On-line learning, Learning Using Privileged Information (LUPI) and transfer learning

Antoine Cornuéjols

AgroParisTech – INRA MIA 518

Équipe LINK



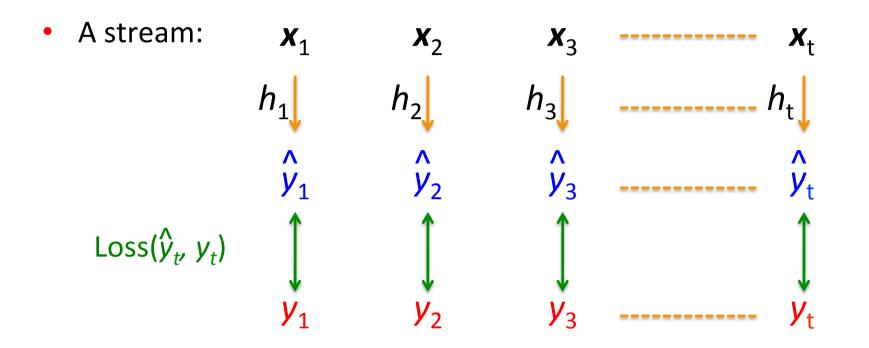


Outline

1. The online learning perspective

- 2. Early classification of time series
- 3. Early classification of time series and transfer learning
- 4. The TransBoost algorithm
- 5. Conclusion

The online learning scenario



E.g. *Choice of melons*. I see one, I make a prediction about its tastiness, then I eat it and know the answer.

- **1**. Learning and testing are intermingled
 - No distinction between training set, validation set and test set

Novelty wrt. Statistical learning

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 - The learner should adapt

Novelty wrt. Statistical learning

- 1. Learning and testing are intermingled
 - No distinction between training set, validation set and test set
- 2. The environment may change over time
 - The learner should adapt
- 3. Dilemma
 - Keep as much as possible memory from the past to gain in precision
 - But **be ready to adapt** to changes (and reduce the size of the memory)

Desirable properties of a system that handle concept drift

• Adapt to concept drift as soon as possible

- Distinguish **noise** from **true changes**
 - Robust to noise but adaptive to changes
- Recognize and react to recurring contexts

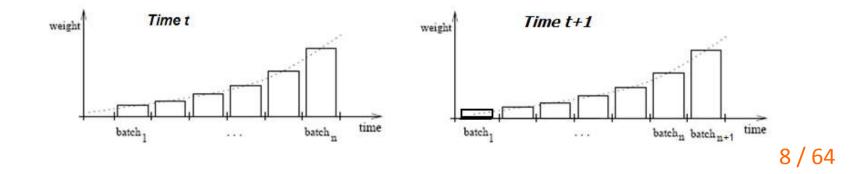
Adapt with limited resources (time and memory)





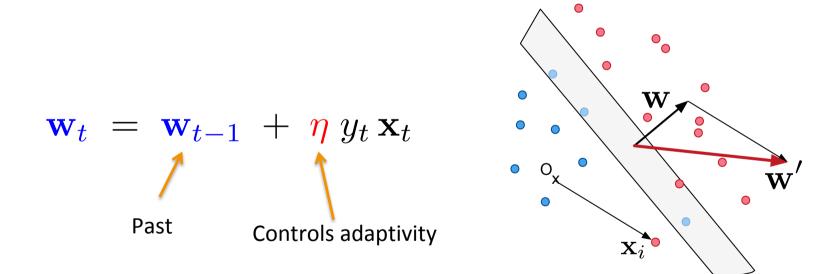
Two main approaches

1. Directly control the memory - Adapt the window size [Widmer G. & Kubat M. (1996). Learning in the presence of concept drift and hidden contexts. Mach. Learning, 23, 69-101] - Weight the past examples $w(\mathbf{x}) = e^{-\lambda t_{\mathbf{x}}}$



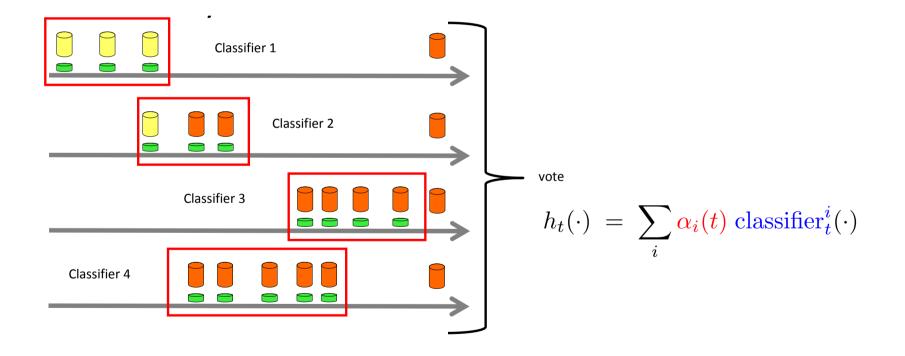
Two main approaches

2. Adapt the hypothesis at each time step



Two main approaches

2. Adapt the hypothesis at each time step



- 1. Non Lipschitzian scenario
 - Successive entries are independent, possibly adversarial
 - Online learning theory [Cesa-Bianchi & Lugosi, 2006]

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- 2. Temporal **consistency**
 - Heuristic online learning methods (sliding windows, adaptation, ...)
 - Tracking: Adapt to the past and always be behind the changes

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- 4. **Transduction**: take into account the future "questions" x_{t+1}
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- 4. **Transduction**: take into account the future "question(s)" x_{t+1}
 - Learn h_t using x_{t+1} as well. (As in semi-supervised learning)
- 5. **Both** (3) and (4)

Online learning and transfer learning

- Each step implies a "small" transfer
 - From the environment at time t-1 to the environment at time t

• Use "source knowledge" (h_{t-1}) and the current batch $\{(\mathbf{x}_t^i, y_t^i)\}_{1 \le i \le m}$

to learn target h_t by adapting from past to current environment

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Learning Using Privileged Information

Inspired by learning at school

- The goal is to learn a function $h: \mathbf{x} \in \mathcal{X} o y \in \{-1, +1\}$
- Suppose that at learning time there is more available information than at test time $S^* = \int (\mathbf{x} \cdot \mathbf{x}^* \cdot \mathbf{u} \cdot) \int d\mathbf{x} \cdot \mathbf{x} \cdot \mathbf{x}^* \cdot \mathbf{u} \cdot d\mathbf{x} \cdot \mathbf{x}^* \cdot \mathbf{u} \cdot d\mathbf{x} \cdot \mathbf{x}^* \cdot \mathbf{u} \cdot d\mathbf{x} \cdot \mathbf{x}^* \cdot \mathbf{u} \cdot \mathbf{u}$

$$\mathcal{S}^* = \{(\mathbf{x}_i, \mathbf{x}_i^*, \mathbf{y}_i)\}_{1 \leq i \leq m}$$

• Can we then improve the generalization performance wrt. the one obtained with *S* only?

