
  A neutron ends its life by capture (with or without detection) or induced fission.
  Neutrons are produced by induced fission and by source which can be a Poisson or compound Poisson process (spontaneous fission).

We consider the following outputs

- Average count number

$$\bar{C}(T) = \frac{\varepsilon_F S}{-\rho \bar{\nu}} T$$

- Feynman moment of order 2

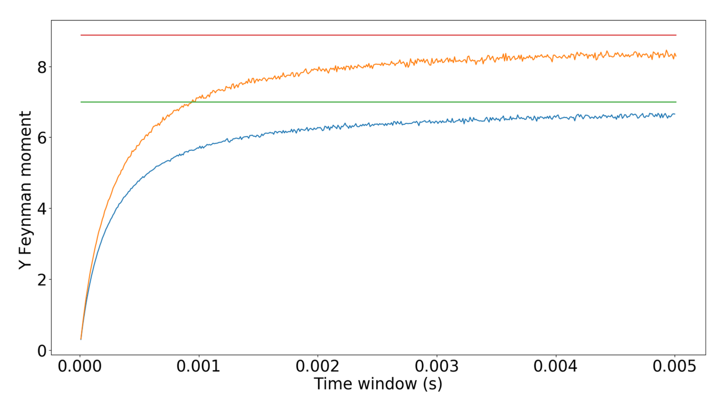
$$Y_2(T) = \frac{\varepsilon_F D_2}{\rho^2}\left(1 - x\rho \frac{\bar{\nu}_S D_{2S}}{\bar{\nu} D_2}\right)\left(1 - \frac{1 - e^{-\alpha T}}{\alpha T}\right)$$

- Feynman moment of order 3

$$Y_3(T) = 3\left(\frac{\varepsilon_F D_2}{-\rho^2}\right)^2\left(1 - x\rho \frac{\bar{\nu}_S D_{2S}}{\bar{\nu} D_2}\right)\left(1 + e^{-\alpha T} - 2\frac{1 - e^{-\alpha T}}{\alpha T}\right) - \frac{\varepsilon_F D_3}{\rho^3}\left(1 - x\rho \frac{\bar{\nu}_S^2 D_{3S}}{\bar{\nu}^2 D_3}\right)\left(1 - \frac{3 - 4e^{-\alpha T} + 2e^{-2\alpha T}}{\alpha T}\right)$$

Check of the point model with Monte Carlo codes

Test problem : Fissile solution (UHE+water)



- Feynman moment of order 2 in function of the time window (Tripoli-4)
- Asymptotic value of the Feynman moment of order 2 for Tripoli-4
- Feynman moment of order 2 in function of the time window (MCNP)
- Asymptotic value of the Feynman moment of order 2 for MCNP

FIGURE – Feynman moment of order 2 for the fissile solution (UHE+water)

- Tripoli-4 Freya model detailled fission
- MCNP Terrell type distribution (gaussian)

Test problem :

- Cf^{252} : Spontaneous fission source, Poisson compound

- Feynman moment of order 2 in function of the time window (Tripoli-4)
- Feynman moment of order 2 in function of the time window (MCNP)
- Asymptotic value of the Feynman moment of order 2

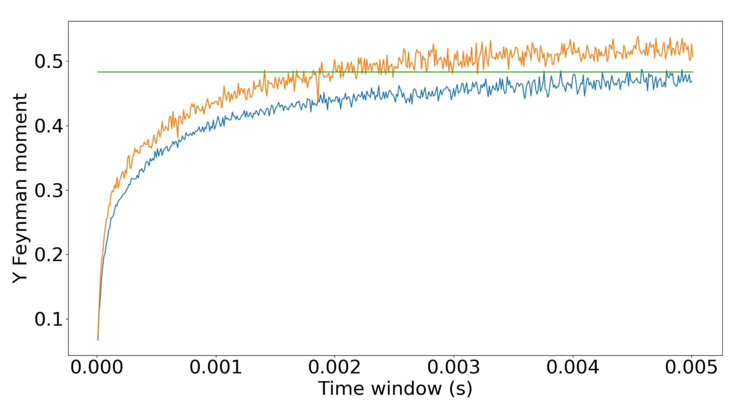


FIGURE – Feynman moment of order 2 for the Cf^{252}

The point model is valid, it can be used for the inverse problem.

