

Building a Hybrid Systems Modeler from Synchronous Language Principles

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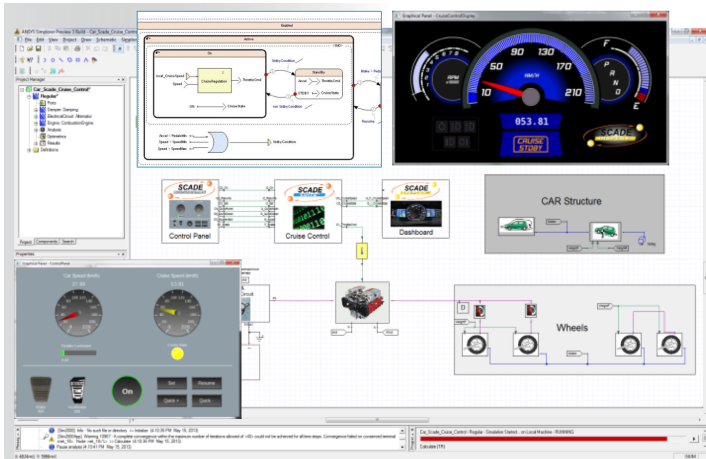
¹Joint work with A. Benveniste, T. Bourke, B. Caillaud (INRIA); J-L. Colaco, B. Pagano, C. Pasteur (Esterel-Tech./ANSYS Core Team)

But modern systems need more

The Current Practice of Hybrid Systems Modeling

Embedded software interacts with physical devices.

The whole system has to be modeled: the controller and the plant.²



²Image by Esterel-Technologies/ANSYS.

Current Practice and Objective

Current Practice

- ▶ Simulink, Modelica used to **model**, rarely to **implement** critical soft.
- ▶ Software must be reimplemented in SCADE or imperative code.
- ▶ Interconnect tools (Simulink+Modelica+SCADE+Simplorer+...)
- ▶ Interchange format for co-simulation: S-functions, FMU/FMI

Objective

- ▶ **Increase the confidence** in what is simulated
- ▶ Reduce compiler bugs
- ▶ Synchronous code for both the controller and the plant

Hybrid System Modelers

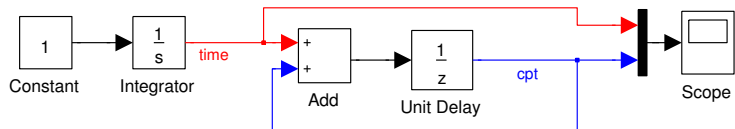
Simulink / FMI	Simplorer / Modelica
Ordinary differential equation $\dot{y} = f(y, t)$	Differential algebraic equation $f(y, \dot{y}, t) = 0$
Explicit	Implicit
Causal	Acausal

Hybrid System Modelers

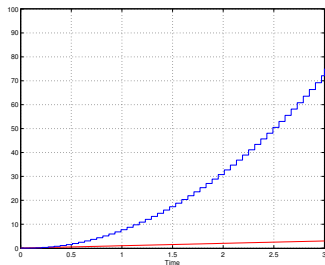
Simulink / FMI / Zélus / Scade Hybrid	Simplorer / Modelica
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Explicit	Implicit
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Strange beasts...

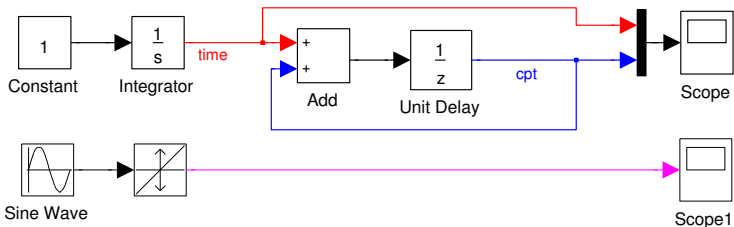
Typing issue 1: Mixing continuous & discrete components