V. Vapnik and A. Vashist (2009) "A new learning paradigm: Learning using privileged information". Neural Networks, vol. 22, no. 5, pp. 544–557, 2009 18 / 64

One solution: SVM+

• The classical optimization problem

$$\min \frac{1}{2} \langle \omega, \omega \rangle + C \sum_{i=1}^{m} \xi_i$$

s.t. $y_i[\langle \omega, x_i \rangle + b] \ge 1 - \xi_i, \qquad i = 1, \dots, m.$

• is changed into

$$\min \frac{1}{2} \left[\langle \omega, \omega \rangle + \gamma \langle \omega^*, \omega^* \rangle \right] + C \sum_{i=1}^m \left[\langle \omega^*, x^* \rangle + b^* \right]$$
s.t. $y_i [\langle \omega, x_i \rangle + b] \ge 1 - \left[\langle \omega^*, x_i^* \rangle + b^* \right], \quad i = 1, \dots, m,$
 $\left[\langle \omega^*, x_i^* \rangle + b^* \right] \ge 0, \quad i = 1, \dots, m,$

C and γ are hyperparameters

- Intuition:
 - Identify the difficult examples
 - And relax / tighten the SVM constraints accordingly -> better generalization performances

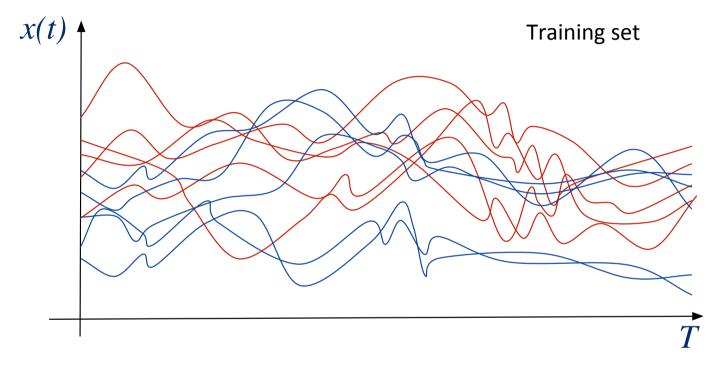
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Classification of time series

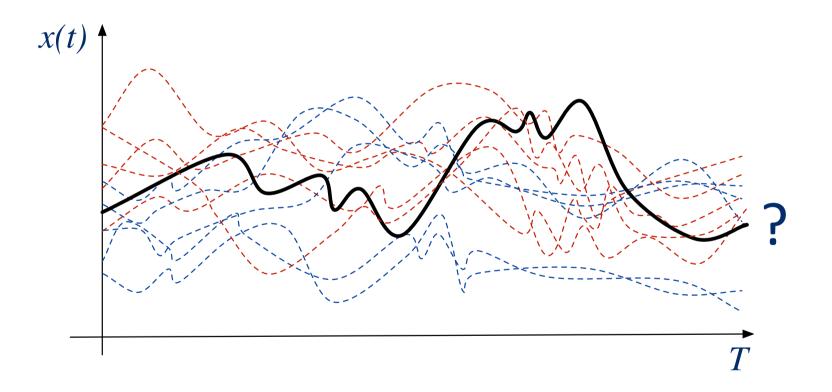


- Monitoring of *consumer actions on a web site*:
- Monitoring of a *patient state*:
- Early prediction of daily *electrical consumption*:

will buy or not critical or not high or low

Standard classification of time series

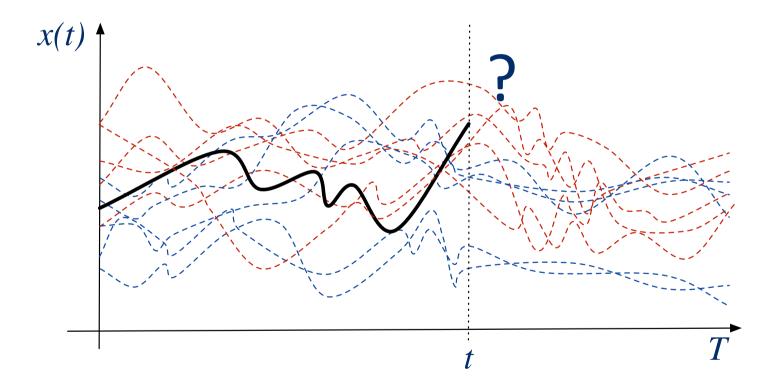
• What is the class of the new time series x_7 ?



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Early classification of time series

• What is the class of the new **incomplete** time series x_t ?



New set of decision problems : early classification

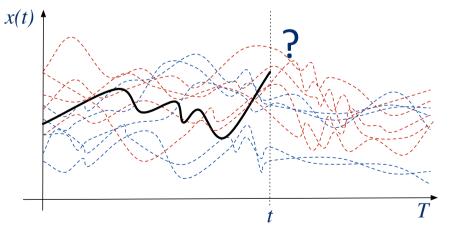
- Data stream
- Classification task
- As **early** as possible
- A trade-off
 - Classification **performance** (better if $t \nearrow$)
 - Cost of **delaying** prediction (better if $t \searrow$)

Early classification of time series

Online decision problem

- With option to defer at each time step
 - If the **expected future performance** overcomes

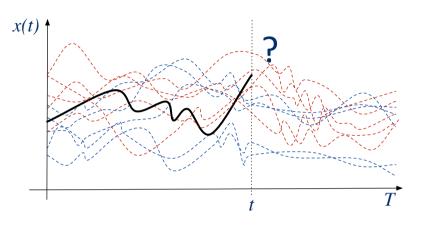
the cost of delaying decision



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Early classification and LUPI

• This is a LUPI setting



• How to take advantage of this?

Decision making (1)

- Given an incoming sequence $\mathbf{x}_t = \langle x_1, x_2, \dots, x_t \rangle$ where $x_t \in \mathbb{R}$
- And given:
 - A miss-classification cost function
 - A delaying decision cost function

$$egin{aligned} C_t(\hat{y}|y) : \mathcal{Y} imes \mathcal{Y} \longrightarrow \mathbb{R} \ C(t) : \mathbb{N} \longrightarrow \mathbb{R} \end{aligned}$$

• What is the optimal time to make a decision?