Physical parameters

Parameters of the model

- $\varepsilon_F = \frac{\text{count number}}{\text{induced fission number}}$
- S intensity of the source
- ρ reactivity of the system
- x proportion of spontaneous fission source neutrons

Nuclear Datas

- $\bar{\nu}$ average number of neutron emitted by a fission
- $D_i = \frac{\overline{\nu(\nu-1)\cdots(\nu-i)}}{\bar{\nu}^i}, i \in \mathbb{N}^*$ Diven factor (idem for $D_{i,S}, i \in \mathbb{N}^*$)

Measurements

- Average count number and Feynman moment

$$\bar{C}(T) = M_1$$
$$Y_2(T) = \frac{2M_2}{M_1} - M_1$$
$$Y_3(T) = \frac{6M_3}{M_1} - 6M_2 + 2M_1^2$$

where $M_i, i \in \mathbb{N}$, moment of order i of neutrons detected in $[0, T]$.

The inverse problem, point estimation

We are looking for the $\mathbf{u}$ which minimizes the mean square error function

$$||\mathbf{y}_{obs} - \mathbf{f}(\mathbf{u})||_\Gamma^2$$

where $\mathbf{y}_{obs} = \begin{pmatrix} \bar{C}(T) \\ Y_2(T) \\ Y_3(T) \end{pmatrix}$ are the measures, $\mathbf{u} = \begin{pmatrix} \varepsilon_F \\ \rho \\ S \\ x \end{pmatrix}$ the parameters, and Γ is the covariance matrix of the empirical Feynman moments.
We obtain the best estimation $\hat{\mathbf{u}}$ using optimization methods like simulated annealing.

	Parameters			Feynman moments		
	ε_F	S	ρ	$\bar{C}$	Y_2	Y_3
Given values	0.481	1000	−0.258	769.75272	6.9939	117.2060
Estimated values	0.4614	1020.4884	−0.2526	769.7501	6.9746	117.2123

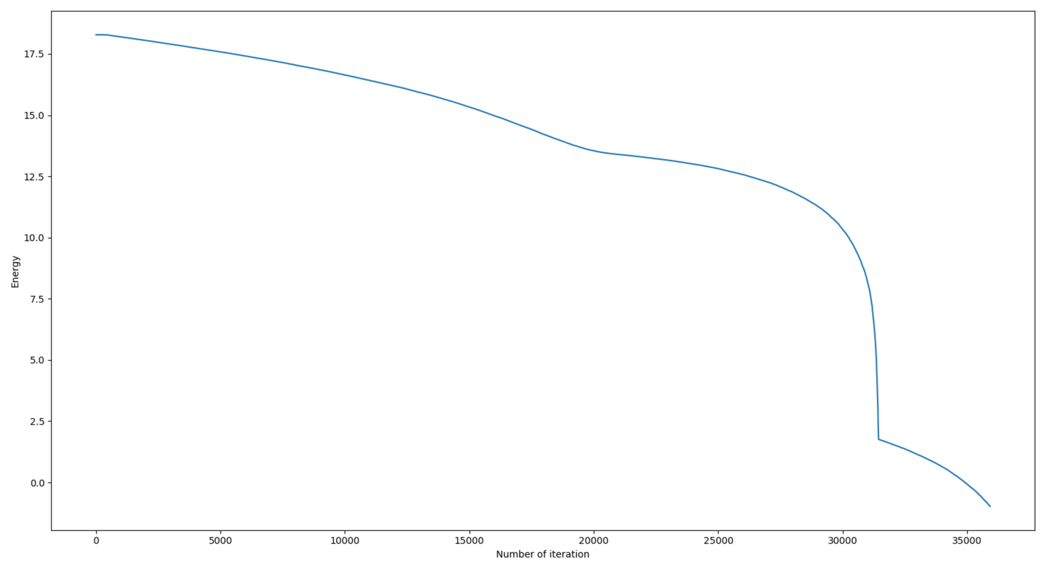


FIGURE – Evolution of the mean square error function

In a second part, we will use a Bayesian approach.

The inverse problem, bayesian inference

The bayesian approach returns the whole distribution.

$$\underset{a\ posteriori\ distribution}{\mathbb{P}(\mathbf{u}|\mathbf{y}_{obs})} \propto \underset{likelihood}{\mathbb{P}(\mathbf{y}_{obs}|\mathbf{u})} \underset{a\ priori\ distribution}{\mathbb{P}(\mathbf{u})}$$

We will use Markov Chain Monte Carlo methods, to obtain the posterior distributions.

Conclusions

- Point model is good enough to serve as a forward model in the solution of the inverse problem.
- The ultimate goal is to apply a bayesian approach to estimate the a posteriori distribution of the nuclear matter parameters.

References

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[4] Hage & Cifarelli, *Correlation Analysis with Neutron Count Distributions in Randomly or Signal Trigerred Time Intervals for Assay of Special Fissile Material*, NUCLEAR SCIENCE AND ENGINEERING, Vol. 112, 1992

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