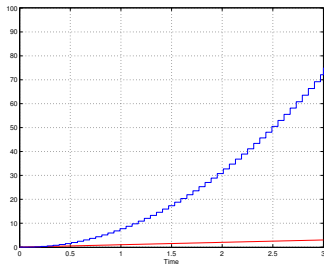
Basic model



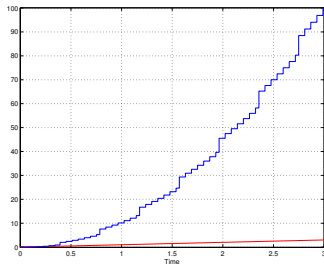
Typing issue 1: Mixing continuous & discrete components



Basic model

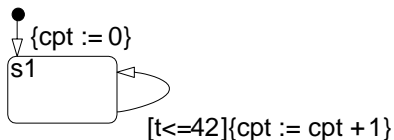
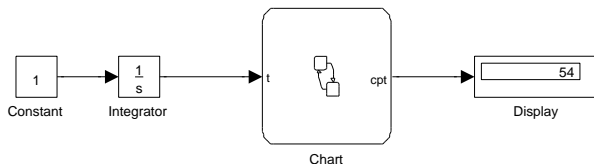


with Sine Wave



- ▶ The shape of `cpt` depends on the steps chosen by the solver.
- ▶ Putting another component in parallel can change the result.

Typing issue 2: Boolean guards in continuous automata

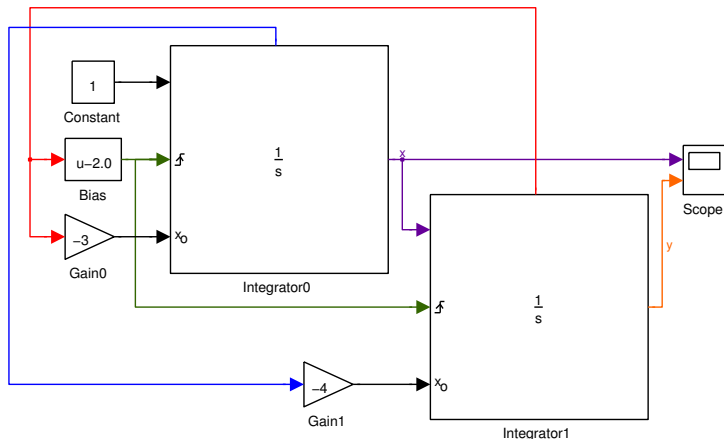


How long is a discrete step?

- ▶ Adding a parallel component changes the result.
- ▶ No warning by the compiler.
- ▶ The manual says: “A single transition is taken per major step”.

Discrete time is not logical: it is that of the simulation engine.

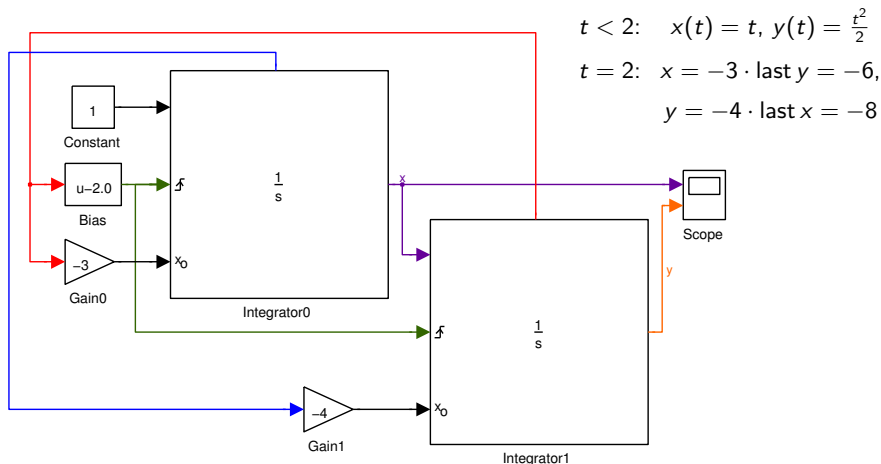
Causality issue: the Simulink state port



The output of the state port is the same as the output of the block's standard output port except for the following case. If the block is reset in the current time step, the output of the state port is the value that would have appeared at the block's standard output if the block had not been reset.