Expected cost for a decision at time t

$$f(\mathbf{x}_t) = \underbrace{\sum_{y \in \mathcal{Y}} P(y|\mathbf{x}_t) \sum_{\hat{y} \in \mathcal{Y}} P(\hat{y}|y, \mathbf{x}_t) C_t(\hat{y}|y)}_{\hat{y} \in \mathcal{Y}} + C(t)$$

expected miss-classification cost given \mathbf{x}_t

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Expected cost for a decision at time t

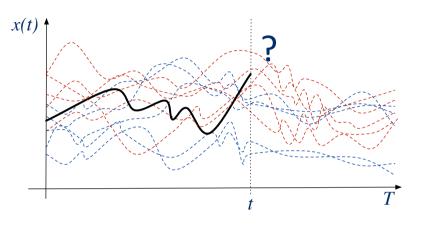
$$f(\mathbf{x}_t) = \underbrace{\sum_{y \in \mathcal{Y}} P(y|\mathbf{x}_t) \sum_{\hat{y} \in \mathcal{Y}} P(\hat{y}|y, \mathbf{x}_t) C_t(\hat{y}|y)}_{\hat{y} \in \mathcal{Y}} + C(t)$$

expected miss-classification cost given \mathbf{x}_t

Optimal time:
$$t^* = \underset{t \in \{1,...,T\}}{\operatorname{ArgMin}} f(\mathbf{x}_t)$$
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Early classification and LUPI

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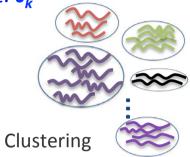


- How to take advantage of this?
 - 1. Knowledge of **possible future sequences**
 - 2. Possibility to learn classifiers for all time steps

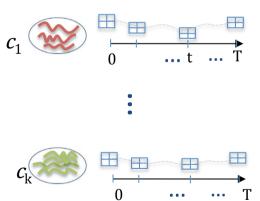
1. During **training**:

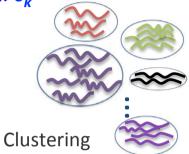
 \rightarrow identify **meaningful subsets** of time sequences in the training set: c_k

$$P(y|\mathbf{x}_t) \rightarrow P(y|\mathbf{c}_k)$$



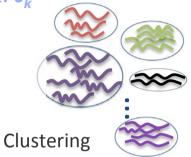
- 1. During **training**:
 - identify **meaningful subsets** of time sequences in the training set: c_k
 - For each of these subsets c_k , and for each time step t—
 - Estimate the confusion matrices ٠
- T classifiers are learnt $h_t(\mathbf{x}_t) : \mathcal{X}_t \to \mathcal{Y}$ And their confusion matrices $P_t(\hat{y}|y, \mathbf{c}_k)$ are estimated on a test set





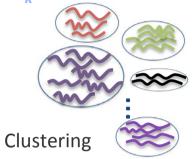
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 - For each of these subsets c_k , and for each time step t
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- **2. Testing**: For any new incomplete incoming sequence x_t
 - \rightarrow Identify the most likely subset: the closer class of shapes to x_t

- **1.** During **training**:
 - identify **meaningful subsets** of time sequences in the training set: c_k
 - For each of these subsets c_k , and for each time step t
 - Estimate the confusion matrices $P_t(\hat{y}|y, \mathfrak{c}_k)$



- **2. Testing**: For any new incomplete incoming sequence x_t
 - Identify the most likely subset: the closer shape to x_t
 - Compute the expected cost of decision for all future time steps

$$f_{\tau}(\mathbf{x}_{t}) = \underbrace{\sum_{\boldsymbol{\mathfrak{c}}_{k} \in \mathcal{C}} P(\boldsymbol{\mathfrak{c}}_{k} | \mathbf{x}_{t}) \sum_{y \in \mathcal{Y}} P(y | \boldsymbol{\mathfrak{c}}_{k}) \sum_{\hat{y} \in \mathcal{Y}} P_{t+\tau}(\hat{y} | y, \boldsymbol{\mathfrak{c}}_{k}) C(\hat{y} | y)}_{\hat{y} \in \mathcal{Y}} + C(t+\tau)}_{\text{expected miss-classification cost given } \mathbf{x}_{t}}$$

A non myopic decision process

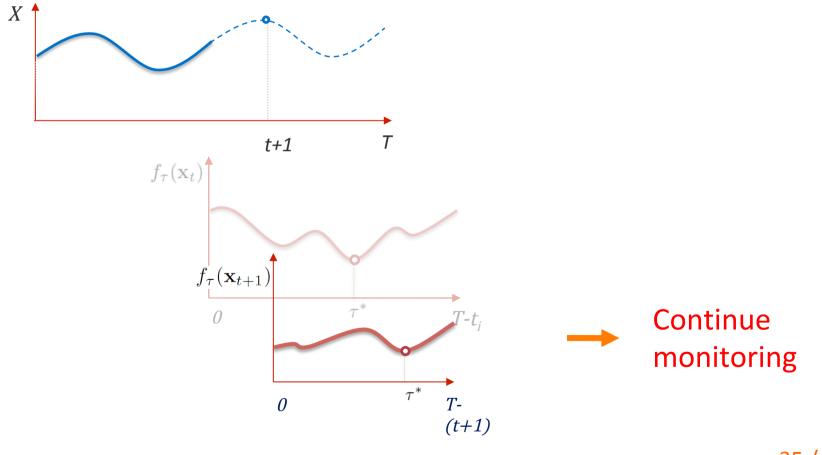
٠

 $\tau^* = \operatorname*{ArgMin}_{\tau \in \{0, \dots, T-t\}} f_{\tau}(\mathbf{x}_t)$ Optimal estimated time relative to current time *t* X^{\prime} Т t $f_{\tau}(\mathbf{x}_t)$ Continue monitoring τ^* 0 T-t

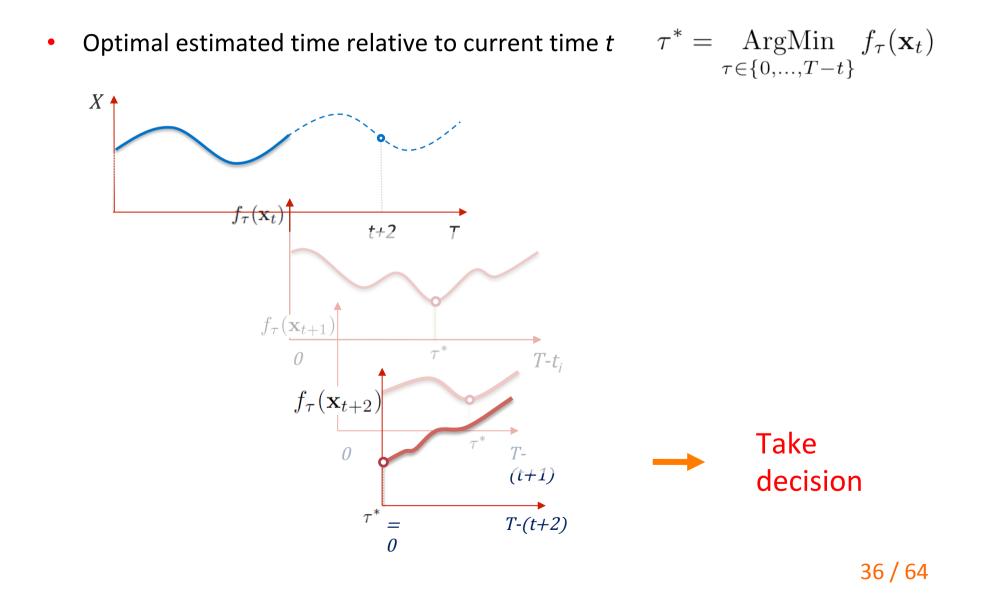
A non myopic decision process

• Optimal estimated time relative to current time *t*

$$\tau^* = \operatorname*{ArgMin}_{\tau \in \{0, \dots, T-t\}} f_{\tau}(\mathbf{x}_t)$$