-Simulink Reference (2-685)

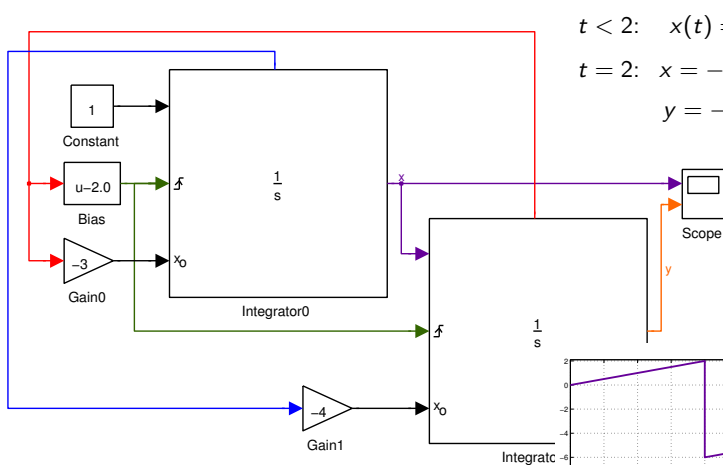
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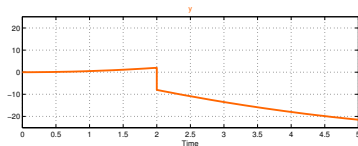
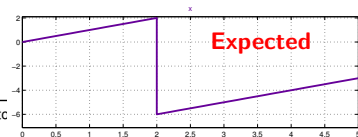
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Causality issue: the Simulink state port



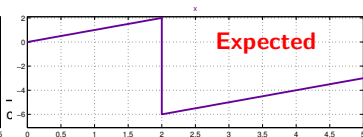
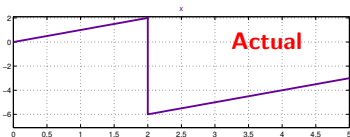
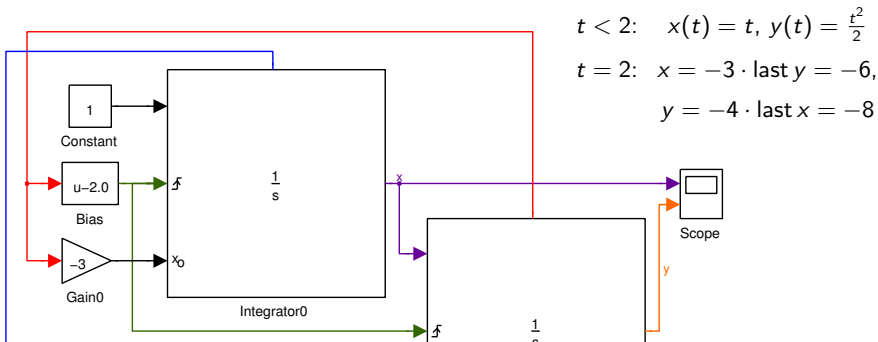
$$t < 2: \quad x(t) = t, \quad y(t) = \frac{t^2}{2}$$
$$t = 2: \quad x = -3 \cdot \text{last } y = -6, \\ y = -4 \cdot \text{last } x = -8$$



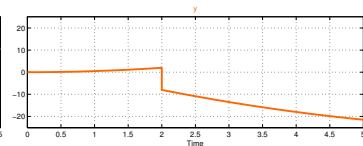
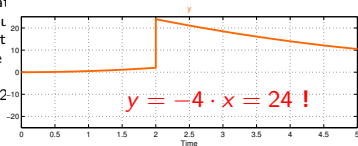
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Causality issue: the Simulink state port



The output of the state block's standard output is reset in t state port is the value standard output if the -Simulink Reference (2