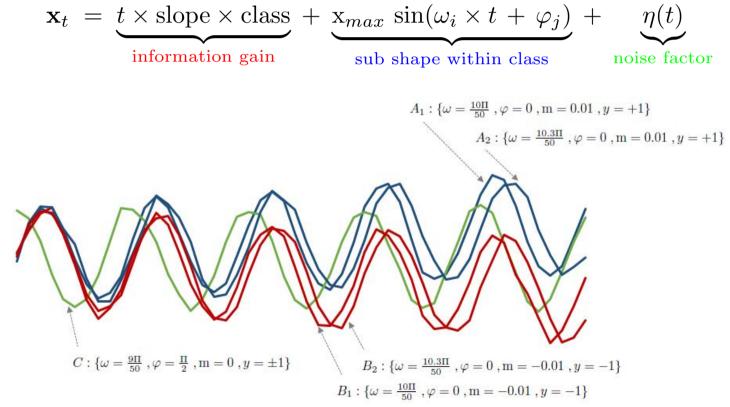
A non myopic decision process



Controlled data

• Control of

- The time-dependent information provided to distinguish between classes
- The shapes of **time series** within each class
- The noise level



Results: effect of the noise level

	$C(t) \stackrel{\pm b}{(t)}$		0.02			0.05			0.07		
		$\varepsilon(t)$	$\overline{ au}^{\star}$	$\sigma(\tau^{\star})$	AUC	$\overline{\tau}^{\star}$	$\sigma(\tau^{\star})$	AUC	$\overline{ au}^{\star}$	$\sigma(\tau^{\star})$	AUC
		0.2	9.0	2.40	0.99	9.0	2.40	0.99	10.0	0.0	1.00
		0.5	13.0	4.40	0.98	13.0	4.40	0.98	15.0	0.18	1.00
	0.01	1.5	24.0	10.02	0.98	32.0	2.56	1.00	30.0	12.79	0.99
		5.0	26.0	7.78	0.84	30.0	18.91	0.87	30.0	19.14	0.88
		10.0	38.0	18.89	0.70	48.0	1.79	0.74	46.0	5.27	0.75
		15.0	23.0	15.88	0.61	32.0	13.88	0.64	29.0	17.80	0.62
		20.0	7.0	8.99	0.52	11.0	11.38	0.55	4.0	1.22	0.52
Increasing the noise		0.2	8.0	2.00	0.98	8.0	2.00	0.98	9.0	0.0	1.00
		0.5	10.0	2.80	0.96	8.0	4.0	0.98	14.0	0.41	0.99
level increases the	0.05	1.5	5.0	0.40	0.68	20.0	0.42	0.95	14.0	4.80	0.88
level increases the		5.0	8.0	3.87	0.68	6.0	1.36	0.64	5.0	0.50	0.65
		10.0	4.0	0.29	0.56	4.0	0.25	0.56	4.0	0.34	0.57
waiting time, and then		15.0	4.0	0.0	0.54	4.0	0.25	0.56	4.0	0.0	0.55
0 ,		20.0	4.0	0.0	0.52	4.0	0.0	0.52	4.0	0.0	0.52
it's no longer worth it											
it sho longer worth it		0.2	6.0	0.80	0.95	7.0	1.60	0.94	8.0	0.40	0.96
		0.5	6.0	0.80	0.84	9.0	2.40	0.93	10.0	0.0	0.95
	0.10	1.5	4.0	0.0	0.67	5.0	0.43	0.68	6.0	0.80	0.74
		5.0	4.0	0.07	0.64	4.0	0.05	0.64	4.0	0.11	0.64
		10.0	4.0	0.0	0.56	48.0	1.79	0.74	4.0	0.22	0.56
		15.0	4.0	0.0	0.55	4.0	0.0	0.55	4.0	0.0	0.55
		20.0	4.0	0.0	0.52	11.0	11.38	0.55	4.0	0.0	0.52

Table 1. Experimental results in function of the waiting cost $C(t) = \{0.01, 0.05, 0.1\} \times t$, the noise level $\varepsilon(t)$ and the trend parameter b.

Results: effect of the waiting cost

	C(t)	$\pm b$	0.02			0.05			0.07		
	O(t)	$\varepsilon(t)$	$\overline{\tau}^{\star}$	$\sigma(\tau^{\star})$	AUC	$\overline{ au}^{\star}$	$\sigma(\tau^{\star})$	AUC	$\overline{\tau}^{\star}$	$\sigma(\tau^{\star})$	AUC
				. ,			. ,			. ,	
	1	0.2	9.0	2.40	0.99	9.0	2.40	0.99	10.0	0.0	1.00
		0.5	13.0	4.40	0.98	13.0	4.40	0.98	15.0	0.18	1.00
	0.01	1.5	24.0	10.02	0.98	32.0	2.56	1.00	30.0	12.79	0.99
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		15.0	23.0	15.88	0.61	32.0	13.88	0.64	29.0	17.80	0.62
		20.0	7.0	8.99	0.52	11.0	11.38	0.55	4.0	1.22	0.52
Increasing the		0.2	8.0	2.00	0.98	8.0	2.00	0.98	9.0	0.0	1.00
		0.5	10.0	2.80	0.96	8.0	4.0	0.98	14.0	0.41	0.99
waiting cost	0.05	1.5	5.0	0.40	0.68	20.0	0.42	0.95	14.0	4.80	0.88
-		5.0	8.0	3.87	0.68	6.0	1.36	0.64	5.0	0.50	0.65
reduces the waiting		10.0	4.0	0.29	0.56	4.0	0.25	0.56	4.0	0.34	0.57
9		15.0	4.0	0.0	0.54	4.0	0.25	0.56	4.0	0.0	0.55
time		20.0	4.0	0.0	0.52	4.0	0.0	0.52	4.0	0.0	0.52
		0.2	6.0	0.80	0.95	7.0	1.60	0.94	8.0	0.40	0.96
		0.5	6.0	0.80	0.84	9.0	2.40	0.93	10.0	0.0	0.95
	0.10	1.5	4.0	0.0	0.67	5.0	0.43	0.68	6.0	0.80	0.74
		5.0	4.0	0.07	0.64	4.0	0.05	0.64	4.0	0.11	0.64
	\checkmark	10.0	4.0	0.0	0.56	48.0	1.79	0.74	4.0	0.22	0.56
	*	15.0	4.0	0.0	0.55	4.0	0.0	0.55	4.0	0.0	0.55
		20.0	4.0	0.0	0.52	11.0	11.38	0.55	4.0	0.0	0.52

Table 2. Experimental results in function of the waiting cost $C(t) = \{0.01, 0.05, 0.1\} \times t$, the noise level $\varepsilon(t)$ and the trend parameter b.

Results: effect of the difference between classes

											\rightarrow
	C(t)	$\pm b$		0.02			0.05			0.07	
Increase of the		$\varepsilon(t)$	$\overline{\tau}^{\star}$	$\sigma(\tau^{\star})$	AUC	$\overline{\tau}^{\star}$	$\sigma(\tau^{\star})$	AUC	$\overline{\tau}^{\star}$	$\sigma(\tau^{\star})$	AUC
				a 10			a 1a				
difference between		0.2	<i>9.0</i>	2.40	0.99	<i>9.0</i>	2.40	0.99	10.0	0.0	1.00
classes	0.01	$\begin{array}{c} 0.5 \\ 1.5 \end{array}$	13.0 24.0	$\begin{array}{c} 4.40 \\ 10.02 \end{array}$	$\begin{array}{c} 0.98 \\ 0.98 \end{array}$	13.0 32.0	$\begin{array}{c} 4.40 \\ 2.56 \end{array}$	$\begin{array}{c} 0.98 \\ 1.00 \end{array}$	$15.0 \\ 30.0$	$0.18 \\ 12.79$	$\begin{array}{c} 1.00 \\ 0.99 \end{array}$
6103563	0.01	5.0	24.0 26.0	$\frac{10.02}{7.78}$	$\begin{array}{c} 0.98\\ 0.84 \end{array}$	32.0 30.0	$\frac{2.50}{18.91}$	0.87	30.0	12.79 19.14	0.99 0.88
		10.0	38.0	18.89	0.70	48.0	10.91 1.79	$0.01 \\ 0.74$	46.0	5.27	$0.00 \\ 0.75$
		15.0	23.0	15.88	0.61	32.0	13.88	0.64	29.0	17.80	0.62
		20.0	7.0	8.99	0.52	11.0	11.38	0.55	4.0	1.22	0.52
The performance											
		0.2	8.0	2.00	0.98	8.0	2.00	0.98	9.0	0.0	1.00
increases (AUC)		0.5	10.0	2.80	0.96	8.0	4.0	0.98	14.0	0.41	0.99
	0.05	1.5	5.0	0.40	0.68	20.0	0.42	0.95	14.0	4.80	0.88
		$\begin{array}{c} 5.0 \\ 10.0 \end{array}$	8.0 4.0	$\begin{array}{c} 3.87 \\ 0.29 \end{array}$	$\begin{array}{c} 0.68 \\ 0.56 \end{array}$	6.0 4.0	$\begin{array}{c} 1.36 \\ 0.25 \end{array}$	$\begin{array}{c} 0.64 \\ 0.56 \end{array}$	$5.0 \\ 4.0$	$\begin{array}{c} 0.50 \\ 0.34 \end{array}$	$\begin{array}{c} 0.65 \\ 0.57 \end{array}$
		$10.0 \\ 15.0$	4.0	0.29 0.0	0.50 0.54	4.0	$0.25 \\ 0.25$	$\begin{array}{c} 0.50\\ 0.56\end{array}$	4.0	$\begin{array}{c} 0.34 \\ 0.0 \end{array}$	$\begin{array}{c} 0.57 \\ 0.55 \end{array}$
The <i>waiting time</i> is not		20.0	4.0	0.0	$0.51 \\ 0.52$	4.0	0.20	$0.50 \\ 0.52$	4.0	0.0	$0.50 \\ 0.52$
•											
much changed in these		0.2	6.0	0.80	0.95	7.0	1.60	0.94	8.0	0.40	0.96
experiments		0.5	6.0	0.80	0.84	9.0	2.40	0.93	10.0	0.0	0.95
experiments	0.10	1.5	4.0	0.0	0.67	5.0	0.43	0.68	6.0	0.80	0.74
		5.0	4.0	0.07	0.64	4.0	0.05	0.64	4.0	0.11	0.64
		10.0	4.0	0.0	0.56	48.0	1.79	0.74	4.0	0.22	0.56
		$\begin{array}{c} 15.0 \\ 20.0 \end{array}$	4.0 4.0	$\begin{array}{c} 0.0 \\ 0.0 \end{array}$	$\begin{array}{c} 0.55 \\ 0.52 \end{array}$	4.0	$\begin{array}{c} 0.0\\ 11.38\end{array}$	$\begin{array}{c} 0.55 \\ 0.55 \end{array}$	4.0 4.0	$\begin{array}{c} 0.0 \\ 0.0 \end{array}$	$\begin{array}{c} 0.55 \\ 0.52 \end{array}$

Table 3. Experimental results in function of the waiting cost $C(t) = \{0.01, 0.05, 0.1\} \times t$, the noise level $\varepsilon(t)$ and the trend parameter b.

[Dachraoui, A., Bondu, A., & Cornuéjols, A. (2015).] [Dachraoui, A., Bondu, A., & Cornuéjols, A. (2016).]

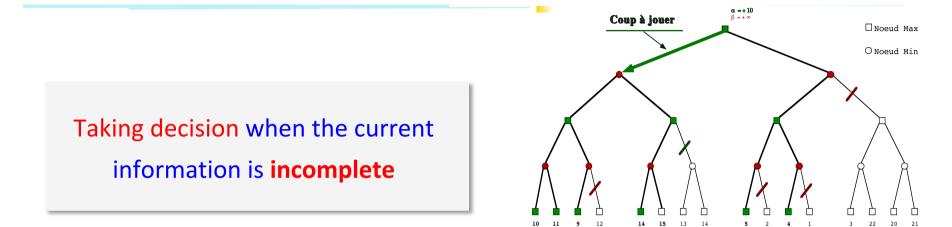
1. Formalized the problem as a sequential decision making problem

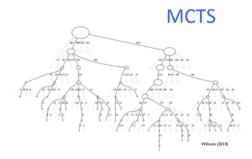
- Explicit trade-off
 - Classification **performance**
 - Cost of delaying the decision
- 2. Proposed an algorithm which is
 - Adaptive
 - Takes into account the peculiarities of *x*_t
 - Non myopic
 - At each time step, estimates the expected future time for optimal decision
- 3. Showed promising experimental results
 - The delay before decision **exhibited** what should be **expected**

Outline

- **1.** The online learning perspective
- 2. Early classification of time series
- 3. Early classification of time series and transfer learning
- 4. The TransBoost algorithm
- 5. Conclusion

Algorithms for games





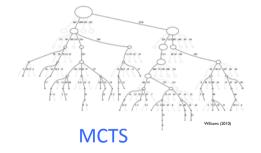
Algorithms for games

Taking decision when the current information is incomplete

• Which move to play?