Excerpt of C code produced by RTW (release R2009)

```
static void mdlOutputs(SimStruct * S, int_T tid)
{ _rtX = (ssGetContStates(S));
  ...
  _rtB = (_ssGetBlockIO(S));
  _rtB->B_0_0_0 = _rtX->Integrator1_CSTATE + _rtP->P_0;
  _rtB->B_0_1_0 = _rtP->P_1 * _rtX->Integrator1_CSTATE;
  if (ssIsMajorTimeStep (S))
  { ...
    if (zcEvent || ...)
    { (ssGetContStates (S))->Integrator0_CSTATE =
      _ssGetBlockIO (S)->B_0_1_0;
    }
    ...
    (_ssGetBlockIO (S))->B_0_2_0 =
    (ssGetContStates (S))->Integrator0_CSTATE;
    _rtB->B_0_3_0 = _rtP->P_2 * _rtX->Integrator0_CSTATE;
    if (ssIsMajorTimeStep (S))
    { ...
      if (zcEvent || ...)
      { (ssGetContStates (S))-> Integrator1_CSTATE =
        (ssGetBlockIO (S))->B_0_3_0;
      }
    }
  } ... }
```

Before assignment:
integrator state contains 'last' value

$$x = -3 \cdot \text{last } y$$

After assignment: integrator
state contains the new value

$$y = -4 \cdot x$$

So, y is updated with the new value of x

There is a problem in the treatment of causality.

Causality: Modelica example

model scheduling

Real x(start = 0);

Real y(start = 0);

equation

der(x) = 1;

der(y) = x;

when x >= 2 then

reinit(x, -3 * y)

end when;

when x >= 2 then

reinit(y, -4 * x);

end when;

end scheduling;

Causality: Modelica example

model scheduling

Real x(start = 0);

Real y(start = 0);

equation

$\text{der}(x) = 1;$

$\text{der}(y) = x;$

when $x \geq 2$ then

reinit(x, $-3 * y$)

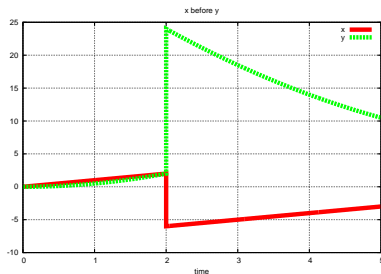
end when;

when $x \geq 2$ then

reinit(y, $-4 * x$);

end when;

end scheduling;



Causality: Modelica example

model scheduling

Real x(start = 0);

Real y(start = 0);

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$\text{der}(y) = x;$

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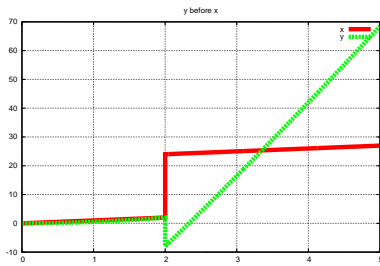
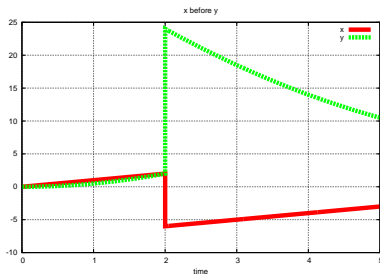
end when;

when $x \geq 2$ then

reinit(y, $-4 * x$);

end when;

end scheduling;



Background: [Benveniste et al., 2010 - 2014]

“Build a hybrid modeler on synchronous language principles”

Milestones

- ▶ Do as if time was global and discrete [JCSS'12]
- ▶ Lustre with ODEs [LCTES'11]
- ▶ Hierarchical automata, both discrete and hybrid [EMSOFT'11]
- ▶ Causality analysis [HSCC'14]

This was experimented in the language Zélus [HCSS'13]

The validation on an industrial compiler remained to be done.