The evaluation function is **insufficiently informed** at the root (current situation)

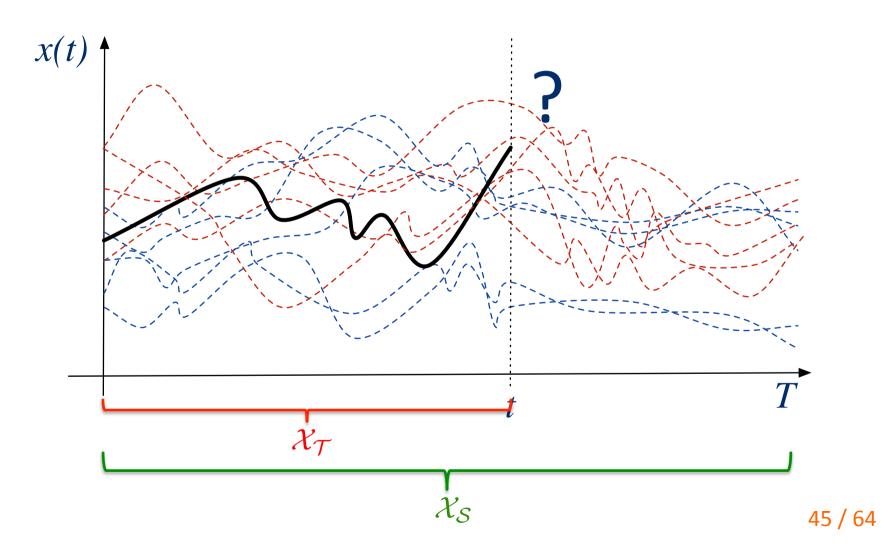
- Query experts that have more information about potential outcomes
- 2. Combination of the estimates through MinMax



"Experts" may live in *input spaces* that are *different*

Early classification of time series

• What is the class of the new incomplete time series x_t ?



• Learn a classifier over the training set of **complete times series**

$$S_{\mathcal{S}} = \{ (\mathbf{x}_i^{\mathcal{S}}, y_i^{\mathcal{S}}) \}_{1 \le i \le m} \to h_{\mathcal{S}}$$

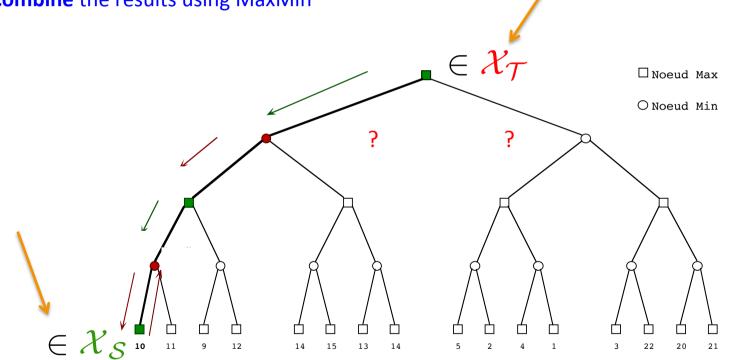
 Try to make use of this classifier to predict the class of incomplete series

$$h_{\mathcal{T}}$$
 = Function using $h_{\mathcal{S}}$

Algorithms for games and transfer learning

Which move?

- Better evaluation function in X_s
- Backup it (by transfer) for $X_{T'}$
- Combine the results using MaxMin



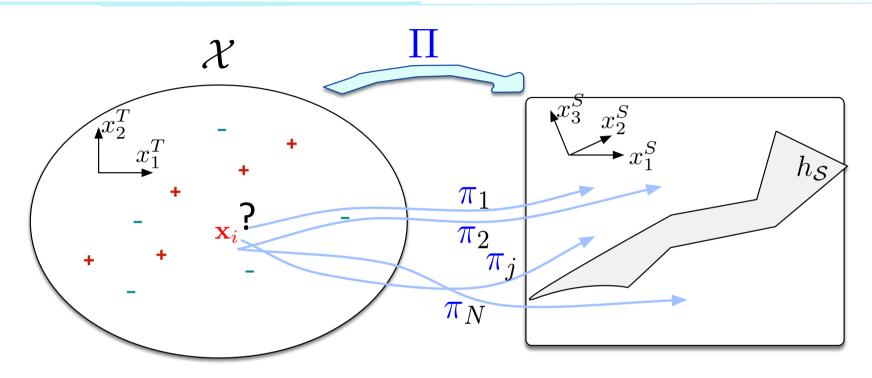
Outline

- **1.** The online learning perspective
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4. The TransBoost algorithm

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TransBoost



Target Domain

Source Domain

$$H_{\mathcal{T}}(\mathbf{x}^{\mathcal{T}}) = \operatorname{sign}\left\{\sum_{n=1}^{N} \alpha_n h_{\mathcal{S}}(\pi_n(\mathbf{x}^{\mathcal{T}}))\right\}$$

TransBoost

- Principle:
 - Learn "weak projections": $\pi_i : \mathcal{X}_S \rightarrow \mathcal{X}_T$

• From:
$$S_{\mathcal{S}} = \{(\mathbf{x}_i^{\mathcal{S}}, y_i^{\mathcal{S}})\}_{1 \leq i \leq m}$$

- Using boosting

- Projection π_n such that : $\varepsilon_n \doteq \mathbf{P}_{i \sim D_n}[h_{\mathcal{S}}(\pi_n(\mathbf{x}_i)) \neq y_i] < 0.5$
- Re-weight the training time series and loop until termination

Result

$$H_{\mathcal{T}}(\mathbf{x}^{\mathcal{T}}) = \operatorname{sign}\left\{\sum_{n=1}^{N} \alpha_n h_{\mathcal{S}}(\pi_n(\mathbf{x}^{\mathcal{T}}))\right\}$$

TransBoost

Algorithm 1: Transfer learning by boosting

Input: $h_{\mathcal{S}} : \mathcal{X}_{\mathcal{S}} \to \mathcal{Y}_{\mathcal{S}}$ the source hypothesis $\mathcal{S}_{\mathcal{T}} = \{(\mathbf{x}_i^{\mathcal{T}}, y_i^{\mathcal{T}}\}_{1 \leq i \leq m}: \text{ the target training set } \}$

Initialization of the distribution on the training set: $D_1(i) = 1/m$ for i = 1, ..., m;

for n = 1, ..., N do Find a projection $\pi_i : \mathcal{X}_{\mathcal{T}} \to \mathcal{X}_{\mathcal{S}}$ st. $h_{\mathcal{S}}(\pi_i(\cdot))$ performs better than random on $D_n(\mathcal{S}_{\mathcal{T}})$; Let ε_n be the error rate of $h_{\mathcal{S}}(\pi_i(\cdot))$ on $D_n(\mathcal{S}_{\mathcal{T}}) : \varepsilon_n \doteq \mathbf{P}_{i\sim D_n}[h_{\mathcal{S}}(\pi_n(\mathbf{x}_i)) \neq y_i]$ (with $\varepsilon_n < 0.5$); Computes $\alpha_i = \frac{1}{2}\log_2(\frac{1-\varepsilon_i}{\varepsilon_i})$; Update, for i = 1..., m: $D_{n+1}(i) = \frac{D_n(i)}{Z_n} \times \begin{cases} e^{-\alpha_n} & \text{if } h_{\mathcal{S}}(\pi_n(\mathbf{x}_i^{\mathcal{T}})) = y_i^{\mathcal{T}} \\ e^{\alpha_n} & \text{if } h_{\mathcal{S}}(\pi_n(\mathbf{x}_i^{\mathcal{T}})) \neq y_i^{\mathcal{T}} \end{cases}$ $= \frac{D_n(i) \exp(-\alpha_n y_i^{(\mathcal{T})} h_{\mathcal{S}}(\pi_n(\mathbf{x}_i^{(\mathcal{T})})))}{Z_n}$

where Z_n is a normalization factor chosen so that D_{n+1} be a distribution on $\mathcal{S}_{\mathcal{T}}$; end