SCADE Hybrid (summer 2014)

- ▶ Prototype based on KCG 6.4 (now KCG 6.5 - 2015)
- ▶ SCADE Hybrid = full SCADE + ODEs
- ▶ Generates FMI 1.0 model-exchange FMUs with Simplorer

In the sequel, we give examples in the concrete syntax of Zélus.
Examples in SCADE Hybrid and generated C code at:

zelus.di.ens.fr/cc2015

Synchronous languages in a slide

- ▶ Compose stream functions; basic values are streams.
- ▶ Operation apply pointwise + unit delay (`fbv`) + automata.

(computes $[x(n) + y(n) + 1]$ at every instant $[n]$ *)*

```
fun add (x,y) = x + y + 1
```

(returns $[true]$ when the number of $[t]$ has reached $[bound]$ *)*

```
node after (bound, t) = (c = bound) where
```

```
  rec c = 0 fby (min(tick, bound))
```

```
  and tick = if t then c + 1 else c
```

The counter can be instantiated twice in a two state automaton,

```
node blink (n, m, t) = x where
```

```
  automaton
```

```
  | On → do x = true  until (after(n, t)) then Off
```

```
  | Off → do x = false until (after(m, t)) then On
```

From it, a synchronous compiler produces **sequential loop-free code** that compute a single **step** of the system.

A Simple Hybrid System

Yet, time was discrete. Now, a simple heat controller. ³

(a model of the heater defined by an ODE with two modes *)*

hybrid heater(active) = temp where

rec der temp = if active then c -. k *. temp else -. k *. temp init temp0

(an hysteresis controller for a heater *)*

hybrid hysteresis_controller(temp) = active where

rec automaton

| Idle → do active = false until (up(t_min -. temp)) then Active

| Active → do active = true until (up(temp -. t_max)) then Idle


(The controller and the plant are put parallel *)*

hybrid main() = temp where

rec active = hysteresis_controller(temp)

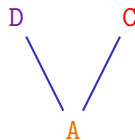
and temp = heater(active)

Three syntactic novelties: keyword **hybrid**, **der** and **up**.

³Hybrid version of N. Halbwachs's example in Lustre at Collège de France, Jan.10 

From Discrete to Hybrid

The type language [LCTES'11]

$$\begin{aligned} bt &::= \text{float} \mid \text{int} \mid \text{bool} \mid \text{zero} \mid \dots \\ \sigma &::= bt \times \dots \times bt \xrightarrow{k} bt \times \dots \times bt \\ k &::= D \mid C \mid A \end{aligned}$$


Function Definition: `fun f(x1,...) = (y1,...)`

- ▶ **Combinatorial functions** (A); usable anywhere.

Node Definition: `node f(x1,...) = (y1,...)`

- ▶ **Discrete-time constructs** (D) of SCADE/Lustre: `pre`, `->`, `fby`.

Hybrid Definition: `hybrid f(x1,...) = (y1,...)`

- ▶ **Continuous-time constructs** (C): `der x = ...`, `up`, `down`, etc.

Mixing continuous/discrete parts

Zero-crossing events

- ▶ They correspond to event indicators/state events in FMI
- ▶ Detected by the solver when a given signal crosses zero

Design choices

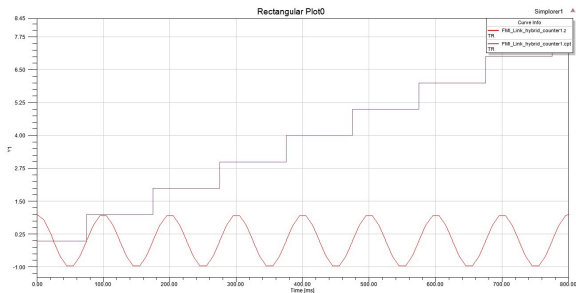
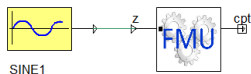
- ▶ A discrete computation can only be triggered by a zero-crossing
- ▶ Discrete state only changes at a zero-crossing event
- ▶ A continuous state can be reset at a zero-crossing event

Example

node counter() = cpt where
rec cpt = 1 \rightarrow pre cpt + 1

hybrid hybrid_counter() = cpt where
rec cpt = present up(z) \rightarrow counter() init 0
and z = sinus()

Output with SCADE Hybrid + Simplorer



Mix of synchronous code and a continuous model

E.g., implement the hysteresis controller by a synchronous function.

(a discrete-time model of the controller; periodically sampled *)*

```
node hysteresis_controller(temp) = active where
  rec automaton
    | Idle → do active = false until (temp <= t_min) then Active
    | Active → do active = true until (temp >= t_max) then Idle
```

(The controller and the plant are put parallel *)*

```
hybrid main() = temp where
  rec active = present (period (0.1)) → hysteresis_controller(temp) init false
  and temp = heater(active)
```

Continuous and discrete PI controller [demo]

```
hybrid integr(x, y0) = y where  
  rec der y = x init y0
```

```
hybrid pi(kp, ki, i) = cmd where  
  rec cmd = kp *. i +. ki *. integr(i, 0.0)  
tel
```

```
let ts = 0.05
```

```
node disc_integr(x, y0) = y where  
  rec init y = y0 and y = last y + Ts * x
```

```
node disc_pi(kp, ki, i) = cmd where  
  rec init cmd = Kp*i  
  and cmd = kp *. i +. ki *. disc_integr(i, 0.0)
```

How to communicate between continuous and discrete time?

E.g., the bouncing ball

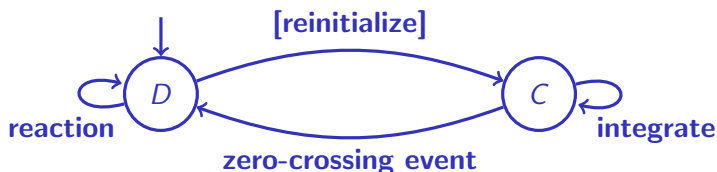
```
hybrid ball(y0) = y where
  rec der y = y_v init y0
  and der y_v = -. g init 0.0 reset z → 0.8 *. last y_v
  and z = up(-. y)
```

- ▶ Replacing `last y_v` by `y_v` would lead to a deadlock.
- ▶ In SCADE and Zélus, `last y_v` is the previous value of `y_v`.
- ▶ It coincides with the **left limit** of `y_v` when `y_v` is left continuous.

Internals

The Simulation Engine of Hybrid Systems

Alternate discrete steps and integration steps



$$\sigma', y' = \text{next}_\sigma(t, y) \quad \text{upz} = g_\sigma(t, y) \quad \dot{y} = f_\sigma(t, y)$$

Properties of the three functions

- ▶ next_σ gathers all discrete changes.
- ▶ g_σ defines signals for zero-crossing detection.
- ▶ f_σ is the function to integrate.

Compilation

The Compiler has to produce:

1. Initialization function *init* to define $y(0)$ and $\sigma(0)$.
2. Functions *f* and *g*.
3. Function *next*.

The Runtime System

1. Program the simulation loop, using a black-box solver (e.g., SUNDIALS CVODE);
2. Or rely on an existing infrastructure.

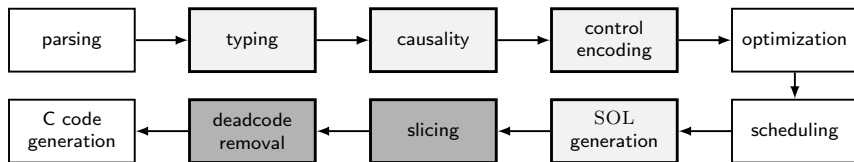
Zélus follows (1); SCADE Hybrid follows (2), targetting Simplorer FMIs.

Compiler Architecture

Two implementations: Zélus and KCG 6.4 (Release 2014) of SCADE.

KCG 6.4 of SCADE

- ▶ Generates FMI 1.0 model-exchange FMUs for Simplorer.
- ▶ Only 5% of the compiler modified. Small changes in:
 - ▶ static analysis (typing, causality).
 - ▶ automata translation; code generation.
 - ▶ FMU generation (XML description, wrapper).
- ▶ FMU integration loop: about 1000 LoC.