Output: the final target hypothesis $H_{\mathcal{T}} : \mathcal{X}_{\mathcal{T}} \to \mathcal{Y}_{\mathcal{T}}$:

$$H_{\mathcal{T}}(\mathbf{x}^{\mathcal{T}}) = \operatorname{sign}\left\{\sum_{n=1}^{N} \alpha_n h_{\mathcal{S}}(\pi_n(\mathbf{x}^{\mathcal{T}}))\right\}$$
(2)

Kesuits										
	On the source									
	Learnin	g from 🛛 🛏			domain					
	target da	ata only	Trans	Boost	1	Naïve transfert				
	,l			4	. ↓					
slope, noise, $t_{\mathcal{T}}$	$h_{\mathcal{T}}$ (train)	$h_{\mathcal{T}}$ (test)	$H_{\mathcal{T}}$ (train)	$H_{\mathcal{T}}$ (test)	$h_{\mathcal{S}}$ (test)	$H'_{\mathcal{T}}$ (test)				
0.001, 0.001, 20	0.46 ± 0.02	0.50 ± 0.08	0.08 ± 0.03	$\textbf{0.08} \pm 0.02$	0.05	0.49 ± 0.01				
0.005, 0.001, 20	0.46 ± 0.02	0.49 ± 0.01	0.01 ± 0.01	$\textbf{0.01} \pm 0.01$	0.01	0.45 ± 0.01				
0.005, 0.002, 20	0.46 ± 0.02	0.49 ± 0.03	0.03 ± 0.02	$\textbf{0.04} \pm 0.02$	0.02	0.43 ± 0.01				
0.005, 0.02, 20	0.44 ± 0.02	0.48 ± 0.03	0.09 ± 0.01	$\textbf{0.10}\pm0.01$	0.01	0.47 ± 0.01				
0.001, 0.2, 20	0.46 ± 0.02	0.50 ± 0.01	0.46 ± 0.02	0.51 ± 0.02	0.11	0.49 ± 0.01				
0.01, 0.2, 20	0.42 ± 0.03	0.47 ± 0.03	0.34 ± 0.02	0.35 ± 0.02	0.02	0.35 ± 0.01				
0.001, 0.001, 50	0.46 ± 0.02	0.50 ± 0.01	0.08 ± 0.03	$\textbf{0.08} \pm 0.02$	0.06	0.41 ± 0.01				
0.005, 0.001, 50	0.25 ± 0.07	0.28 ± 0.09	0.01 ± 0.01	$\textbf{0.01} \pm 0.01$	0.01	0.28 ± 0.01				
0.005, 0.002, 50	0.27 ± 0.07	0.30 ± 0.08	0.02 ± 0.01	$\textbf{0.02}\pm0.01$	0.02	0.28 ± 0.01				
0.005, 0.02, 50	0.26 ± 0.07	0.30 ± 0.08	0.04 ± 0.01	$\textbf{0.04} \pm 0.01$	0.01	0.31 ± 0.01				
0.001, 0.2, 50	0.44 ± 0.02	0.50 ± 0.01	0.38 ± 0.03	0.44 ± 0.02	0.15	0.43 ± 0.01				
0.01, 0.2, 50	0.10 ± 0.03	0.12 ± 0.04	0.10 ± 0.02	0.11 ± 0.02	0.03	0.15 ± 0.02				
0.001, 0.001, 100	0.43 ± 0.03	0.47 ± 0.03	0.07 ± 0.02	0.07 ± 0.02	0.02	0.23 ± 0.01				
0.005, 0.001, 100	0.06 ± 0.03	0.07 ± 0.03	0.01 ± 0.01	$\textbf{0.01} \pm 0.01$	0.01	0.07 ± 0.02				
0.005, 0.002, 100	0.08 ± 0.03	0.10 ± 0.04	0.02 ± 0.01	$\textbf{0.02}\pm0.01$	0.02	0.07 ± 0.01				
0.005, 0.02, 100	0.08 ± 0.03	0.09 ± 0.03	0.02 ± 0.01	$\textbf{0.03} \pm 0.01$	0.01	0.07 ± 0.01				
0.001, 0.2, 100	0.04 ± 0.03	0.46 ± 0.02	0.28 ± 0.02	0.31 ± 0.01	0.16	0.31 ± 0.01				
0.01, 0.2, 100	0.03 ± 0.01	0.05 ± 0.02	0.04 ± 0.01	0.05 ± 0.01	0.02	0.05 ± 0.01				

Doculto

Table 1: Comparison of learning directly in the target domain (columns $h_{\mathcal{T}}$ (train) and $h_{\mathcal{T}}$ (test)), using TransBoost (columns $H_{\mathcal{T}}$ (train) and $H_{\mathcal{T}}$ (test)), learning in the source domain (column $h_{\mathcal{S}}$ (test)) and, finally, completing the time series with a SVR regression and using $h_{\mathcal{S}}$ (naïve transfer). Test errors are highlighted in the orange columns. Bold numbers indicates where TransBoost significantly dominates both learning without transfer and learning with naïve transfer.

Results

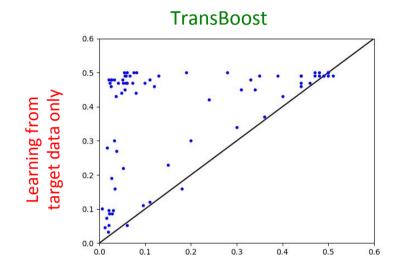


Figure 3: Comparison of error rates. *y*-axis: test error of the SVM classifier (without transfer). *x*-axis : test error of the TransBoost classifier with 10 boosting steps. The results of 75 experiments (each one repeated 100 times) are summed up in this graph.

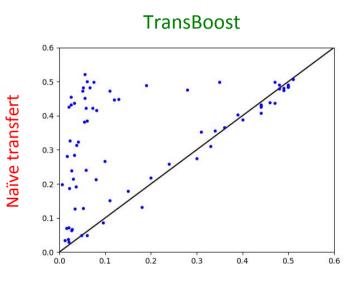


Figure 4: Comparison of error rates. *y*-axis: test error of the "naïve" transfer method. *x*-axis : test error of the TransBoost classifier with 10 boosting steps. The results of 75 experiments (each one repeated 100 times) are summed up in this graph.