A SCADE-like Input Language

Essentially SCADE with three syntax extensions (in red).

```
d ::= const x = e | k f(pi) = pi where E | d; d
k ::= fun | node | hybrid
e ::= x | v | op(e, ..., e) | v fby e | last x | f(e, ..., e) | up(e)
p ::= x | (x, ..., x)
pi ::= xi | xi, ..., xi
xi ::= x | x last e | x default e
E ::= p = e | der x = e
      | if e then E else E
      | reset E every e
      | local pi in E | do E and ... E done
```

A Clocked Data-flow Internal Language

The internal language is extended with three extra operations.
Translation based on Colaco et al. [EMSOFT'05].

$$d ::= \text{const } x = c \mid k f(p) = a \text{ where } C \mid d; d$$
$$k ::= \text{fun} \mid \text{node} \mid \text{hybrid}$$
$$C ::= (x_i = a_i)_{x_i \in I} \text{ with } \forall i \neq j. x_i \neq x_j$$
$$a ::= e^{ck}$$
$$e ::= x \mid v \mid \text{op}(a, \dots, a) \mid v \text{ fby } a \mid \text{pre}(a) \\ \mid f(a, \dots, a) \\ \mid \text{merge}(a, a, a) \mid a \text{ when } a \\ \mid \text{integr}(a, a) \mid \text{up}(a)$$
$$p ::= x \mid (x, \dots, x)$$
$$ck ::= \text{base} \mid ck \text{ on } a$$

Clocked Equations Put in Normal Form

Name the result of every stateful operation. Separate into syntactic categories.

- ▶ *se*: strict expressions
- ▶ *de*: delayed expressions
- ▶ *ce*: controlled expressions.

Equation $lx = \text{integr}(x', x)$ defines lx to be the continuous state variable; possibly reset with x .

$$eq ::= x = ce^{ck} \mid x = f(sa, \dots, sa)^{ck} \mid x = de^{ck}$$

$$sa ::= se^{ck}$$

$$ca ::= ce^{ck}$$

$$se ::= x \mid v \mid op(sa, \dots, sa) \mid sa \text{ when } sa$$

$$ce ::= se \mid \text{merge}(sa, ca, ca) \mid ca \text{ when } sa$$

$$de ::= \text{pre}(ca) \mid v \text{ fby } ca \mid \text{integr}(ca, ca) \mid \text{up}(ca)$$

Well Scheduled Form

Equations are statically scheduled.

$Read(a)$: set of variables read by a .

Given $C = (x_i = a_i)_{x_i \in I}$, a valid schedule is a one-to-one function

$$Schedule(.) : I \rightarrow \{1 \dots |I|\}$$

such that, for all $x_i \in I, x_j \in Read(a_i) \cap I$:

1. if a_i is strict, $Schedule(x_j) < Schedule(x_i)$ and
2. if a_i is delayed, $Schedule(x_i) \leq Schedule(x_j)$.

From the data-dependence point-of-view, **integr**(ca_1, ca_2) and **up**(ca) break instantaneous loops.

A Sequential Object Language (SOL)

- ▶ Translation into an intermediate imperative language [Colaco et al., LCTES'08]
- ▶ Instead of producing two methods step and reset, produce more.
- ▶ Mark memory variables with a kind *m*

$$md ::= \begin{array}{l} | \text{const } x = c \\ | \text{const } f = \text{class} \langle M, I, (\text{method}_i(p_i) = e_i \text{ where } S_i)_{i \in [1..n]} \rangle \end{array}$$
$$M ::= [x : m [= v]; \dots; x : m [= v]]$$
$$I ::= [o : f; \dots; o : f]$$
$$m ::= \textit{Discrete} \mid \textit{Zero} \mid \textit{Cont}$$
$$e ::= v \mid lv \mid op(e, \dots, e) \mid o.\textit{method}(e, \dots, e)$$
$$S ::= () \mid lv \leftarrow e \mid S ; S \mid \text{var } x, \dots, x \text{ in } S \mid \text{if } c \text{ then } S \text{ else } S$$
$$R, L ::= S; \dots; S$$
$$lv ::= x \mid lv.\textit{field} \mid \text{state}(x)$$

State Variables

Discrete State Variables (sort *Discrete*)

- ▶ Read with `state(x)`;
- ▶ modified with `state(x) ← c`

Zero-crossing State Variables (sort *Zero*)

- ▶ A pair with two fields.
- ▶ The field `state(x).zin` is a boolean, true when a zero-crossing on x has been detected, false otherwise.
- ▶ The field `state(x).zout` is the value for which a zero-crossing must be detected.