Transfer learning

Illustrations





FIGURE 1: Trained model on the data source : is it a picture of a dog or a cat?





FIGURE 2: Model source transferred on the data target : is it a clip-art of a dog or a cat?

Transfer learning

Illustrations

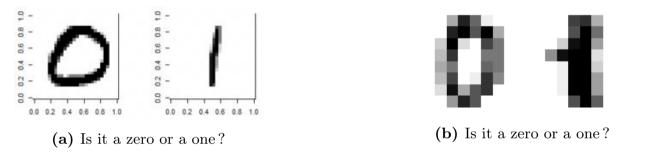
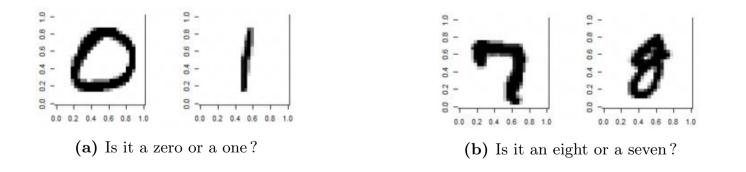
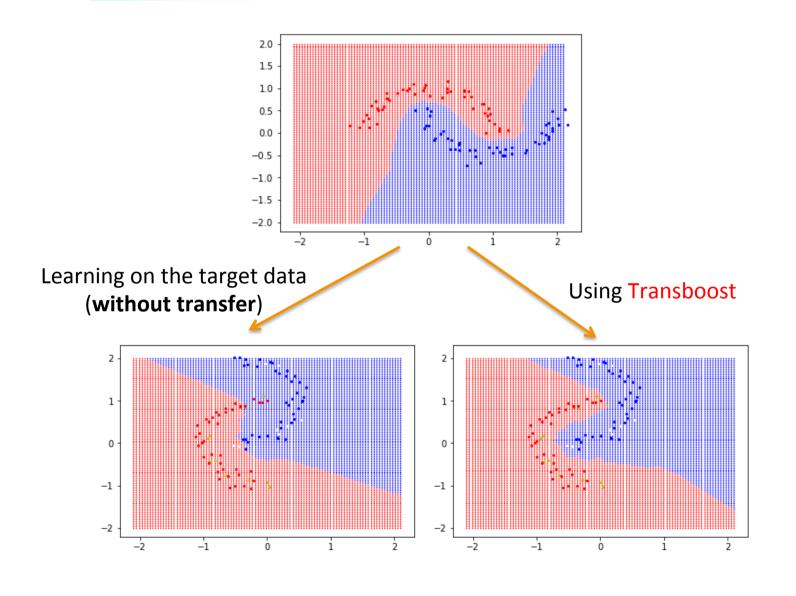


FIGURE 15: Transfer learning of the source model 0/1 mnist so that it can distinguish 0/1 sklearn digits



Transfer learning



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Conclusion

- Ensemble method for transfer learning
 - Learn weak translator from target to source!!
 - The learning problem now becomes the problem of choosing a good set of (weak) projections
 - Theoretical guarantees exist:
 - from the theory for boosting and for transfer as well

Theoretical guarantees

Theorem 1. Let $\omega : \mathbb{R} \to \mathbb{R}$ be a non-decreasing function. Suppose that P_S , P_T , h_S , $h_T = \hat{h}_S \circ \pi(\pi \in \Pi)$, \hat{h}_S and Π have the property given by Equation (2). Let $\hat{\pi} := \operatorname{ArgMin}_{\pi \in \Pi} \hat{R}_T(\hat{h}_S \circ \pi)$, be the best apparent projection. Then, with probability at least $1 - \delta$ ($\delta \in (0, 1)$) over pairs of training sets for tasks S and T:

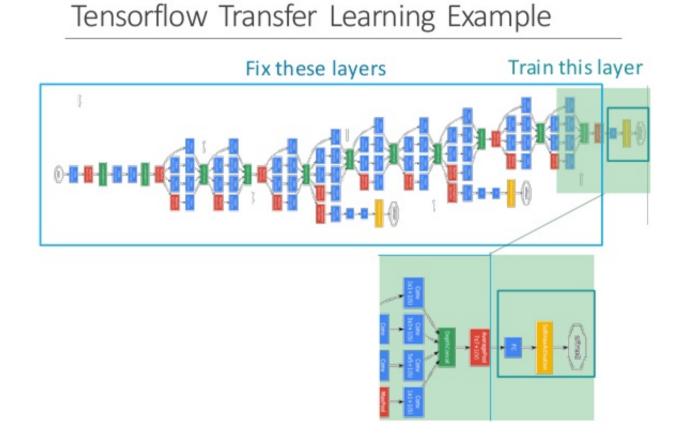
$$R_{\mathcal{T}}(\hat{h}_{\mathcal{T}}) \leq \omega \left(\widehat{R}_{\mathcal{S}}(\hat{h}_{\mathcal{S}}) \right) + 2\sqrt{\frac{2 \left(\mathcal{H}_{\mathcal{S}} \right) \log(2em_{\mathcal{S}}/d_{\mathcal{H}_{\mathcal{S}}}) + 2\log(8/\delta)}{m_{\mathcal{S}}}} + 4\sqrt{\frac{2 \left(\mathcal{H}_{\mathcal{S}} \right) \log(2em_{\mathcal{T}}/d_{h_{\mathcal{S}} \circ \Pi}) + 2\log(8/\delta)}{m_{\mathcal{T}}}}$$
(3)

$$\forall \, \widehat{h}_{\mathcal{S}} \in \mathcal{H}_{\mathcal{S}} : \quad \min_{\pi \in \Pi} R_{\mathcal{T}}(\widehat{h}_{\mathcal{S}} \circ \pi) \leq \omega \left(R_{\mathcal{S}}(h_{\mathcal{S}}) \right) \quad (2)$$

where $\omega : \mathbb{R} \to \mathbb{R}$ is a non-decreasing function.

Transfer learning for deep neural networks

Illustration



Outline

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Conclusion

1. **Online** learning

- Links with transfer learning
- New scenarios must be explored
 - Extrapolate likely changes of *h*
 - Transduction ("weak LUPI")
- 2. Early classification of time series
 - Can be solved as a LUPI framework
 - Can be seen as involving **transfer learning**

3. The **Transboost** algorithm

- From LUPI to transfer learning

Online learning: back to the future

- Central question
 - Controlling the memory
 - What to keep from the past?
 - How to adapt the current hypothesis?

• Can TransBoost help?

Online learning

- Suppose
 - Online with small batches at each time step
 - The current batch is labeled (after prediction has been performed)
 - The source hypothesis is kNN ($k \ge 3$) with (all) past examples
- Use Transboost to learn projections
 - To past points
 - With constraints preventing to project on points close to the point projected
 - Make statistics about the most useful points

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Supplementary material