Continuous State Variables (sort *Cont*)

- ▶ `state(x).der` is its instantaneous derivative;
- ▶ `state(x).pos` its value

Example: translation of the bouncing ball

```
let bouncing = machine(continuous) {
  memories disc init_25 : bool = true;
             zero result_17 : bool = false;
             cont y_v_15 : float = 0.; cont y_14 : float = 0.

  method reset =
    init_25 <- true; y_v_15.pos <- 0.

  method step time_23 y0_9 =
    (if init_25 then (y_14.pos <- y0_9; ()) else ());
    init_25 <- false;
    result_17.zout <- (~-. ) y_14.pos;
    if result_17.zin
      then (y_v_15.pos <- ( *. ) 0.8 y_v_15.pos);
    y_14.der <- y_v_15.pos;
    y_v_15.der <- (~-. ) g; y_14.pos }
```


Finally

1. Translate as usual to produce a function step.
2. For hybrid nodes, **copy-and-paste** the step method.
3. Either into a **cont** method activated during the continuous mode, or two extra methods **derivatives** and **crossings**.
4. Apply the following:
 - ▶ During the continuous mode (method **cont**), all zero-crossings (variables of type *zero*, e.g., `state(x).zin`) are surely false. All zero-crossing outputs (`state(x).zout ← ...`) are useless.
 - ▶ During the discrete step (method **step**), all derivative changes (`state(x).der ← ...`) are useless.
 - ▶ Remove dead-code by calling an existing pass.
5. That's all!

Examples (both Zélus and SCADE) at: zelus.di.ens.fr/cc2015

Example: translation of the bouncing ball

```
let bouncing = machine(continuous) {
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  method step time_23 y0_9 =
    (if init_25 then (y_14.pos <- y0_9; ()) else ());
    init_25 <- false;
    if result_17.zin
      then (y_v_15.pos <- ( *. ) 0.8 y_v_15.pos);
    y_14.pos
  method cont time_23 y0_9 =
    result_17.zout <- (~-. ) y_14.pos;
    y_14.der <- y_v_15.pos;
    y_v_15.der <- (~-. ) g }
```

Conclusion

Two full scale prototypes

- ▶ The **Zélus** academic language and compiler.
- ▶ The industrial **KCG 6.5** (Release 2015) code generator of SCADE.
- ▶ For KCG, **less than 5%** of extra LOC, in all.
- ▶ The extension is **fully conservative** w.r.t existing SCADE.
- ▶ The very same code is used both for simulation and embedded code.

Lessons

- ▶ The existing compiler architecture of SCADE KCG, based on successive rewriting, helped a lot.
- ▶ The discipline to make the extension compatible with existing compile-time checks and semantics helped a lot.
- ▶ Is-it helpful for identifying a safe subset of Simulink?
- ▶ Replace SUNDIALS by a guaranteed solver working with intervals?

Zélus

A synchronous language with ODEs



Compiler

Zélus is a synchronous language extended with Ordinary Differential Equations (ODEs) to model systems with complex interaction between discrete-time and continuous-time dynamics. It shares the basic principles of [Lustre](#) with features from [Lucid Synchronic](#) (type inference, hierarchical automata, and signals). The compiler is written

Research

Zélus is used to experiment with new techniques for building hybrid modelers like [Simulink/Stateflow](#) and [Modelica](#) on top of a synchronous language. The language exploits novel techniques for defining the semantics of hybrid modelers, it provides dedicated type systems to ensure the absence of discontinuities during integration and the

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